E4-1 (a) The time to pass between the plates is $t = x/v_x = (2.3 \text{ cm})/(9.6{\times}10^8 \text{ cm/s}) = 2.4{\times}10^{-9}\text{s}$.

(b) The vertical displacement of the beam is then $y = a_y t^2/2 == (9.4 \times 10^{16} \text{ cm/s}^2)(2.4 \times 10^{-9}\text{s})^2/2 = 0.27$ cm.

(c) The velocity components are $v_x = 9.6 \times 10^8$ cm/s and $v_y = a_y t = (9.4 \times 10^{16} \text{ cm/s}^2)(2.4 \times 10^{-9}\text{s}) = 2.3{\times}10^8\text{cm/s}$.

E4-2 $\vec{\mathbf{a}} = \Delta\vec{\mathbf{v}}/\Delta t = -(6.30\hat{\mathbf{i}} - 8.42\hat{\mathbf{j}})(\text{m/s})/(3\,\text{s}) = (-2.10\hat{\mathbf{i}} + 2.81\hat{\mathbf{j}})(\text{m/s}^2)$.

E4-3 (a) The velocity is given by

$$\begin{aligned} \frac{d\vec{\mathbf{r}}}{dt} &= \frac{d}{dt}\left(A\hat{\mathbf{i}}\right) + \frac{d}{dt}\left(Bt^2\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(Ct\hat{\mathbf{k}}\right), \\ \vec{\mathbf{v}} &= (0) + 2Bt\hat{\mathbf{j}} + C\hat{\mathbf{k}}. \end{aligned}$$

(b) The acceleration is given by

$$\begin{aligned} \frac{d\vec{\mathbf{v}}}{dt} &= \frac{d}{dt}\left(2Bt\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(C\hat{\mathbf{k}}\right), \\ \vec{\mathbf{a}} &= (0) + 2B\hat{\mathbf{j}} + (0). \end{aligned}$$

(c) Nothing exciting happens in the x direction, so we will focus on the yz plane. The trajectory in this plane is a parabola.

E4-4 (a) Maximum x is when $v_x = 0$. Since $v_x = a_x t + v_{x,0}$, $v_x = 0$ when $t = -v_{x,0}/a_x = -(3.6\,\text{m/s})/(-1.2\,\text{m/s}^2) = 3.0\,\text{s}$.

(b) Since $v_x = 0$ we have $|\vec{\mathbf{v}}| = |v_y|$. But $v_y = a_y t + v_{y,0} = -(1.4\,\text{m/s})(3.0\,\text{s}) + (0) = -4.2\,\text{m/s}$. Then $|\vec{\mathbf{v}}| = 4.2\,\text{m/s}$.

(c) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [-(0.6\,\text{m/s}^2)\hat{\mathbf{i}} - (0.7\,\text{m/s}^2)\hat{\mathbf{j}}](3.0\,\text{s})^2 + [(3.6\,\text{m/s})\hat{\mathbf{i}}](3.0\,\text{s}) = (5.4\,\text{m})\hat{\mathbf{i}} - (6.3\,\text{m})\hat{\mathbf{j}}.$$

E4-5 $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (3.7\,\text{N})\hat{\mathbf{j}} + (4.3\,\text{N})\hat{\mathbf{i}}$. Then $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (0.71\,\text{m/s}^2)\hat{\mathbf{j}} + (0.83\,\text{m/s}^2)\hat{\mathbf{i}}$.

E4-6 (a) The acceleration is $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (2.2\,\text{m/s}^2)\hat{\mathbf{j}}$. The velocity after 15 seconds is $\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_{\mathbf{0}}$, or

$$\vec{\mathbf{v}} = [(2.2\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s}) + [(42\,\text{m/s})\hat{\mathbf{i}}] = (42\,\text{m/s})\hat{\mathbf{i}} + (33\,\text{m/s})\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [(1.1\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s})^2 + [(42\,\text{m/s})\hat{\mathbf{i}}](15\,\text{s}) = (630\,\text{m})\hat{\mathbf{i}} + (250\,\text{m})\hat{\mathbf{j}}.$$

E4-7 The block has a weight $W = mg = (5.1\text{ kg})(9.8\text{ m/s}^2) = 50$ N.

