E9-1 (a) First, $\vec{\mathbf{F}} = (5.0\text{ N})\hat{\mathbf{i}}$.

$$\begin{aligned}\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}},\\ &= [y(0) - (0)(0)]\hat{\mathbf{i}} + [(0)F_x - x(0)]\hat{\mathbf{j}} + [x(0) - yF_x]\hat{\mathbf{k}},\\ &= [-yF_x]\hat{\mathbf{k}} = -(3.0\text{ m})(5.0\text{ N})\hat{\mathbf{k}} = -(15.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}.\end{aligned}$$

(b) Now $\vec{\mathbf{F}} = (5.0\text{ N})\hat{\mathbf{j}}$. Ignoring all zero terms,

$$\vec{\tau} = [xF_y]\hat{\mathbf{k}} = (2.0\text{ m})(5.0\text{ N})\hat{\mathbf{k}} = (10\text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

(c) Finally, $\vec{\mathbf{F}} = (-5.0\text{ N})\hat{\mathbf{i}}$.

$$\vec{\tau} = [-yF_x]\hat{\mathbf{k}} = -(3.0\text{ m})(-5.0\text{ N})\hat{\mathbf{k}} = (15.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

E9-2 (a) Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is $\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2$.

(b) $\tau = (1.30\,\text{m})(4.20\,\text{N})\sin(75.0^\circ) - (2.15\,\text{m})(4.90\,\text{N})\sin(58.0^\circ) = -3.66\text{N}\cdot\text{m}$.

E9-4 Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is

$$\tau = (8.0\,\text{m})(10\,\text{N})\sin(45^\circ) - (4.0\,\text{m})(16\,\text{N})\sin(90^\circ) + (3.0\,\text{m})(19\,\text{N})\sin(20^\circ) = 12\text{N}\cdot\text{m}.$$

E9-5 Since $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ lie in the xy plane, then $\vec{\mathbf{t}} = \vec{\mathbf{r}}\times\vec{\mathbf{s}}$ must be perpendicular to that plane, and can only point along the z axis.

The angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ is $320^\circ - 85^\circ = 235^\circ$. So $|\vec{\mathbf{t}}| = rs|\sin\theta| = (4.5)(7.3)|\sin(235^\circ)| = 27$.

Now for the direction of $\vec{\mathbf{t}}$. The smaller rotation to bring $\vec{\mathbf{r}}$ into $\vec{\mathbf{s}}$ is through a counterclockwise rotation; the right hand rule would then show that the cross product points along the *positive* z direction.

E9-6 $\vec{\mathbf{a}} = (3.20)[\cos(63.0^\circ)\hat{\mathbf{j}} + \sin(63.0^\circ)\hat{\mathbf{k}}]$ and $\vec{\mathbf{b}} = (1.40)[\cos(48.0^\circ)\hat{\mathbf{i}} + \sin(48.0^\circ)\hat{\mathbf{k}}]$. Then

$$\begin{aligned}\vec{\mathbf{a}}\times\vec{\mathbf{b}} &= (3.20)\cos(63.0^\circ)(1.40)\sin(48.0^\circ)\hat{\mathbf{i}}\\ &\quad +(3.20)\sin(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{j}}\\ &\quad -(3.20)\cos(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{k}}\\ &= 1.51\hat{\mathbf{i}} + 2.67\hat{\mathbf{j}} - 1.36\hat{\mathbf{k}}.\end{aligned}$$

E9-7 $\vec{\mathbf{b}}\times\vec{\mathbf{a}}$ has magnitude $ab\sin\phi$ and points in the negative z direction. It is then perpendicular to $\vec{\mathbf{a}}$, so $\vec{\mathbf{c}}$ has magnitude $a^2b\sin\phi$. The direction of $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$ but lies in the plane containing vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. Then it makes an angle $\pi/2 - \phi$ with $\vec{\mathbf{b}}$.

E9-8 (a) In unit vector notation,

$$\begin{aligned}\vec{\mathbf{c}} &= [(-3)(-3) - (-2)(1)]\hat{\mathbf{i}} + [(1)(4) - (2)(-3)]\hat{\mathbf{j}} + [(2)(-2) - (-3)(4)]\hat{\mathbf{k}},\\ &= 11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 8\hat{\mathbf{k}}.\end{aligned}$$

(b) Evaluate $\arcsin[|\vec{\mathbf{a}}\times b|/(ab)]$, finding magnitudes with the Pythagoras relationship:

$$\phi = \arcsin{(16.8)}/[(3.74)(5.39)] = 56^\circ.$$

E9-9 This exercise is a three dimensional generalization of Ex. 9-1, except nothing is zero.

$$\begin{aligned}\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}},\\ &= [(-2.0\text{ m})(4.3\text{ N}) - (1.6\text{ m})(-2.4\text{ N})]\hat{\mathbf{i}} + [(1.6\text{ m})(3.5\text{ N}) - (1.5\text{ m})(4.3\text{ N})]\hat{\mathbf{j}}\\ &\quad +[(1.5\text{ m})(-2.4\text{ N}) - (-2.0\text{ m})(3.5\text{ N})]\hat{\mathbf{k}},\\ &= [-4.8\,\text{N}\cdot\text{m}]\hat{\mathbf{i}} + [-0.85\,\text{N}\cdot\text{m}]\hat{\mathbf{j}} + [3.4\,\text{N}\cdot\text{m}]\hat{\mathbf{k}}.\end{aligned}$$

E9-10 (a) $\vec{\mathbf{F}} = (2.6\,\text{N})\hat{\mathbf{i}}$, then $\vec{\tau} = (0.85\,\text{m})(2.6\,\text{N})\hat{\mathbf{j}} - (-0.36\,\text{m})(2.6\,\text{N})\hat{\mathbf{k}} = 2.2\,\text{N}\cdot\text{m}\hat{\mathbf{j}} + 0.94\,\text{N}\cdot\text{m}\hat{\mathbf{k}}$.

