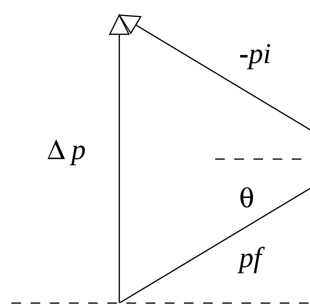
E6-1 (a) $v_1 = (m_2/m_1)v_2 = (2650\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 52.0\text{ km/h}$.
(b) $v_1 = (m_2/m_1)v_2 = (9080\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 178\text{ km/h}$.

E6-2 $\vec{\mathbf{p}}_\text{i} = (2000\,\text{kg})(40\text{ km/h})\hat{\mathbf{j}} = 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\vec{\mathbf{p}}_\text{f} = (2000\,\text{kg})(50\text{ km/h})\hat{\mathbf{i}} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}}$. $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}} - 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\Delta p = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = 1.28\times10^5\text{kg}\cdot\text{km/h}$. The direction is 38.7° south of east.

E6-3 The figure below shows the initial and final momentum vectors arranged to geometrically show $\vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = \Delta\vec{\mathbf{p}}$. We can use the cosine law to find the length of $\Delta\vec{\mathbf{p}}$.



The angle $\alpha = 42^\circ + 42^\circ$, $p_\text{i} = mv = (4.88\,\text{kg})(31.4\,\text{m/s}) = 153\,\text{kg}\cdot\text{m/s}$. Then the magnitude of $\Delta\vec{\mathbf{p}}$ is

$$\Delta p = \sqrt{(153\,\text{kg}\cdot\text{m/s})^2 + (153\,\text{kg}\cdot\text{m/s})^2 - 2(153\,\text{kg}\cdot\text{m/s})^2\cos(84^\circ)} = 205\,\text{kg}\cdot\text{m/s},$$

directed up from the plate. By symmetry it must be perpendicular.

E6-4 The change in momentum is $\Delta p = -mv = -(2300\,\text{kg})(15\,\text{m/s}) = -3.5\times10^4\text{kg}\cdot\text{m/s}$. The average force is $F = \Delta p/\Delta t = (-3.5\times10^4\text{kg}\cdot\text{m/s})/(0.54\,\text{s}) = -6.5\times10^4\text{N}$.

E6-5 (a) The change in momentum is $\Delta p = (-mv) - mv$; the average force is $F = \Delta p/\Delta t = -2mv/\Delta t$.
(b) $F = -2(0.14\,\text{kg})(7.8\,\text{m/s})/(3.9\times10^{-3}\text{s}) = 560\,\text{N}$.

E6-6 (a) $J = \Delta p = (0.046\,\text{kg})(52.2\,\text{m/s}) - 0 = 2.4\,\text{N}\cdot\text{s}$.
(b) The impulse imparted to the club is opposite that imparted to the ball.
(c) $F = \Delta p/\Delta t = (2.4\,\text{N}\cdot\text{s})/(1.20\times10^{-3}\text{s}) = 2000\,\text{N}$.

E6-7 Choose the coordinate system so that the ball is only moving along the x axis, with away from the batter as positive. Then $p_{\text{f}x} = mv_{\text{f}x} = (0.150\,\text{kg})(61.5\,\text{m/s}) = 9.23$ kg·m/s and $p_{\text{i}x} = mv_{\text{i}x} = (0.150\,\text{kg})(-41.6\,\text{m/s}) = -6.24$ kg·m/s. The impulse is given by $J_x = p_{\text{f}x} - p_{\text{i}x} = 15.47$ kg·m/s. We can find the average force by application of Eq. 6-7:

$$F_{\text{av},x} = \frac{J_x}{\Delta t} = \frac{(15.47\text{ kg}\cdot\text{m/s})}{(4.7\times10^{-3}\text{ s})} = 3290\,\text{N}.$$

E6-8 The magnitude of the impulse is $J = F\delta t = (-984\,\mathrm{N})(0.0270\,\mathrm{s}) = -26.6\,\mathrm{N}\cdot\mathrm{s}$. Then $p_\mathrm{f} = p_\mathrm{i} + \Delta p$, so

$$v_\mathrm{f} = \frac{(0.420\,\mathrm{kg})(13.8\,\mathrm{m/s}) + (-26.6\,\mathrm{N}\cdot\mathrm{s})}{(0.420\,\mathrm{kg})} = -49.5\,\mathrm{m/s}.$$

The ball moves backward!

E6-9 The change in momentum of the ball is $\Delta p = (mv) - (-mv) = 2mv = 2(0.058\,\mathrm{kg})(32\,\mathrm{m/s}) = 3.7\,\mathrm{kg}\cdot\mathrm{m/s}$. The impulse is the area under a force - time graph; for the trapezoid in the figure this area is $J = F_\mathrm{max}(2\,\mathrm{ms} + 6\,\mathrm{ms})/2 = (4\,\mathrm{ms})F_\mathrm{max}$. Then $F_\mathrm{max} = (3.7\,\mathrm{kg}\cdot\mathrm{m/s})/(4\,\mathrm{ms}) = 930\,\mathrm{N}$.