(a) Initially $P = 12$ N, so $P_y = (12\,\text{N})\sin(25°) = 5.1$ N and $P_x = (12\,\text{N})\cos(25°) = 11$ N. Since the upward component is less than the weight, the block doesn't leave the floor, and a normal force will be present which will make $\sum F_y = 0$. There is only one contribution to the horizontal force, so $\sum F_x = P_x$. Newton's second law then gives $a_x = P_x/m = (11\text{ N})/(5.1\text{ kg}) = 2.2\text{ m/s}^2$.

(b) As P is increased, so is P_y; eventually P_y will be large enough to overcome the weight of the block. This happens just after $P_y = W = 50$ N, which occurs when $P = P_y/\sin\theta = 120$ N.

(c) Repeat part (a), except now $P = 120$ N. Then $P_x = 110$ N, and the acceleration of the block is $a_x = P_x/m = 22\text{ m/s}^2$.

E4-8 (a) The block has weight $W = mg = (96.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 942\,\mathrm{N}$. $P_x = (450\,\mathrm{N})\cos(38^\circ) = 355\,\mathrm{N}$; $P_y = (450\,\mathrm{N})\sin(38^\circ) = 277\,\mathrm{N}$. Since $P_y < W$ the crate stays on the floor and there is a normal force $N = W - P_y$. The net force in the x direction is $F_x = P_x - (125\,\mathrm{N}) = 230\,\mathrm{N}$. The acceleration is $a_x = F_x/m = (230\,\mathrm{N})/(96.0\,\mathrm{kg}) = 2.40\,\mathrm{m/s^2}$.

(b) The block has mass $m = W/g = (96.0\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 9.79\,\mathrm{kg}$. $P_x = (450\,\mathrm{N})\cos(38^\circ) = 355\,\mathrm{N}$; $P_y = (450\,\mathrm{N})\sin(38^\circ) = 277\,\mathrm{N}$. Since $P_y > W$ the crate lifts off of the floor! The net force in the x direction is $F_x = P_x - (125\,\mathrm{N}) = 230\,\mathrm{N}$. The x acceleration is $a_x = F_x/m = (230\,\mathrm{N})/(9.79\,\mathrm{kg}) = 23.5\,\mathrm{m/s^2}$. The net force in the y direction is $F_y = P_y - W = 181\,\mathrm{N}$. The y acceleration is $a_y = F_y/m = (181\,\mathrm{N})/(9.79\,\mathrm{kg}) = 18.5\,\mathrm{m/s^2}$. Wow.

E4-9 Let y be perpendicular and x be parallel to the incline. Then $P = 4600\,\mathrm{N}$;

$$P_x = (4600\,\mathrm{N})\cos(27^\circ) = 4100\,\mathrm{N};$$

$$P_y = (4600\,\mathrm{N})\sin(27^\circ) = 2090\,\mathrm{N}.$$

The weight of the car is $W = mg = (1200\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 11800\,\mathrm{N}$;

$$W_x = (11800\,\mathrm{N})\sin(18^\circ) = 3650\,\mathrm{N};$$

$$W_y = (11800\,\mathrm{N})\cos(18^\circ) = 11200\,\mathrm{N}.$$

Since $W_y > P_y$ the car stays on the incline. The net force in the x direction is $F_x = P_x - W_x = 450\,\mathrm{N}$. The acceleration in the x direction is $a_x = F_x/m = (450\,\mathrm{N})/(1200\,\mathrm{kg}) = 0.375\,\mathrm{m/s^2}$. The distance traveled in 7.5 s is $x = a_x t^2/2 = (0.375\,\mathrm{m/s^2})(7.5\,\mathrm{s})^2/2 = 10.5\,\mathrm{m}$.

E4-10 Constant speed means zero acceleration, so net force is zero. Let y be perpendicular and x be parallel to the incline. The weight is $W = mg = (110\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1080\,\mathrm{N}$; $W_x = W\sin(34^\circ)$; $W_y = W\cos(34^\circ)$. The push F has components $F_x = F\cos(34^\circ)$ and $F_y = -F\sin(34^\circ)$. The y components will balance after a normal force is introduced; the x components will balance if $F_x = W_x$, or $F = W\tan(34^\circ) = (1080\,\mathrm{N})\tan(34^\circ) = 730\,\mathrm{N}$.