(b) $\vec{\mathbf{F}} = (-2.6\,\text{N})\hat{\mathbf{k}}$, then $\vec{\tau} = (-0.36\,\text{m})(-2.6\,\text{N})\hat{\mathbf{i}} - (0.54\,\text{m})(-2.6\,\text{N})\hat{\mathbf{j}} = 0.93\,\text{N}\cdot\text{m}\hat{\mathbf{i}} + 1.4\,\text{N}\cdot\text{m}\hat{\mathbf{j}}$.

E9-11 (a) The rotational inertia about an axis through the origin is

$$I = mr^2 = (0.025\,\text{kg})(0.74\,\text{m})^2 = 1.4\times 10^{-2}\text{kg}\cdot\text{m}^2.$$

(b) $\alpha = (0.74\,\text{m})(22\,\text{N})\sin(50°)/(1.4\times 10^{-2}\text{kg}\cdot\text{m}^2) = 890$ rad/s.

E9-12 (a) $I_0 = (0.052\,\text{kg})(0.27\,\text{m})^2 + (0.035\,\text{kg})(0.45\,\text{m})^2 + (0.024\,\text{kg})(0.65\,\text{m})^2 = 2.1\times 10^{-2}\text{kg}\cdot\text{m}^2$.

(b) The center of mass is located at

$$x_{\text{cm}} = \frac{(0.052\,\text{kg})(0.27\,\text{m}) + (0.035\,\text{kg})(0.45\,\text{m}) + (0.024\,\text{kg})(0.65\,\text{m})}{(0.052\,\text{kg}) + (0.035\,\text{kg}) + (0.024\,\text{kg})} = 0.41\,\text{m}.$$

Applying the parallel axis theorem yields $I_{\text{cm}} = 2.1\times 10^{-2}\text{kg}\cdot\text{m}^2 - (0.11\,\text{kg})(0.41\,\text{m})^2 = 2.5\times 10^{-3}\text{kg}\cdot\text{m}^2$.

E9-13 (a) Rotational inertia is additive so long as we consider the inertia about the same axis. We can use Eq. 9-10:

$$I = \sum m_n r_n^2 = (0.075\,\text{kg})(0.42\,\text{m})^2 + (0.030\,\text{kg})(0.65\,\text{m})^2 = 0.026\,\text{kg}\cdot\text{m}^2.$$

(b) No change.

E9-14 $\vec{\tau} = [(0.42\,\text{m})(2.5\,\text{N}) - (0.65\,\text{m})(3.6\,\text{N})]\hat{\mathbf{k}} = -1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}}$. Using the result from E9-13, $\vec{\alpha} = (-1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}})/(0.026\,\text{kg}\cdot\text{m}^2) = 50\text{ rad/s}^2\hat{\mathbf{k}}$. That's clockwise if viewed from above.

E9-15 (a) $F = m\omega^2 r = (110\,\text{kg})(33.5\text{ rad/s})^2(3.90\,\text{m}) = 4.81\times 10^5\text{N}$.

(b) The angular acceleration is $\alpha = (33.5\text{ rad/s})/(6.70\,\text{s}) = 5.00\text{ rad/s}^2$. The rotational inertia about the axis of rotation is $I = (110\,\text{kg})(7.80\,\text{m})^2/3 = 2.23\times 10^3\text{kg}\cdot\text{m}^2$. $\tau = I\alpha = (2.23\times 10^3\text{kg}\cdot\text{m}^2)(5.00\text{ rad/s}^2) = 1.12\times 10^4\text{N}\cdot\text{m}$.

E9-16 We can add the inertias for the three rods together,

$$I = 3\left(\frac{1}{3}ML^2\right) = (240\,\text{kg})(5.20\,\text{m})^2 = 6.49\times 10^3\text{kg}\cdot\text{m}^2.$$

E9-17 The diagonal distance from the axis through the center of mass and the axis through the edge is $h = \sqrt{(a/2)^2 + (b/2)^2}$, so

$$I = I_{cm} + Mh^2 = \frac{1}{12}M\left(a^2 + b^2\right) + M\left((a/2)^2 + (b/2)^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M\left(a^2 + b^2\right).$$

Simplifying, $I = \frac{1}{3}M\left(a^2 + b^2\right)$.

E9-18 $I = I_{cm} + Mh^2 = (0.56\,\text{kg})(1.0\,\text{m})^2/12 + (0.56)(0.30\,\text{m})^2 = 9.7\times 10^{-2}\text{kg}\cdot\text{m}^2$.

E9-19 For particle one $I_1 = mr^2 = mL^2$; for particle two $I_2 = mr^2 = m(2L)^2 = 4mL^2$. The rotational inertia of the rod is $I_{\text{rod}} = \frac{1}{3}(2M)(2L)^2 = \frac{8}{3}ML^2$. Add the three inertias:

$$I = \left(5m + \frac{8}{3}M\right)L^2.$$

E9-20 (a) $I = MR^2/2 = M(R/\sqrt{2})^2$.

(b) Let I be the rotational inertia. Assuming that k is the radius of a hoop with an equivalent rotational inertia, then $I = Mk^2$, or $k = \sqrt{I/M}$.

E9-21 Note the mistakes in the equation in the part (b) of the exercise text.

(a) $m_n = M/N$.

(b) Each piece has a thickness $t = L/N$, the distance from the end to the nth piece is $x_n = (n - 1/2)t = (n - 1/2)L/N$. The axis of rotation is the center, so the distance from the center is $r_n = x_n - L/2 = nL/N - (1 + 1/2N)L$.

(c) The rotational inertia is

$$\begin{aligned}
I &= \sum_{n=1}^{N} m_n r_n^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}(n - 1/2 - N)^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}\left(n^2 - (2N+1)n + (N+1/2)^2\right), \\
&= \frac{ML^2}{N^3}\left(\frac{N(N+1)(2N+1)}{6} - (2N+1)\frac{N(N+1)}{2} + (N+1/2)^2 N\right), \\
&\approx \frac{ML^2}{N^3}\left(\frac{2N^3}{6} - \frac{2N^3}{2} + N^3\right), \\
&= ML^2/3.
\end{aligned}$$

E9-22 $F = (46\,\text{N})(2.6\text{ cm})/(13\text{ cm}) = 9.2\,\text{N}$.

E9-23 Tower topples when center of gravity is no longer above base. Assuming center of gravity is located at the very center of the tower, then when the tower leans 7.0 m then the tower falls. This is 2.5 m farther than the present.