E6-10 The final speed of each object is given by $v_i = J/m_i$, where i refers to which object (as opposed to "initial"). The object are going in different directions, so the relative speed will be the sum. Then

$$v_\mathrm{rel} = v_1 + v_2 = (300\,\mathrm{N}\cdot\mathrm{s})[1/(1200\,kg) + 1/(1800\,\mathrm{kg})] = 0.42\,\mathrm{m/s}.$$

E6-11 Use Simpson's rule. Then the area is given by

$$\begin{aligned} J_x &= \frac{1}{3}h\left(f_0 + 4f_1 + 2f_2 + 4f_3 + ... + 4f_13 + f_14\right), \\ &= \frac{1}{3}(0.2\,\mathrm{ms})\left(200 + 4\cdot 800 + 2\cdot 1200...\,\mathrm{N}\right) \end{aligned}$$

which gives $J_x = 4.28$ kg·m/s.

Since the impulse is the change in momentum, and the ball started from rest, $p_{\mathrm{f}x} = J_x + p_{\mathrm{i}x} = 4.28$ kg·m/s. The final velocity is then found from $v_x = p_x/m = 8.6$ m/s.

E6-12 (a) The average speed during the the time the hand is in contact with the board is half of the initial speed, or $v_\mathrm{av} = 4.8\,\mathrm{m/s}$. The time of contact is then $t = y/v_\mathrm{av} = (0.028\,\mathrm{m})/(4.8\,\mathrm{m/s}) = 5.8\,\mathrm{ms}$.

(b) The impulse given to the board is the same as the magnitude in the change in momentum of the hand, or $J = (0.54\,\mathrm{kg})(9.5\,\mathrm{m/s}) = 5.1\,\mathrm{N}\cdot\mathrm{s}$. Then $F_\mathrm{av} = (5.1\,\mathrm{N}\cdot\mathrm{s})/(5.8\,\mathrm{ms}) = 880\,\mathrm{N}$.

E6-13 $\Delta p = J = F\Delta t = (3000\,\mathrm{N})(65.0\,\mathrm{s}) = 1.95\times 10^5\mathrm{N}\cdot\mathrm{s}$. The direction of the thrust relative to the velocity doesn't matter in this exercise.

E6-14 (a) $p = mv = (2.14\times 10^{-3}\mathrm{kg})(483\,\mathrm{m/s}) = 1.03\,\mathrm{kg}\cdot\mathrm{m/s}$.

(b) The impulse imparted to the wall in one second is ten times the above momentum, or $J = 10.3\,\mathrm{kg}\cdot\mathrm{m/s}$. The average force is then $F_\mathrm{av} = (10.3\,\mathrm{kg}\cdot\mathrm{m/s})/(1.0\,\mathrm{s}) = 10.3\,\mathrm{N}$.

(c) The average force for each individual particle is $F_\mathrm{av} = (1.03\,\mathrm{kg}\cdot\mathrm{m/s})/(1.25\times 10^{-3}\,\mathrm{s}) = 830\,\mathrm{N}$.

E6-15 A transverse direction means at right angles, so the thrusters have imparted a momentum sufficient to direct the spacecraft 100+3400 = 3500 km to the side of the original path. The spacecraft is half-way through the six-month journey, so it has three months to move the 3500 km to the side. This corresponds to a transverse speed of $v = (3500\times 10^3\mathrm{m})/(90\times 86400\,\mathrm{s}) = 0.45\,\mathrm{m/s}$. The required time for the rocket to fire is $\Delta t = (5400\,\mathrm{kg})(0.45\,\mathrm{m/s})/(1200\,\mathrm{N}) = 2.0\,\mathrm{s}$.

E6-16 Total initial momentum is zero, so

$$v_\mathrm{m} = -\frac{m_\mathrm{s}}{m_\mathrm{m}}v_\mathrm{s} = -\frac{m_\mathrm{s}g}{m_\mathrm{m}g}v_\mathrm{s} = -\frac{(0.158\ \mathrm{lb})}{(195\ \mathrm{lb})}(12.7\ \mathrm{ft/s}) = -1.0\times 10^{-2}\ \mathrm{ft/s}.$$

E6-17 Conservation of momentum:

$$\begin{aligned}
p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c},\\
m_{\rm m}v_{\rm f,m} + m_{\rm c}v_{\rm f,c} &= m_{\rm m}v_{\rm i,m} + m_{\rm c}v_{\rm i,c},\\
v_{\rm f,c} - v_{\rm i,c} &= \frac{m_{\rm m}v_{\rm i,m} - m_{\rm m}v_{\rm f,m}}{m_{\rm c}},\\
\Delta v_{\rm c} &= \frac{(75.2\text{ kg})(2.33\text{ m/s}) - (75.2\text{ kg})(0)}{(38.6\text{ kg})},\\
&= 4.54\,\text{m/s}.
\end{aligned}$$

The answer is positive; the cart speed *increases*.

E6-18 Conservation of momentum:

$$\begin{aligned}
p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c},\\
m_{\rm m}(v_{\rm f,c} - v_{\rm rel}) + m_{\rm c}v_{\rm f,c} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c},\\
(m_{\rm m} + m_{\rm c})v_{\rm f,c} - m_{\rm m}v_{\rm rel} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c},\\
\Delta v_{\rm c} &= m_{\rm m}v_{\rm rel}/(m_{\rm m} + m_{\rm c}),\\
&= wv_{\rm rel}/(w + W).
\end{aligned}$$