E4-11 If the x axis is parallel to the river and the y axis is perpendicular, then $\vec{\mathbf{a}} = 0.12\hat{\mathbf{i}}\,\mathrm{m/s^2}$. The net force on the barge is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (9500\,\mathrm{kg})(0.12\hat{\mathbf{i}}\,\mathrm{m/s^2}) = 1100\hat{\mathbf{i}}\,\mathrm{N}.$$

The force exerted on the barge by the horse has components in both the x and y direction. If $P = 7900$ N is the magnitude of the pull and $\theta = 18^\circ$ is the direction, then $\vec{\mathbf{P}} = P\cos\theta\hat{\mathbf{i}} + P\sin\theta\hat{\mathbf{j}} = (7500\hat{\mathbf{i}} + 2400\hat{\mathbf{j}})$ N.

Let the force exerted on the barge by the water be $\vec{\mathbf{F_w}} = F_{w,x}\hat{\mathbf{i}} + F_{w,y}\hat{\mathbf{j}}$. Then $\sum F_x = (7500\ \mathrm{N}) + F_{w,x}$ and $\sum F_y = (2400\ \mathrm{N}) + F_{w,y}$. But we already found $\sum \vec{\mathbf{F}}$, so

$$\begin{aligned} F_x = 1100\ \mathrm{N} &= 7500\ \mathrm{N} + F_{w,x}, \\ F_y = 0 &= 2400\ \mathrm{N} + F_{w,y}. \end{aligned}$$

Solving, $F_{w,x} = -6400$ N and $F_{w,y} = -2400$ N. The magnitude is found by $F_w = \sqrt{F_{w,x}^2 + F_{w,y}^2} = 6800$ N.

E4-12 (a) Let y be perpendicular and x be parallel to the direction of motion of the plane. Then $W_x = mg\sin\theta$; $W_y = mg\cos\theta$; $m = W/g$. The plane is accelerating in the x direction, so $a_x = 2.62\,\mathrm{m/s^2}$; the net force is in the x direction, where $F_x = ma_x$. But $F_x = T - W_x$, so

$$T = F_x + W_x = W\frac{a_x}{g} + W\sin\theta = (7.93\times10^4\mathrm{N})\left[\frac{(2.62\,\mathrm{m/s^2})}{(9.81\,\mathrm{m/s^2})} + \sin(27^\circ)\right] = 5.72\times10^4\mathrm{N}.$$

(b) There is no motion in the y direction, so

$$L = W_y = (7.93\times10^4\mathrm{N})\cos(27^\circ) = 7.07\times10^4\mathrm{N}.$$

E4-13 (a) The ball rolled off horizontally so $v_{0y} = 0$. Then

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2, \\ (-4.23\ \mathrm{ft}) &= (0)t - \frac{1}{2}(32.2\ \mathrm{ft/s^2})t^2, \end{aligned}$$

which can be solved to yield $t = 0.514$ s.

(b) The initial velocity in the x direction can be found from $x = v_{0x}t$; rearranging, $v_{0x} = x/t = (5.11\ \mathrm{ft})/(0.514\ \mathrm{s}) = 9.94\ \mathrm{ft/s}$. Since there is no y component to the velocity, then the initial speed is $v_0 = 9.94\ \mathrm{ft/s}$.

E4-14 The electron travels for a time $t = x/v_x$. The electron "falls" vertically through a distance $y = -gt^2/2$ in that time. Then

$$y = -\frac{g}{2}\left(\frac{x}{v_x}\right)^2 = -\frac{(9.81\,\mathrm{m/s^2})}{2}\left(\frac{(1.0\,\mathrm{m})}{(3.0\times10^7\mathrm{m/s})}\right)^2 = -5.5\times10^{-15}\,\mathrm{m}.$$

E4-15 (a) The dart "falls" vertically through a distance $y = -gt^2/2 = -(9.81\,\mathrm{m/s^2})(0.19\,\mathrm{s})^2/2 = -0.18\,\mathrm{m}$.

(b) The dart travels horizontally $x = v_xt = (10\,\mathrm{m/s})(0.19\,\mathrm{s}) = 1.9\,\mathrm{m}$.

E4-16 The initial velocity components are

$$v_{x,0} = (15\,\mathrm{m/s})\cos(20^\circ) = 14\,\mathrm{m/s}$$

and

$$v_{y,0} = -(15\,\mathrm{m/s})\sin(20^\circ) = -5.1\,\mathrm{m/s}.$$

(a) The horizontal displacement is $x = v_xt = (14\,\mathrm{m/s})(2.3\,\mathrm{s}) = 32\,\mathrm{m}$.