(b) $\theta = \arcsin(7.0\,\text{m}/55\,\text{m}) = 7.3°$.

E9-24 If the torque from the force is sufficient to lift edge the cube then the cube will tip. The net torque about the edge which stays in contact with the ground will be $\tau = Fd - mgd/2$ if F is sufficiently large. Then $F \geq mg/2$ is the minimum force which will cause the cube to tip.

The minimum force to get the cube to slide is $F \geq \mu_s mg = (0.46)mg$. The cube will slide first.

E9-25 The ladder slips if the force of static friction required to keep the ladder up exceeds $\mu_s N$. Equations 9-31 give us the normal force in terms of the masses of the ladder and the firefighter, $N = (m + M)g$, and is independent of the location of the firefighter on the ladder. Also from Eq. 9-31 is the relationship between the force from the wall and the force of friction; the condition at which slipping occurs is $F_w \geq \mu_s(m + M)g$.

Now go straight to Eq. 9-32. The $a/2$ in the second term is the location of the firefighter, who in the example was halfway between the base of the ladder and the top of the ladder. In the exercise we don't know where the firefighter is, so we'll replace $a/2$ with x. Then

$$-F_w h + Mgx + \frac{mga}{3} = 0$$

is an expression for rotational equilibrium. Substitute in the condition of F_w when slipping just starts, and we get

$$-\left(\mu_s(m + M)g\right) h + Mgx + \frac{mga}{3} = 0.$$

Solve this for x,

$$x = \mu_s \left(\frac{m}{M} + 1\right) h - \frac{ma}{3M} = (0.54)\left(\frac{45 \text{ kg}}{72 \text{ kg}} + 1\right)(9.3 \text{ m}) - \frac{(45 \text{ kg})(7.6 \text{ m})}{3(72 \text{ kg})} = 6.6 \text{ m}$$

This is the horizontal distance; the fraction of the total length along the ladder is then given by $x/a = (6.6 \text{ m})/(7.6 \text{ m}) = 0.87$. The firefighter can climb $(0.87)(12\,\text{m}) = 10.4\,\text{m}$ up the ladder.

E9-26 (a) The net torque about the rear axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(3.05\,\text{m} - 1.78\,\text{m}) - F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 5.55\times10^3\text{N}$. Each of the front tires support half of this, or $2.77\times10^3\text{N}$.

(b) The net torque about the front axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(1.78\,\text{m}) - F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 7.78\times10^3\text{N}$. Each of the front tires support half of this, or $3.89\times10^3\text{N}$.

E9-27 The net torque on the bridge about the end closest to the person is

$$(160 \text{ lb})L/4 + (600 \text{ lb})L/2 - F_\text{f} L = 0,$$

which has a solution for the supporting force on the far end of $F_\text{f} = 340$ lb.

The net force on the bridge is $(160 \text{ lb})L/4 + (600 \text{ lb})L/2 - (340 \text{ lb}) - F_\text{c} = 0$, so the force on the close end of the bridge is $F_\text{c} = 420$ lb.

E9-28 The net torque on the board about the left end is

$$F_\text{r}(1.55\,\text{m}) - (142\,\text{N})(2.24\,\text{m}) - (582\,\text{N})(4.48\,\text{m}) = 0,$$

which has a solution for the supporting force for the right pedestal of $F_\text{r} = 1890\,\text{N}$. The force on the board from the pedestal is up, so the force on the pedestal from the board is down (compression).

The net force on the board is $F_\text{l} + (1890\,\text{N}) - (142\,\text{N}) - (582\,\text{N}) = 0$, so the force from the pedestal on the left is $F_\text{l} = -1170\,\text{N}$. The negative sign means up, so the pedestal is under tension.

E9-29 We can assume that both the force $\vec{\mathbf{F}}$ and the force of gravity $\vec{\mathbf{W}}$ act on the center of the wheel. Then the wheel will just start to lift when

$$\vec{\mathbf{W}} \times \vec{\mathbf{r}} + \vec{\mathbf{F}} \times \vec{\mathbf{r}} = 0,$$

or

$$W \sin\theta = F \cos\theta,$$

where θ is the angle between the vertical (pointing down) and the line between the center of the wheel and the point of contact with the step. The use of the sine on the left is a straightforward application of Eq. 9-2. Why the cosine on the right? Because

$$\sin(90^\circ - \theta) = \cos\theta.$$

Then $F = W \tan\theta$. We can express the angle θ in terms of trig functions, h, and r. $r\cos\theta$ is the vertical distance from the center of the wheel to the top of the step, or $r - h$. Then

$$\cos\theta = 1 - \frac{h}{r} \text{ and } \sin\theta = \sqrt{1 - \left(1 - \frac{h}{r}\right)^2}.$$

Finally by combining the above we get

$$F = W\frac{\sqrt{\frac{2h}{r} - \frac{h^2}{r^2}}}{1 - \frac{h}{r}} = W\frac{\sqrt{2hr - h^2}}{r - h}.$$

E9-30 (a) Assume that each of the two support points for the square sign experience the same tension, equal to half of the weight of the sign. The net torque on the rod about an axis through the hinge is

$$(52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(0.95\,\text{m}) + (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(2.88\,\text{m}) - (2.88\,\text{m})T\sin\theta = 0,$$

where T is the tension in the cable and θ is the angle between the cable and the rod. The angle can be found from $\theta = \arctan(4.12\,\text{m}/2.88\,\text{m}) = 55.0^\circ$, so $T = 416\,\text{N}$.

(b) There are two components to the tension, one which is vertical, $(416\,\text{N})\sin(55.0^\circ) = 341\,\text{N}$, and another which is horizontal, $(416\,\text{N})\cos(55.0^\circ) = 239\,\text{N}$. The horizontal force exerted by the wall must then be 239 N. The net vertical force on the rod is $F + (341\,\text{N}) - (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2) = 0$, which has solution $F = 172\,\text{N}$ as the vertical upward force of the wall on the rod.