E6-19 Conservation of momentum. Let m refer to motor and c refer to command module:

$$\begin{aligned}
p_{\rm f,m} + p_{\rm f,c} &= p_{\rm i,m} + p_{\rm i,c},\\
m_{\rm m}(v_{\rm f,c} - v_{\rm rel}) + m_{\rm c}v_{\rm f,c} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c},\\
(m_{\rm m} + m_{\rm c})v_{\rm f,c} - m_{\rm m}v_{\rm rel} &= (m_{\rm m} + m_{\rm c})v_{\rm i,c},\\
v_{\rm f,c} &= \frac{m_{\rm m}v_{\rm rel} + (m_{\rm m} + m_{\rm c})v_{\rm i,c}}{(m_{\rm m} + m_{\rm c})},\\
&= \frac{4m_{\rm c}(125\text{ km/h}) + (4m_{\rm c} + m_{\rm c})(3860\text{ km/h})}{(4m_{\rm c} + m_{\rm c})} = 3960\text{ km/h}.
\end{aligned}$$

E6-20 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned}
m_1v_{1,\rm f} + m_2v_{2,\rm f} &= m_1v_{1,\rm i} + m_2v_{2,\rm i},\\
v_{1,\rm f} &= v_{1,\rm i} + \frac{m_2}{m_1}(v_{2,\rm i} - v_{2,\rm f}),\\
&= (5.5\,\text{m/s}) + \frac{(2.4\,\text{kg})}{(1.6\,\text{kg})}[(2.5\,\text{m/s}) - (4.9\,\text{m/s})],\\
&= 1.9\,\text{m/s}.
\end{aligned}$$

E6-21 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned}
m_1v_{1,\rm f} + m_2v_{2,\rm f} &= m_1v_{1,\rm i} + m_2v_{2,\rm i},\\
v_{1,\rm f} &= v_{1,\rm i} + \frac{m_2}{m_1}(v_{2,\rm i} - v_{2,\rm f}),\\
&= (5.5\,\text{m/s}) + \frac{(2.4\,\text{kg})}{(1.6\,\text{kg})}[(-2.5\,\text{m/s}) - (4.9\,\text{m/s})],\\
&= -5.6\,\text{m/s}.
\end{aligned}$$

E6-22 Assume a completely inelastic collision. Call the Earth 1 and the meteorite 2. Then

$$\begin{aligned}
m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} &= m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}, \\
v_{1,\mathrm{f}} &= \frac{m_2 v_{2,\mathrm{i}}}{m_1 + m_2}, \\
&= \frac{(5\times 10^{10}\mathrm{kg})(7200\,\mathrm{m/s})}{(5.98\times 10^{24}\mathrm{kg}) + (5\times 10^{10}\mathrm{kg})} = 7\times 10^{-11}\mathrm{m/s}.
\end{aligned}$$

That's 2 mm/y!

E6-23 Conservation of momentum is used to solve the problem:

$$\begin{aligned}
P_\mathrm{f} &= P_\mathrm{i}, \\
p_{\mathrm{f,bl}} + p_{\mathrm{f,bu}} &= p_{\mathrm{i,bl}} + p_{\mathrm{i,bu}}, \\
m_\mathrm{bl} v_{\mathrm{f,bl}} + m_\mathrm{bu} v_{\mathrm{f,bu}} &= m_\mathrm{bl} v_{\mathrm{i,bl}} + m_\mathrm{bu} v_{\mathrm{i,bu}}, \\
(715\ \mathrm{g}) v_{\mathrm{f,bl}} + (5.18\ \mathrm{g})(428\ \mathrm{m/s}) &= (715\ \mathrm{g})(0) + (5.18\ \mathrm{g})(672\ \mathrm{m/s}),
\end{aligned}$$

which has solution $v_{\mathrm{f,bl}} = 1.77\,\mathrm{m/s}$.

E6-24 The y component of the initial momentum is zero; therefore the magnitudes of the y components of the two particles must be equal after the collision. Then

$$\begin{aligned}
m_\alpha v_\alpha \sin\theta_\alpha &= m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}, \\
v_\alpha &= \frac{m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}}{m_\alpha v_\alpha \sin\theta_\alpha}, \\
&= \frac{(16\ \mathrm{u})(1.20\times 10^5\mathrm{m/s})\sin(51^\circ)}{(4.00\ \mathrm{u})\sin(64^\circ)} = 4.15\times 10^5\mathrm{m/s}.
\end{aligned}$$

E6-25 The total momentum is

$$\begin{aligned}
\vec{\mathbf{p}} &= (2.0\,\mathrm{kg})[(15\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}] + (3.0\,\mathrm{kg})[(-10\,\mathrm{m/s})\hat{\mathbf{i}} + (5\,\mathrm{m/s})\hat{\mathbf{j}}], \\
&= 75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.
\end{aligned}$$

The final velocity of B is

$$\begin{aligned}
\vec{\mathbf{v}}_{B\mathrm{f}} &= \frac{1}{m_B}(\vec{\mathbf{p}} - m_A \vec{\mathbf{v}}_{A\mathrm{f}}), \\
&= \frac{1}{(3.0\,\mathrm{kg})}\{(75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (2.0\,\mathrm{kg})[(-6.0\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}]\}, \\
&= (4.0\,\mathrm{m/s})\hat{\mathbf{i}} + (5.0\,\mathrm{m/s})\hat{\mathbf{j}}.
\end{aligned}$$

E6-26 Assume electron travels in $+x$ direction while neutrino travels in $+y$ direction. Conservation of momentum requires that

$$\vec{\mathbf{p}} = -(1.2\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{i}} - (6.4\times 10^{-23}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{j}}$$

be the momentum of the nucleus after the decay. This has a magnitude of $p = 1.4\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s}$ and be directed 152° from the electron.

E6-27 What we know:

$$\begin{aligned}\vec{\mathbf{p}}_{1,\mathrm{i}} &= (1.50\times10^5\mathrm{kg})(6.20\,\mathrm{m/s})\hat{\mathbf{i}} = 9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}},\\ \vec{\mathbf{p}}_{2,\mathrm{i}} &= (2.78\times10^5\mathrm{kg})(4.30\,\mathrm{m/s})\hat{\mathbf{j}} = 1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}},\\ \vec{\mathbf{p}}_{2,\mathrm{f}} &= (2.78\times10^5\mathrm{kg})(5.10\,\mathrm{m/s})[\sin(18^\circ)\hat{\mathbf{i}} + \cos(18^\circ)\hat{\mathbf{j}}],\\ &= 4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} + 1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.\end{aligned}$$

Conservation of momentum then requires

$$\begin{aligned}\vec{\mathbf{p}}_{1,\mathrm{f}} &= (9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}) - (4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}})\\ &\quad+(1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}),\\ &= 4.92\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} - 1.50\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}.\end{aligned}$$

This corresponds to a velocity of

$$\vec{\mathbf{v}}_{1,\mathrm{f}} = 3.28\,\mathrm{m/s}\,\hat{\mathbf{i}} - 1.00\,\mathrm{m/s}\,\hat{\mathbf{j}},$$

which has a magnitude of 3.43 m/s directed 17° to the right.

E6-28 $v_\mathrm{f} = -2.1\,\mathrm{m/s}$.

E6-29 We want to solve Eq. 6-24 for m_2 given that $v_{1,\mathrm{f}} = 0$ and $v_{1,\mathrm{i}} = -v_{2,\mathrm{i}}$. Making these substitutions

$$\begin{aligned}(0) &= \frac{m_1 - m_2}{m_1 + m_2}v_{1,\mathrm{i}} + \frac{2m_2}{m_1 + m_2}(-v_{1,\mathrm{i}}),\\ 0 &= (m_1 - m_2)v_{1,\mathrm{i}} - (2m_2)v_{1,\mathrm{i}},\\ 3m_2 &= m_1\end{aligned}$$

so $m_2 = 100$ g.

E6-30 (a) Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (0.342\,\mathrm{kg})\frac{(1.24\,\mathrm{m/s}) - (0.636\,\mathrm{m/s})}{(1.24\,\mathrm{m/s}) + (0.636\,\mathrm{m/s})} = 0.110\,\mathrm{kg}.$$

(b) $v_{2\mathrm{f}} = 2(0.342\,\mathrm{kg})(1.24\,\mathrm{m/s})/(0.342\,\mathrm{kg} + 0.110\,\mathrm{kg}) = 1.88\,\mathrm{m/s}$.

E6-31 Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (2.0\,\mathrm{kg})\frac{v_{1\mathrm{i}} - v_{1\mathrm{i}}/4}{v_{1\mathrm{i}} + v_{1\mathrm{i}}/4} = 1.2\,\mathrm{kg}.$$

E6-32 I'll multiply all momentum equations by g, then I can use weight directly without converting to mass.

(a) $v_\mathrm{f} = [(31.8\ \mathrm{T})(5.20\ \mathrm{ft/s}) + (24.2\ \mathrm{T})(2.90\ \mathrm{ft/s})]/(31.8\ \mathrm{T} + 24.2\ \mathrm{T}) = 4.21\ \mathrm{ft/s}$.

(b) Evaluate:

$$v_{1\mathrm{f}} = \frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) + \frac{2(24.2\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) = 3.21\ \mathrm{ft/s}.$$

$$v_{2\mathrm{f}} = -\frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) + \frac{2(31.8\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) = 5.51\ \mathrm{ft/s}.$$

E6-33 Let the initial momentum of the first object be $\vec{\mathbf{p}}_{1,\mathrm{i}} = m\vec{\mathbf{v}}_{1,\mathrm{i}}$, that of the second object be $\vec{\mathbf{p}}_{2,\mathrm{i}} = m\vec{\mathbf{v}}_{2,\mathrm{i}}$, and that of the combined final object be $\vec{\mathbf{p}}_{\mathrm{f}} = 2m\vec{\mathbf{v}}_{\mathrm{f}}$. Then

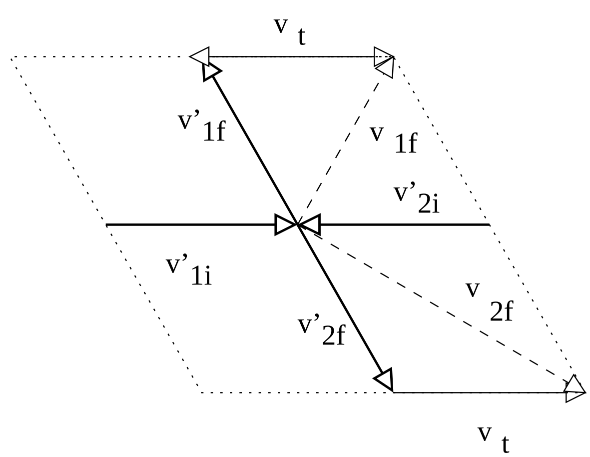
$$\vec{\mathbf{p}}_{1,\mathrm{i}} + \vec{\mathbf{p}}_{2,\mathrm{i}} = \vec{\mathbf{p}}_{\mathrm{f}},$$

implies that we can find a triangle with sides of length $p_{1,\mathrm{i}}$, $p_{2,\mathrm{i}}$, and p_{f}. These lengths are

$$\begin{aligned} p_{1,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{2,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{\mathrm{f}} &= 2mv_{\mathrm{f}} = 2mv_{\mathrm{i}}/2 = mv_{\mathrm{i}}, \end{aligned}$$

so this is an equilateral triangle. This means the angle between the initial velocities is 120°.