E9-31 (a) The net torque on the rod about an axis through the hinge is

$$\tau = W(L/2)\cos(54.0^\circ) - TL\sin(153.0^\circ) = 0.$$

or $T = (52.7\text{ lb}/2)(\sin 54.0^\circ/\sin 153.0^\circ) = 47.0$ lb.

(b) The vertical upward force of the wire on the rod is $T_y = T\cos(27.0^\circ)$. The vertical upward force of the wall on the rod is $P_y = W - T\cos(27.0^\circ)$, where W is the weight of the rod. Then

$$P_y = (52.7\text{ lb}) - (47.0\text{ lb})\cos(27.0^\circ) = 10.8\text{ lb}$$

The horizontal force from the wall is balanced by the horizontal force from the wire. Then $P_x = (47.0\text{ lb})\sin(27.0^\circ) = 21.3$ lb.

E9-32 If the ladder is not slipping then the torque about an axis through the point of contact with the ground is

$$\tau = (WL/2)\cos\theta - Nh/\sin\theta = 0,$$

where N is the normal force of the edge on the ladder. Then $N = WL\cos\theta\,\sin\theta\,/(2h)$.

N has two components; one which is vertically up, $N_y = N\cos\theta$, and another which is horizontal, $N_x = N\sin\theta$. The horizontal force must be offset by the static friction.

The normal force on the ladder from the ground is given by

$$N_\text{g} = W - N\cos\theta = W[1 - L\cos^2\theta\,\sin\theta\,/(2h)].$$

The force of static friction can be as large as $f = \mu_s N_g$, so

$$\mu_s = \frac{WL\cos\theta\,\sin^2\theta\,/(2h)}{W[1 - L\cos^2\theta\,\sin\theta\,/(2h)]} = \frac{L\cos\theta\,\sin^2\theta}{2h - L\cos^2\theta\,\sin\theta}.$$

Put in the numbers and $\theta = 68.0°$. Then $\mu_s = 0.407$.

E9-33 Let out be positive. The net torque about the axis is then

$$\tau = (0.118\,\text{m})(5.88\,\text{N}) - (0.118\,\text{m})(4.13\,\text{m}) - (0.0493\,\text{m})(2.12\,\text{N}) = 0.102\,\text{N}\cdot\text{m}.$$

The rotational inertia of the disk is $I = (1.92\,\text{kg})(0.118\,\text{m})^2/2 = 1.34\times10^{-2}\text{kg}\cdot\text{m}^2$. Then $\alpha = (0.102\,\text{N}\cdot\text{m})/(1.34\times10^{-2}\text{kg}\cdot\text{m}^2) = 7.61\ \text{rad/s}^2$.

E9-34 (a) $I = \tau/\alpha = (960\,\text{N}\cdot\text{m})/(6.23\ \text{rad/s}^2) = 154\,\text{kg}\cdot\text{m}^2$.
(b) $m = (3/2)I/r^2 = (1.5)(154\,\text{kg}\cdot\text{m}^2)/(1.88\,\text{m})^2 = 65.4\,\text{kg}$.

E9-35 (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{6.20\ \text{rad/s}}{0.22\ \text{s}} = 28.2\ \text{rad/s}^2$$

(b) From Eq. 9-11, $\tau = I\alpha = (12.0\,\text{kg}\cdot\text{m}^2)(28.2\ \text{rad/s}^2) = 338\,\text{N}\cdot\text{m}$.

E9-36 The angular acceleration is $\alpha = 2(\pi/2\ \text{rad})/(30\,\text{s})^2 = 3.5\times10^{-3}\ \text{rad/s}^2$. The required force is then

$$F = \tau/r = I\alpha/r = (8.7\times10^4\text{kg}\cdot\text{m}^2)(3.5\times10^{-3}\ \text{rad/s}^2)/(2.4\,\text{m}) = 127\,\text{N}.$$

Don't let the door slam...

E9-37 The torque is $\tau = rF$, the angular acceleration is $\alpha = \tau/I = rF/I$. The angular velocity is

$$\omega = \int_0^t \alpha\,dt = \frac{rAt^2}{2I} + \frac{rBt^3}{3I},$$

so when $t = 3.60\,\text{s}$,

$$\omega = \frac{(9.88\times10^{-2}\text{m})(0.496\,\text{N/s})(3.60\,\text{s})^2}{2(1.14\times10^{-3}\text{kg}\cdot\text{m}^2)} + \frac{(9.88\times10^{-2}\text{m})(0.305\,\text{N/s}^2)(3.60\,\text{s})^3}{3(1.14\times10^{-3}\text{kg}\cdot\text{m}^2)} = 690\ \text{rad/s}.$$

E9-38 (a) $\alpha = 2\theta/t^2$.
(b) $a = \alpha R = 2\theta R/t^2$.
(c) T_1 and T_2 are *not* equal. Instead, $(T_1 - T_2)R = I\alpha$. For the hanging block $Mg - T_1 = Ma$. Then

$$T_1 = Mg - 2MR\theta/t^2,$$

and

$$T_2 = Mg - 2MR\theta/t^2 - 2(I/R)\theta/t^2.$$

E9-39 Apply a kinematic equation from chapter 2 to find the acceleration:

$$\begin{aligned} y &= v_{0y}t + \frac{1}{2}a_y t^2, \\ a_y &= \frac{2y}{t^2} = \frac{2(0.765\ \text{m})}{(5.11\ \text{s})^2} = 0.0586\ \text{m/s}^2 \end{aligned}$$

Closely follow the approach in Sample Problem 9-10. For the heavier block, $m_1 = 0.512$ kg, and Newton's second law gives

$$m_1 g - T_1 = m_1 a_y,$$

where a_y is positive and *down*. For the lighter block, $m_2 = 0.463$ kg, and Newton's second law gives

$$T_2 - m_2 g = m_2 a_y,$$

where a_y is positive and *up*. We do know that $T_1 > T_2$; the net force on the pulley creates a torque which results in the pulley rotating toward the heavier mass. That net force is $T_1 - T_2$; so the rotational form of Newton's second law gives

$$(T_1 - T_2)\,R = I\alpha_z = Ia_T/R,$$

where $R = 0.049$ m is the radius of the pulley and a_T is the tangential acceleration. But this acceleration is equal to a_y, because everything— both blocks and the pulley— are moving together.