E6-34 We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\mathrm{cm}} = v_{1\mathrm{i}}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = v'_{1\mathrm{i}} = v'_{2\mathrm{i}} = v'_{1\mathrm{f}} = v'_{2\mathrm{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\mathrm{f}}$ and $v_{2\mathrm{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

(a) The target proton moves off at 90° to the direction the incident proton moves after the collision, or 26° away from the incident protons original direction.

(b) The y components of the final momenta must be equal, so $v_{2\mathrm{f}}\sin(26°) = v_{1\mathrm{f}}\sin(64°)$, or $v_{2\mathrm{f}} = v_{1\mathrm{f}}\tan(64°)$. The x components must add to the original momentum, so $(514\,\mathrm{m/s}) = v_{2\mathrm{f}}\cos(26°) + v_{1\mathrm{f}}\cos(64°)$, or

$$v_{1\mathrm{f}} = (514\,\mathrm{m/s})/\{\tan(64°)\cos(26°) + \cos(64°)\} = 225\,\mathrm{m/s},$$

and

$$v_{2\mathrm{f}} = (225\,\mathrm{m/s})\tan(64°) = 461\,\mathrm{m/s}.$$

E6-35 $v_{\mathrm{cm}} = \{(3.16\,\mathrm{kg})(15.6\,\mathrm{m/s}) + (2.84\,\mathrm{kg})(-12.2\,\mathrm{m/s})\}/\{(3.16\,\mathrm{kg}) + (2.84\,\mathrm{kg})\} = 2.44\,\mathrm{m/s}$, positive means to the left.

P6-1 The force is the change in momentum over change in time; the momentum is the mass time velocity, so

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \Delta v \frac{m}{\Delta t} = 2u\mu,$$

since μ is the mass per unit time.

P6-2 (a) The initial momentum is $\vec{\mathbf{p}}_{\text{i}} = (1420\,\text{kg})(5.28\,\text{m/s})\hat{\mathbf{j}} = 7500\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}$. After making the right hand turn the final momentum is $\vec{\mathbf{p}}_{\text{f}} = 7500\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{i}}$. The impulse is $\vec{\mathbf{J}} = 7500\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{i}} - 7500\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}$, which has magnitude $J = 10600\,\text{kg}\cdot\text{m/s}$.

(b) During the collision the impulse is $\vec{\mathbf{J}} = 0 - 7500\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{i}}$. The magnitude is $J = 7500\,\text{kg}\cdot\text{m/s}$.

(c) The average force is $F = J/t = (10600\,\text{kg}\cdot\text{m/s})/(4.60\,\text{s}) = 2300\,\text{N}$.

(d) The average force is $F = J/t = (7500\,\text{kg}\cdot\text{m/s})/(0.350\,\text{s}) = 21400\,\text{N}$.

P6-3 (a) Only the component of the momentum which is perpendicular to the wall changes. Then

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = -2(0.325\,\text{kg})(6.22\,\text{m/s})\sin(33^\circ)\hat{\mathbf{j}} = -2.20\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{F}} = -\vec{\mathbf{J}}/t = -(-2.20\,\text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}})/(0.0104\,\text{s}) = 212\,\text{N}$.

P6-4 The change in momentum of one bullet is $\Delta p = 2mv = 2(0.0030\,\text{kg})(500\,\text{m/s}) = 3.0\,\text{kg}\cdot\text{m/s}$. The average force is the total impulse in one minute divided by one minute, or

$$F_{\text{av}} = 100(3.0\,\text{kg}\cdot\text{m/s})/(60\,\text{s}) = 5.0\,\text{N}.$$

P6-5 (a) The volume of a hailstone is $V = 4\pi r^3/3 = 4\pi(0.5\text{ cm})^3/3 = 0.524\text{ cm}^3$. The mass of a hailstone is $m = \rho V = (9.2\times10^{-4}\text{kg/cm}^3)(0.524\text{ cm}^3) = 4.8\times10^{-4}\text{kg}$.

(b) The change in momentum of one hailstone when it hits the ground is

$$\Delta p = (4.8\times10^{-4}\text{kg})(25\,\text{m/s}) = 1.2\times10^{-2}\text{kg}\cdot\text{m/s}.$$

The hailstones fall at 25 m/s, which means that in one second the hailstones in a column 25 m high hit the ground. Over an area of $10\,\text{m}\times20\,\text{m}$ then there would be $(25\,\text{m})(10\,\text{m})(20\,\text{m}) = 500\,\text{m}^3$ worth of hailstones, or 6.00×10^5 hailstones per second striking the surface. Then

$$F_{\text{av}} = 6.00\times10^5(1.2\times10^{-2}\,\text{kg}\cdot\text{m/s})/(1\,\text{s}) = 7200\,\text{N}.$$

P6-6 Assume the links are *not* connected once the top link is released. Consider the link that starts h above the table; it falls a distance h in a time $t = \sqrt{2h/g}$ and hits the table with a speed $v = gt = \sqrt{2hg}$. When the link hits the table h of the chain is already on the table, and $L - h$ is yet to come. The linear mass density of the chain is M/L, so when this link strikes the table the mass is hitting the table at a rate $dm/dt = (M/L)v = (M/L)\sqrt{2hg}$. The average force required to stop the falling link is then $v\,dm/dt = (M/L)2hg = 2(M/L)hg$. But the weight of the chain that is already on the table is $(M/L)hg$, so the net force on the table is the sum of these two terms, or $F = 3W$.

P6-7 The weight of the marbles in the box after a time t is $mgRt$ because Rt is the number of marbles in the box.

The marbles fall a distance h from rest; the time required to fall this distance is $t = \sqrt{2h/g}$, the speed of the marbles when they strike the box is $v = gt = \sqrt{2gh}$. The momentum each marble imparts on the box is then $m\sqrt{2gh}$. If the marbles strike at a rate R then the force required to stop them is $Rm\sqrt{2gh}$.

The reading on the scale is then

$$W = mR(\sqrt{2gh} + gt).$$

This will give a numerical result of

$$(4.60\times10^{-3}\mathrm{kg})(115\,\mathrm{s}^{-1})\left(\sqrt{2(9.81\,\mathrm{m/s^2})(9.62\,\mathrm{m})} + (9.81\,\mathrm{m/s^2})(6.50\,\mathrm{s})\right) = 41.0\,\mathrm{N}.$$

P6-8 (a) $v = (108\,\mathrm{kg})(9.74\,\mathrm{m/s})/(108\,\mathrm{kg} + 1930\,\mathrm{kg}) = 0.516\,\mathrm{m/s}$.

(b) Label the person as object 1 and the car as object 2. Then $m_1v_1 + m_2v_2 = (108\,\mathrm{kg})(9.74\,\mathrm{m/s})$ and $v_1 = v_2 + 0.520\,\mathrm{m/s}$. Combining,

$$v_2 = [1050\,\mathrm{kg}\cdot\mathrm{m/s} - (0.520\,\mathrm{m/s})(108\,\mathrm{kg})]/(108\,\mathrm{kg} + 1930\,\mathrm{kg}) = 0.488\,\mathrm{m/s}.$$

P6-9 (a) It takes a time $t_1 = \sqrt{2h/g}$ to fall $h = 6.5$ ft. An object will be moving at a speed $v_1 = gt_1 = \sqrt{2hg}$ after falling this distance. If there is an inelastic collision with the pile then the two will move together with a speed of $v_2 = Mv_1/(M+m)$ after the collision.

If the pile then stops within $d = 1.5$ inches, then the time of stopping is given by $t_2 = d/(v_2/2) = 2d/v_2$.

For inelastic collisions this corresponds to an average force of

$$F_{\mathrm{av}} = \frac{(M+m)v_2}{t_2} = \frac{(M+m)v_2^2}{2d} = \frac{M^2v_1^2}{2(M+m)d} = \frac{(gM)^2}{g(M+m)}\frac{h}{d}.$$

Note that we multiply through by g to get weights. The numerical result is $F_{\mathrm{av}} = 130$ t.

(b) For an elastic collision $v_2 = 2Mv_1/(M+m)$; the time of stopping is still expressed by $t_2 = 2d/v_2$, but we now know F_{av} instead of d. Then

$$F_{\mathrm{av}} = \frac{mv_2}{t_2} = \frac{mv_2^2}{2d} = \frac{4Mmv_1^2}{(M+m)d} = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{d}.$$

or

$$d = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{F_{\mathrm{av}}},$$

which has a numerical result of $d = 0.51$ inches.

But wait! The weight, which just had an elastic collision, "bounced" off of the pile, and then hit it again. This drives the pile deeper into the earth. The weight hits the pile a second time with a speed of $v_3 = (M-m)/(M+m)v_1$; the pile will (in this second elastic collision) then have a speed of $v_4 = 2M(M+m)v_3 = [(M-m)/(M+m)]v_2$. In other words, we have an infinite series of distances traveled by the pile, and if $\alpha = [(M-m)/(M+m)] = 0.71$, the depth driven by the pile is

$$d_{\mathrm{f}} = d(1 + \alpha^2 + \alpha^4 + \alpha^6 \cdots) = \frac{d}{1-\alpha^2},$$

or $d = 1.03$.

P6-10 The cat jumps off of sled A; conservation of momentum requires that $Mv_{A,1} + m(v_{A,1} + v_c) = 0$, or

$$v_{A,1} = -mv_c/(m+M) = -(3.63\,\mathrm{kg})(3.05\,\mathrm{m/s})/(22.7\,\mathrm{kg} + 3.63\,\mathrm{kg}) = -0.420\,\mathrm{m/s}.$$

The cat lands on sled B; conservation of momentum requires $v_{B,1} = m(v_{A,1} + v_c)/(m+M)$. The cat jumps off of sled B; conservation of momentum is now

$$Mv_{B,2} + m(v_{B,2} - v_c) = m(v_{A,1} + v_c),$$

or

$$v_{B,2} = 2mv_c/(m+M) = (3.63\,\text{kg})[(-0.420\,\text{m/s}) + 2(3.05\,\text{m/s})]/(22.7\,\text{kg} + 3.63\,\text{kg}) = 0.783\,\text{m/s}.$$

The cat then lands on cart A; conservation of momentum requires that $(M+m)v_{A,2} = -Mv_{B,2}$, or

$$v_{A,2} = -(22.7\,\text{kg})(0.783\,\text{m/s})/(22.7\,\text{kg} + 3.63\,\text{kg}) = -0.675\,\text{m/s}.$$