We then have *three* equations and *three* unknowns. We'll add the first two together,

$$\begin{aligned} m_1 g - T_1 + T_2 - m_2 g &= m_1 a_y + m_2 a_y, \\ T_1 - T_2 &= (g - a_y)m_1 - (g + a_y)m_2, \end{aligned}$$

and then combine this with the third equation by substituting for $T_1 - T_2$,

$$\begin{aligned} (g - a_y)m_1 - (g + a_y)m_2 &= Ia_y/R^2, \\ \left[\left(\frac{g}{a_y} - 1\right)m_1 - \left(\frac{g}{a_y} + 1\right)m_2\right]R^2 &= I. \end{aligned}$$

Now for the numbers:

$$\begin{aligned} \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} - 1\right)(0.512\,\text{kg}) - \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} + 1\right)(0.463\,\text{kg}) &= 7.23\,\text{kg}, \\ (7.23\,\text{kg})(0.049\,\text{m})^2 &= 0.0174\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

E9-40 The wheel turns with an initial angular speed of $\omega_0 = 88.0$ rad/s. The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then $\alpha = -\omega_0/t = -\omega_0^2/(2\phi)$.

The rotational inertia is $I = MR^2/2$, so the torque required to stop the disk is $\tau = I\alpha = -MR^2\omega_0^2/(4\phi)$. The force of friction on the disk is $f = \mu N$, so $\tau = Rf$. Then

$$\mu = \frac{MR\omega_0^2}{4N\phi} = \frac{(1.40\,\text{kg})(0.23\,\text{m})(88.0\ \text{rad/s})^2}{4(130\,\text{N})(17.6\ \text{rad})} = 0.272.$$

E9-41 (a) The automobile has an initial speed of $v_0 = 21.8\,\text{m/s}$. The angular speed is then $\omega_0 = (21.8\,\text{m/s})/(0.385\,\text{m}) = 56.6$ rad/s.

(b) The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then

$$\alpha = -\omega_0/t = -\omega_0^2/(2\phi) = -(56.6\ \text{rad/s})^2/[2(180\ \text{rad})] = -8.90\ \text{rad/s}.$$

(c) The automobile traveled $x = \phi r = (180\ \text{rad})(0.385\,\text{m}) = 69.3\,\text{m}$.

E9-42 (a) The angular acceleration is derived in Sample Problem 9-13,

$$\alpha = \frac{g}{R_0}\frac{1}{1+I/(MR_0^2)} = \frac{(981\text{ cm/s}^2)}{(0.320\text{ cm})}\frac{1}{1+(0.950\,\text{kg}\cdot\text{cm}^2)/[(0.120\,\text{kg})(0.320\text{ cm})^2]} = 39.1\text{ rad/s}^2.$$

The acceleration is $a = \alpha R_0 = (39.1\text{ rad/s}^2)(0.320\text{ cm}) = 12.5\text{ cm/s}^2$.

(b) Starting from rest, $t = \sqrt{2x/a} = \sqrt{2(134\text{ cm})/(12.5\text{ cm/s}^2)} = 4.63\,\text{s}$.

(c) $\omega = \alpha t = (39.1\text{ rad/s}^2)(4.63\,\text{s}) = 181\text{ rad/s}$. This is the same as 28.8 rev/s.

(d) The yo-yo accelerates toward the ground according to $y = at^2 + v_0 t$, where *down* is positive. The time required to move to the end of the string is found from

$$t = \frac{-v_0 + \sqrt{v_0^2 + 4ay}}{2a} = \frac{-(1.30\,\text{m/s}) + \sqrt{(1.30\,\text{m/s})^2 + 4(0.125\,\text{m/s}^2)(1.34\,\text{m})}}{2(0.125\,\text{m/s}^2)} = 0.945\,\text{s}$$

The initial rotational speed was $\omega_0 = (1.30\,\text{m/s})/(3.2\times10^{-3}\text{m}) = 406\text{ rad/s}$. Then

$$\omega = \omega_0 + \alpha t = (406\text{ rad/s}) + (39.1\text{ rad/s}^2)(0.945\,\text{s}) = 443\text{ rad/s},$$

which is the same as 70.5 rev/s.

E9-43 (a) Assuming a perfect hinge at B, the only two vertical forces on the tire will be the normal force from the belt and the force of gravity. Then $N = W = mg$, or $N = (15.0\text{ kg})(9.8\text{ m/s}^2) = 147\text{ N}$.

While the tire skids we have kinetic friction, so $f = \mu_k N = (0.600)(147\text{ N}) = 88.2\text{ N}$. The force of gravity and and the pull from the holding rod AB both act at the axis of rotation, so can't contribute to the net torque. The normal force acts at a point which is parallel to the displacement from the axis of rotation, so it doesn't contribute to the torque either (because the cross product would vanish); so the only contribution to the torque is from the frictional force.

The frictional force is perpendicular to the radial vector, so the magnitude of the torque is just $\tau = rf = (0.300\text{ m})(88.2\text{ N}) = 26.5\text{ N·m}$. This means the angular acceleration will be $\alpha = \tau/I = (26.5\text{ N·m})/(0.750\text{ kg·m}^2) = 35.3\text{ rad/s}^2$.

When $\omega R = v_T = 12.0$ m/s the tire is no longer slipping. We solve for ω and get $\omega = 40$ rad/s.

Now we solve $\omega = \omega_0 + \alpha t$ for the time. The wheel started from rest, so $t = 1.13$ s.

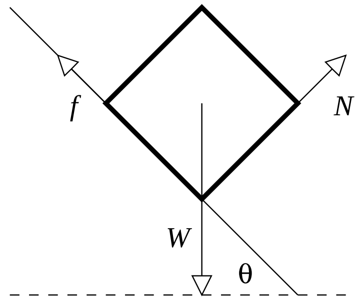
(b) The length of the skid is $x = vt = (12.0\text{ m/s})(1.13\text{ s}) = 13.6$ m long.

P9-1 The problem of sliding down the ramp has been solved (see Sample Problem 5-8); the critical angle θ_s is given by $\tan\theta_s = \mu_s$.