P6-11 We align the coordinate system so that west is $+x$ and south is $+y$. The each car contributes the following to the initial momentum

$$\begin{aligned} A &: \quad (2720\text{ lb}/g)(38.5\text{ mi/h})\hat{\mathbf{i}} = 1.05\times10^5\text{ lb}\cdot\text{mi/h}/g\,\hat{\mathbf{i}}, \\ B &: \quad (3640\text{ lb}/g)(58.0\text{ mi/h})\hat{\mathbf{j}} = 2.11\times10^5\text{ lb}\cdot\text{mi/h}/g\,\hat{\mathbf{j}}. \end{aligned}$$

These become the components of the final momentum. The direction is then

$$\theta = \arctan\frac{2.11\times10^5\text{ lb}\cdot\text{mi/h}/g}{1.05\times10^5\text{ lb}\cdot\text{mi/h}/g} = 63.5^\circ,$$

south of west. The magnitude is the square root of the sum of the squares,

$$2.36\times10^5\text{ lb}\cdot\text{mi/h}/g,$$

and we divide this by the mass (6360 lb/g) to get the final speed after the collision: 37.1 mi/h.

P6-12 (a) Ball A must carry off a momentum of $\vec{\mathbf{p}} = m_B v\,\hat{\mathbf{i}} - m_B v/2\,\hat{\mathbf{j}}$, which would be in a direction $\theta = \arctan(-0.5/1) = 27^\circ$ from the original direction of B, or 117° from the final direction.

(b) No.

P6-13 (a) We assume all balls have a mass m. The collision imparts a "sideways" momentum to the cue ball of $m(3.50\,\text{m/s})\sin(65^\circ) = m(3.17\,\text{m/s})$. The other ball must have an equal, but opposite "sideways" momentum, so $-m(3.17\,\text{m/s}) = m(6.75\,\text{m/s})\sin\theta$, or $\theta = -28.0^\circ$.

(b) The final "forward" momentum is

$$m(3.50\,\text{m/s})\cos(65^\circ) + m(6.75\,\text{m/s})\cos(-28^\circ) = m(7.44\,\text{m/s}),$$

so the initial speed of the cue ball would have been 7.44 m/s.

P6-14 Assuming $M \gg m$, Eq. 6-25 becomes

$$v_{2\text{f}} = 2v_{1\text{i}} - v_{1\text{i}} = 2(13\text{ km/s}) - (-12\text{ km/s}) = 38\text{ km/s}.$$

P6-15 (a) We get

$$v_{2,\text{f}} = \frac{2(220\text{ g})}{(220\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 74.4\,\text{m/s}.$$

(b) Doubling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(440\text{ g})}{(440\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 81.5\,\text{m/s}.$$

(c) Tripling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(660\text{ g})}{(660\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 84.1\,\text{m/s}.$$

Although the heavier club helps some, the maximum speed to get out of the ball will be less than twice the speed of the club.

P6-16 There will always be at least two collisions. The balls are a, b, and c from left to right. After the first collision between a and b one has

$$v_{b,1} = v_0 \text{ and } v_{a,1} = 0.$$

After the first collision between b and c one has

$$v_{c,1} = 2mv_0/(m+M) \text{ and } v_{b,2} = (m-M)v_0/(m+M).$$

(a) If $m \geq M$ then ball b continue to move to the right (or stops) and there are no more collisions.
(b) If $m < M$ then ball b bounces back and strikes ball a which was at rest. Then

$$v_{a,2} = (m-M)v_0/(m+M) \text{ and } v_{b,3} = 0.$$

P6-17 All three balls are identical in mass and radii? Then balls 2 and 3 will move off at 30° to the initial direction of the first ball. By symmetry we expect balls 2 and 3 to have the same speed.

The problem now is to define an elastic three body collision. It is no longer the case that the balls bounce off with the same speed in the center of mass. One can't even treat the problem as two separate collisions, one right after the other. No amount of momentum conservation laws will help solve the problem; we need some additional physics, but at this point in the text we don't have it.

P6-18 The original speed is v_0 in the lab frame. Let α be the angle of deflection in the cm frame and $\vec{\mathbf{v}}'_1$ be the initial velocity in the cm frame. Then the velocity after the collision in the cm frame is $v'_1 \cos\alpha\,\hat{\mathbf{i}} + v'_1 \sin\alpha\,\hat{\mathbf{j}}$ and the velocity in the lab frame is $(v'_1\cos\alpha + v)\hat{\mathbf{i}} + v'_1 \sin\alpha\,\hat{\mathbf{j}}$, where v is the speed of the cm frame. The deflection angle in the lab frame is

$$\theta = \arctan[(v'_1 \sin\alpha)/(v'_1\cos\alpha + v)],$$

but $v = m_1 v_0/(m_1+m_2)$ and $v'_1 = v_0 - v$ so $v'_1 = m_2 v_0/(m_1+m_2)$ and

$$\theta = \arctan[(m_2\sin\alpha)/(m_2\cos\alpha + m_1)].$$

(c) θ is a maximum when $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum, which happens when $\cos\alpha = -m_1/m_2$ if $m_1 \leq m_2$. Then $[(m_2\sin\alpha)/(m_2\cos\alpha+m_1)]$ can have any value between $-\infty$ and ∞, so θ can be between 0 and π.

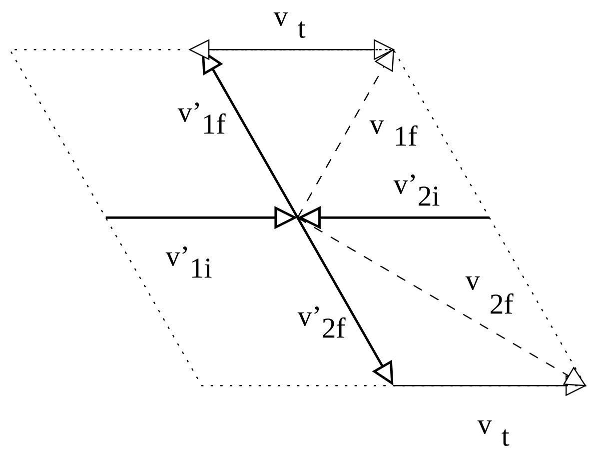
(a) If $m_1 > m_2$ then $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum when $\cos\alpha = -m_2/m_1$, then

$$[(m_2\sin\alpha)/(m_2\cos\alpha+m_1)] = m_2/\sqrt{m_1^2 - m_2^2}.$$

If $\tan\theta = m_2/\sqrt{m_1^2-m_2^2}$ then m_1 is like a hypotenuse and m_2 the opposite side. Then

$$\cos\theta = \sqrt{m_1^2-m_2^2}/m_1 = \sqrt{1-(m_2/m_1)^2}.$$

(b) We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\rm cm} = v_{1\rm i}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = {v'}_{1\text{i}} = {v'}_{2\text{i}} = {v'}_{1\text{f}} = {v'}_{2\text{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\text{f}}$ and $v_{2\text{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

P6-19 (a) The speed of the bullet after leaving the first block but before entering the second can be determined by momentum conservation.

$$\begin{aligned}
P_\text{f} &= P_\text{i},\\
p_\text{f,bl} + p_\text{f,bu} &= p_\text{i,bl} + p_\text{i,bu},\\
m_\text{bl} v_\text{f,bl} + m_\text{bu} v_\text{f,bu} &= m_\text{bl} v_\text{i,bl} + m_\text{bu} v_\text{i,bu},\\
(1.78\text{kg})(1.48\,\text{m/s}) + (3.54\times10^{-3}\text{kg})(1.48\,\text{m/s}) &= (1.78\text{kg})(0) + (3.54\times10^{-3}\text{kg})v_\text{i,bu},
\end{aligned}$$

which has solution $v_\text{i,bl} = 746\,\text{m/s}$.

(b) We do the same steps again, except applied to the first block,

$$\begin{aligned}
P_\text{f} &= P_\text{i},\\
p_\text{f,bl} + p_\text{f,bu} &= p_\text{i,bl} + p_\text{i,bu},\\
m_\text{bl} v_\text{f,bl} + m_\text{bu} v_\text{f,bu} &= m_\text{bl} v_\text{i,bl} + m_\text{bu} v_\text{i,bu},\\
(1.22\text{kg})(0.63\,\text{m/s}) + (3.54\times10^{-3}\text{kg})(746\,\text{m/s}) &= (1.22\text{kg})(0) + (3.54\times10^{-3}\text{kg})v_\text{i,bu},
\end{aligned}$$

which has solution $v_\text{i,bl} = 963\,\text{m/s}$.

P6-20 The acceleration of the block down the ramp is $a_1 = g\sin(22^\circ)$. The ramp has a length of $d = h/\sin(22^\circ)$, so it takes a time $t_1 = \sqrt{2d/a_1} = \sqrt{2h/g}/\sin(22^\circ)$ to reach the bottom. The speed when it reaches the bottom is $v_1 = a_1 t_1 = \sqrt{2gh}$. Notice that it is independent of the angle!

The collision is inelastic, so the two stick together and move with an initial speed of $v_2 = m_1 v_1/(m_1 + m_2)$. They slide a distance x before stopping; the average speed while decelerating is $v_\text{av} = v_2/2$, so the stopping time is $t_2 = 2x/v_2$ and the deceleration is $a_2 = v_2/t_2 = v_2^2/(2x)$. If the retarding force is $f = (m_1 + m_2)a_2$, then $f = \mu_\text{k}(m_1 + m_2)g$. Glue it all together and

$$\mu_\text{k} = \frac{m_1^2}{(m_1 + m_2)^2}\frac{h}{x} = \frac{(2.0\,\text{kg})^2}{(2.0\,\text{kg} + 3.5\,\text{kg})^2}\frac{(0.65\,\text{m})}{(0.57\,\text{m})} = 0.15.$$

P6-21 (a) For an object with initial speed v and deceleration $-a$ which travels a distance x before stopping, the time t to stop is $t = v/a$, the average speed while stopping is $v/2$, and $d = at^2/2$. Combining, $v = \sqrt{2ax}$. The deceleration in this case is given by $a = \mu_k g$.

Then just after the collision

$$v_A = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(8.20\,\mathrm{m})} = 4.57\,\mathrm{m/s},$$

while

$$v_B = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(6.10\,\mathrm{m})} = 3.94\,\mathrm{m/s},$$

(b) $v_0 = [(1100\,\mathrm{kg})(4.57\,\mathrm{m/s}) + (1400\mathrm{kg})(3.94\,\mathrm{m/s})]/(1400\,\mathrm{kg}) = 7.53\,\mathrm{m/s}.$