The problem of tipping is actually not that much harder: an object tips when the center of gravity is no longer over the base. The important angle for tipping is shown in the figure below; we can find that by trigonometry to be

$$\tan\theta_t = \frac{O}{A} = \frac{(0.56\text{ m})}{(0.56\text{ m}) + (0.28\text{ m})} = 0.67,$$

so $\theta_t = 34°$.



(a) If $\mu_s = 0.60$ then $\theta_s = 31°$ and the crate slides.
(b) If $\mu_s = 0.70$ then $\theta_s = 35°$ and the crate tips before sliding; it tips at 34°.

P9-2 (a) The total force up on the chain needs to be equal to the total force down; the force down is W. Assuming the tension at the end points is T then $T \sin\theta$ is the upward component, so $T = W/(2\sin\theta)$.

(b) There is a horizontal component to the tension $T\cos\theta$ at the wall; this *must* be the tension at the horizontal point at the bottom of the cable. Then $T_{\text{bottom}} = W/(2\tan\theta)$.

P9-3 (a) The rope exerts a force on the sphere which has horizontal $T\sin\theta$ and vertical $T\cos\theta$ components, where $\theta = \arctan(r/L)$. The weight of the sphere is balanced by the upward force from the rope, so $T\cos\theta = W$. But $\cos\theta = L/\sqrt{r^2 + L^2}$, so $T = W\sqrt{1 + r^2/L^2}$.

(b) The wall pushes outward against the sphere equal to the inward push on the sphere from the rope, or $P = T\sin\theta = W\tan\theta = Wr/L$.

P9-4 Treat the problem as having two forces: the man at one end lifting with force $F = W/3$ and the two men acting together a distance x away from the first man and lifting with a force $2F = 2W/3$. Then the torque about an axis through the end of the beam where the first man is lifting is $\tau = 2xW/3 - WL/2$, where L is the length of the beam. This expression equal zero when $x = 3L/4$.

P9-5 (a) We can solve this problem with Eq. 9-32 after a few modifications. We'll assume the center of mass of the ladder is at the center, then the third term of Eq. 9-32 is $mga/2$. The cleaner didn't climb half-way, he climbed 3.10/5.12 = 60.5% of the way, so the second term of Eq. 9-32 becomes $Mga(0.605)$. h, L, and a are related by $L^2 = a^2 + h^2$, so $h = \sqrt{(5.12\text{ m})^2 - (2.45\text{ m})^2} = 4.5$ m. Then, putting the correction into Eq. 9-32,

$$\begin{aligned} F_w &= \frac{1}{h}\left[Mga(0.605) + \frac{mga}{2}\right], \\ &= \frac{1}{(4.5\text{ m})}\Big[(74.6\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})(0.605)\,, \\ &\quad + (10.3\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})/2\Big], \\ &= 269\text{ N} \end{aligned}$$

(b) The vertical component of the force of the ground on the ground is the sum of the weight of the window cleaner and the weight of the ladder, or 833 N.

The horizontal component is equal in magnitude to the force of the ladder on the window. Then the net force of the ground on the ladder has magnitude

$$\sqrt{(269\,\text{N})^2 + (833\,\text{N})^2} = 875\,\text{N}$$

and direction

$$\theta = \arctan(833/269) = 72^\circ \text{ above the horizontal.}$$

P9-6 (a) There are no upward forces other than the normal force on the bottom ball, so the force exerted on the bottom ball by the container is $2W$.

(c) The bottom ball must exert a force on the top ball which has a vertical component equal to the weight of the top ball. Then $W = N \sin\theta$ or the force of contact between the balls is $N = W/\sin\theta$.

(b) The force of contact between the balls has a horizontal component $P = N\cos\theta = W/\tan\theta$, this must also be the force of the walls on the balls.

P9-7 (a) There are three forces on the ball: weight $\vec{\mathbf{W}}$, the normal force from the lower plane $\vec{\mathbf{N}}_1$, and the normal force from the upper plane $\vec{\mathbf{N}}_2$. The force from the lower plane has components $N_{1,x} = -N_1 \sin\theta_1$ and $N_{1,y} = N_1 \cos\theta_1$. The force from the upper plane has components $N_{2,x} = N_2 \sin\theta_2$ and $N_{2,y} = -N_2\cos\theta_2$. Then $N_1 \sin\theta_1 = N_2 \sin\theta_2$ and $N_1 \cos\theta_1 = W + N_2\cos\theta_2$.

Solving for N_2 by dividing one expression by the other,

$$\frac{\cos\theta_1}{\sin\theta_1} = \frac{W}{N_2 \sin\theta_2} + \frac{\cos\theta_2}{\sin\theta_2},$$

or

$$\begin{aligned}
N_2 &= \frac{W}{\sin\theta_2}\left(\frac{\cos\theta_1}{\sin\theta_1} - \frac{\cos\theta_2}{\sin\theta_2}\right)^{-1}, \\
&= \frac{W}{\sin\theta_2}\frac{\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1}{\sin\theta_1\,\sin\theta_2}, \\
&= \frac{W\sin\theta_1}{\sin(\theta_2 - \theta_1)}.
\end{aligned}$$

Then solve for N_1,

$$N_1 = \frac{W\sin\theta_2}{\sin(\theta_2 - \theta_1)}.$$

(b) Friction changes everything.

P9-8 (a) The net torque about a line through A is

$$\tau = Wx - TL\sin\theta = 0,$$

so $T = Wx/(L\sin\theta)$.

(b) The horizontal force on the pin is equal in magnitude to the horizontal component of the tension: $T\cos\theta = Wx/(L\tan\theta)$. The vertical component balances the weight: $W - Wx/L$.

(c) $x = (520\,\text{N})(2.75\,\text{m})\sin(32.0^\circ)/(315\,\text{N}) = 2.41\,\text{m}$.

P9-9 (a) As long as the center of gravity of an object (even if combined) is above the base, then the object will not tip.

Stack the bricks from the top down. The center of gravity of the top brick is $L/2$ from the edge of the top brick. This top brick can be offset nor more than $L/2$ from the one beneath. The center of gravity of the top two bricks is located at

$$x_{\text{cm}} = [(L/2) + (L)]/2 = 3L/4.$$

These top two bricks can be offset no more than $L/4$ from the brick beneath. The center of gravity of the top three bricks is located at

$$x_{\text{cm}} = [(L/2) + 2(L)]/3 = 5L/6.$$

These top three bricks can be offset no more than $L/6$ from the brick beneath. The total offset is then $L/2 + L/4 + L/6 = 11L/12$.

(b) Actually, we never need to know the location of the center of gravity; we now realize that each brick is located with an offset $L/(2n)$ with the brick beneath, where n is the number of the brick counting from the top. The series is then of the form

$$(L/2)[(1/1) + (1/2) + (1/3) + (1/4) + \cdots],$$

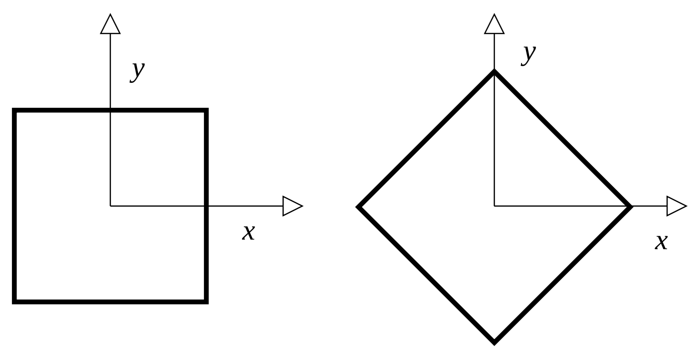
a series (harmonic, for those of you who care) which does not converge.

(c) The center of gravity would be half way between the ends of the two extreme bricks. This would be at NL/n; the pile will topple when this value exceeds L, or when $N = n$.

P9-10 (a) For a planar object which lies in the $x - y$ plane, $I_x = \int x^2 dm$ and $I_y = \int y^2 dm$. Then $I_x + I_y = \int (x^2 + y^2) dm = \int r^2\, dm$. But this is the rotational inertia about the z axis, sent r is the distance from the z axis.

(b) Since the rotational inertia about one diameter (I_x) should be the same as the rotational inertia about any other (I_y) then $I_x = I_y$ and $I_x = I_z/2 = MR^2/4$.

P9-11 Problem 9-10 says that $I_x + I_y = I_z$ for any thin, flat object which lies only in the $x - y$ plane.It doesn't matter in which direction the x and y axes are chosen, so long as they are perpendicular. We can then orient our square as in either of the pictures below:



By symmetry $I_x = I_y$ in either picture. Consequently, $I_x = I_y = I_z/2$ for either picture. It is the same square, so I_z is the same for both picture. Then I_x is also the same for both orientations.

P9-12 Let M_0 be the mass of the plate *before* the holes are cut out. Then $M_1 = M_0(a/L)^2$ is the mass of the part cut out of each hole and $M = M_0 - 9M_1$ is the mass of the plate. The rotational inertia (about an axis perpendicular to the plane through the center of the square) for the large uncut square is $M_0 L^2/6$ and for each smaller cut out is $M_1 a^2/6$.

From the large uncut square's inertia we need to remove $M_1 a^2/6$ for the center cut-out, $M_1 a^2/6 + M_1(L/3)^2$ for each of the four edge cut-outs, and $M_1 a^2/6 + M_1(\sqrt{2}L/3)^2$ for each of the corner sections.

Then

$$\begin{aligned} I &= \frac{M_0L^2}{6} - 9\frac{M_1a^2}{6} - 4\frac{M_1L^2}{9} - 4\frac{2M_1L_2}{9}, \\ &= \frac{M_0L^2}{6} - 3\frac{M_0a^4}{2L^2} - 4\frac{M_0a^2}{3}. \end{aligned}$$

P9-13 (a) From Eq. 9-15, $I = \int r^2 dm$ about some axis of rotation when r is measured from that axis. If we consider the x axis as our axis of rotation, then $r = \sqrt{y^2 + z^2}$, since the distance to the x axis depends only on the y and z coordinates. We have similar equations for the y and z axes, so

$$\begin{aligned} I_x &= \int \left(y^2 + z^2\right) dm, \\ I_y &= \int \left(x^2 + z^2\right) dm, \\ I_z &= \int \left(x^2 + y^2\right) dm. \end{aligned}$$

These three equations can be added together to give

$$I_x + I_y + I_z = 2\int \left(x^2 + y^2 + z^2\right) dm,$$

so if we *now* define r to be measured from the origin (which is not the definition used above), then we end up with the answer in the text.

(b) The right hand side of the equation is integrated over the entire body, regardless of how the axes are defined. So the integral should be the same, no matter how the coordinate system is rotated.

P9-14 (a) Since the shell is spherically symmetric $I_x = I_y = I_z$, so $I_x = (2/3)\int r^2 dm = (2R^2/3)\int dm = 2MR^2/3$.

(b) Since the solid ball is spherically symmetric $I_x = I_y = I_z$, so

$$I_x = \frac{2}{3}\int r^2 \frac{3Mr^2\,dr}{R^3} = \frac{2}{5}MR^2.$$

P9-15 (a) A simple ratio will suffice:

$$\frac{dm}{2\pi r\,dr} = \frac{M}{\pi R^2} \text{ or } dm = \frac{2Mr}{R^2}dr.$$

(b) $dI = r^2 dm = (2Mr^3/R^2)dr$.

(c) $I = \int_0^R (2Mr^3/R^2)dr = MR^2/2$.

P9-16 (a) Another simple ratio will suffice:

$$\frac{dm}{\pi r^2 dz} = \frac{M}{(4/3)\pi R^3} \text{ or } dm = \frac{3M(R^2 - z^2)}{4R^3}dz.$$

(b) $dI = r^2 dm/2 = [3M(R^2 - z^2)^2/8R^3]dz$.

(c) There are a few steps to do here:

$$\begin{aligned} I &= \int_{-R}^{R} \frac{3M(R^2 - z^2)^2}{8R^3} dz, \\ &= \frac{3M}{4R^3} \int_0^R (R^4 - 2R^2 z^2 + z^4) dz, \\ &= \frac{3M}{4R^3}(R^5 - 2R^5/3 + R^5/5) = \frac{2}{5} M R^2. \end{aligned}$$

P9-17 The *rotational* acceleration will be given by $\alpha_z = \sum \tau / I$.

The torque about the pivot comes from the force of gravity on each block. This forces will both originally be at right angles to the pivot arm, so the net torque will be $\sum \tau = mgL_2 - mgL_1$, where clockwise as seen on the page is positive.

The rotational inertia about the pivot is given by $I = \sum m_n r_n^2 = m(L_2^2 + L_1^2)$. So we can now find the rotational acceleration,

$$\alpha = \frac{\sum \tau}{I} = \frac{mgL_2 - mgL_1}{m(L_2^2 + L_1^2)} = g\frac{L_2 - L_1}{L_2^2 + L_1^2} = 8.66 \text{ rad/s}^2.$$

The linear acceleration is the tangential acceleration, $a_T = \alpha R$. For the left block, $a_T = 1.73$ m/s^2; for the right block $a_T = 6.93$ m/s^2.

P9-18 (a) The force of friction on the hub is $\mu_k Mg$. The torque is $\tau = \mu_k Mga$. The angular acceleration has magnitude $\alpha = \tau/I = \mu_k ga/k^2$. The time it takes to stop the wheel will be $t = \omega_0/\alpha = \omega_0 k^2/(\mu_k ga)$.

(b) The average rotational speed while slowing is $\omega_0/2$. The angle through which it turns while slowing is $\omega_0 t/2$ radians, or $\omega_0 t/(4\pi) = \omega_0^2 k^2/(4\pi\mu_k ga)$

P9-19 (a) Consider a differential ring of the disk. The torque on the ring because of friction is

$$d\tau = r\,dF = r\frac{\mu_k Mg}{\pi R^2} 2\pi r\,dr = \frac{2\mu_k Mgr^2}{R^2} dr.$$

The net torque is then

$$\tau = \int d\tau = \int_0^R \frac{2\mu_k Mgr^2}{R^2} dr = \frac{2}{3}\mu_k MgR.$$

(b) The rotational acceleration has magnitude $\alpha = \tau/I = \frac{4}{3}\mu_k g/R$. Then it will take a time

$$t = \omega_0/\alpha = \frac{3R\omega_0}{4\mu_k g}$$

to stop.

P9-20 We need only show that the two objects have the same acceleration.

Consider first the hoop. There is a force $W_{||} = W\sin\theta = mg\sin\theta$ pulling it down the ramp and a frictional force f pulling it up the ramp. The frictional force is just large enough to cause a torque that will allow the hoop to roll without slipping. This means $a = \alpha R$; consequently, $fR = \alpha I = aI/R$. In this case $I = mR^2$.

The acceleration down the plane is

$$ma = mg\sin\theta - f = mg\sin\theta - maI/R^2 = mg\sin\theta - ma.$$

Then $a = g\sin\theta/2$. The mass and radius are irrelevant!

For a block sliding with friction there are also two forces: $W_{||} = W\sin\theta = mg\sin\theta$ and $f = \mu_k mg\cos\theta$. Then the acceleration down the plane will be given by

$$a = g\sin\theta - \mu_k g\cos\theta,$$

which will be equal to that of the hoop if

$$\mu_k = \frac{\sin\theta - \sin\theta/2}{\cos\theta} = \frac{1}{2}\tan\theta.$$

P9-21 This problem is equivalent to Sample Problem 9-11, except that we have a sphere instead of a cylinder. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\text{cm}} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\text{cm}}\alpha.$$

The conditions for accelerating without slipping are $a_{\text{cm}} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\text{cm}}\alpha}{R} = -\frac{I_{\text{cm}}(-a_{\text{cm}})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\text{cm}}(a_{\text{cm}})}{R^2} = Ma_{\text{cm}},$$

and solve for a_{cm}. For fun, let's write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for the sphere. Then, upon some mild rearranging, we get

$$a_{\text{cm}} = g\frac{\sin\theta}{1+\beta}$$

For the sphere, $a_{\text{cm}} = 5/7 g\sin\theta$.

(a) If $a_{\text{cm}} = 0.133g$, then $\sin\theta = 7/5(0.133) = 0.186$, and $\theta = 10.7^\circ$.

(b) A frictionless block has no rotational properties; in this case $\beta = 0$! Then $a_{\text{cm}} = g\sin\theta = 0.186g$.

P9-22 (a) There are three forces on the cylinder: gravity W and the tension from each cable T. The downward acceleration of the cylinder is then given by $ma = W - 2T$.

The ropes unwind according to $\alpha = a/R$, but $\alpha = \tau/I$ and $I = mR^2/2$. Then

$$a = \tau R/I = (2TR)R/(mR^2/2) = 4T/m.$$

Combining the above, $4T = W - 2T$, or $T = W/6$.

(b) $a = 4(mg/6)/m = 2g/3$.

P9-23 The force of friction required to keep the cylinder rolling is given by

$$f = \frac{1}{3}Mg\sin\theta;$$

the normal force is given to be $N = Mg\cos\theta$; so the coefficient of static friction is given by

$$\mu_s \geq \frac{f}{N} = \frac{1}{3}\tan\theta.$$

P9-24 $a = F/M$, since F is the net force on the disk. The torque about the center of mass is FR, so the disk has an angular acceleration of

$$\alpha = \frac{FR}{I} = \frac{FR}{MR^2/2} = \frac{2F}{MR}.$$

P9-25 This problem is equivalent to Sample Problem 9-11, except that we have an unknown rolling object. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\rm cm} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\rm cm}\alpha.$$

The conditions for accelerating without slipping are $a_{\rm cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\rm cm}\alpha}{R} = -\frac{I_{\rm cm}(-a_{\rm cm})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\rm cm}(a_{\rm cm})}{R^2} = Ma_{\rm cm},$$

and solve for $a_{\rm cm}$. Write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for a sphere, $\beta = 1/2$ for a cylinder, and $\beta = 1$ for a hoop. Then, upon some mild rearranging, we get

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

Note that a is largest when β is smallest; consequently the cylinder wins. Neither M nor R entered into the final equation.