(b) The vertical displacement is

$$y = -gt^2/2 + v_{y,0}t = -(9.81\,\mathrm{m/s^2})(2.3\,\mathrm{s})^2/2 + (-5.1\,\mathrm{m/s})(2.3\,\mathrm{s}) = -38\,\mathrm{m}.$$

E4-17 Find the time in terms of the the initial y component of the velocity:

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= v_{0y} - gt, \\ t &= v_{0y}/g. \end{aligned}$$

Use this time to find the highest point:

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2, \\ y_{\max} &= v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2}g\left(\frac{v_{0y}}{g}\right)^2, \\ &= \frac{v_{0y}^2}{2g}. \end{aligned}$$

Finally, we know the initial y component of the velocity from Eq. 2-6, so $y_{\max} = (v_0 \sin\phi_0)^2/2g$.

E4-18 The horizontal displacement is $x = v_x t$. The vertical displacement is $y = -gt^2/2$. Combining, $y = -g(x/v_x)^2/2$. The edge of the nth step is located at $y = -nw$, $x = nw$, where $w = 2/3$ ft. If $|y| > nw$ when $x = nw$ then the ball hasn't hit the step. Solving,

$$\begin{aligned} g(nw/v_x)^2/2 &< nw, \\ gnw/v_x^2 &< 2, \\ n &< 2v_x^2/(gw) = 2(5.0\text{ ft/s})^2/[(32\text{ ft/s}^2)(2/3\text{ ft})] = 2.34. \end{aligned}$$

Then the ball lands on the third step.

E4-19 (a) Start from the observation point 9.1 m above the ground. The ball will reach the highest point when $v_y = 0$, this will happen at a time t after the observation such that $t = v_{y,0}/g = (6.1\,\text{m/s})/(9.81\,\text{m/s}^2) = 0.62\,\text{s}$. The vertical displacement (from the ground) will be

$$y = -gt^2/2 + v_{y,0}t + y_0 = -(9.81\,\text{m/s}^2)(0.62\,\text{s})^2/2 + (6.1\,\text{m/s})(0.62\,\text{s}) + (9.1\,\text{m}) = 11\,\text{m}.$$

(b) The time for the ball to return to the ground from the highest point is $t = \sqrt{2y_{\max}/g} = \sqrt{2(11\,\text{m})/(9.81\,\text{m/s}^2)} = 1.5\,\text{s}$. The total time of flight is twice this, or 3.0 s. The horizontal distance traveled is $x = v_x t = (7.6\,\text{m/s})(3.0\,\text{s}) = 23\,\text{m}$.

(c) The velocity of the ball just prior to hitting the ground is

$$\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_0 = (-9.81\,\text{m/s}^2)\hat{\mathbf{j}}(1.5\,\text{s}) + (7.6\,\text{m/s})\hat{\mathbf{i}} = 7.6\,\text{m/s}\hat{\mathbf{i}} - 15\,\text{m/s}uj.$$

The magnitude is $\sqrt{7.6^2 + 15^2}(\text{m/s}) = 17\,\text{m/s}$. The direction is

$$\theta = \arctan(-15/7.6) = -63^\circ.$$

E4-20 Focus on the time it takes the ball to get to the plate, assuming it traveled in a straight line. The ball has a "horizontal" velocity of 135 ft/s. Then $t = x/v_x = (60.5\text{ ft})/(135\text{ ft/s}) = 0.448\,\text{s}$. The ball will "fall" a vertical distance of $y = -gt^2/2 = -(32\text{ ft/s}^2)(0.448\,\text{s})^2/2 = -3.2$ ft. That's in the strike zone.

E4-21 Since $R \propto 1/g$ one can write $R_2/R_1 = g_1/g_2$, or

$$\Delta R = R_2 - R_1 = R_1\left(1 - \frac{g_1}{g_2}\right) = (8.09\,\text{m})\left[1 - \frac{(9.7999\,\text{m/s}^2)}{(9.8128\,\text{m/s}^2)}\right] = 1.06\text{ cm}.$$

E4-22 If initial position is $\vec{\mathbf{r}}_0 = 0$, then final position is $\vec{\mathbf{r}} = (13 \text{ ft})\hat{\mathbf{i}} + (3 \text{ ft})\hat{\mathbf{j}}$. The initial velocity is $\vec{\mathbf{v}}_0 = v\cos\theta\hat{\mathbf{i}} + v\sin\theta\hat{\mathbf{j}}$. The horizontal equation is $(13 \text{ ft}) = v\cos\theta t$; the vertical equation is $(3 \text{ ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{3 \text{ ft} + (g/2)t^2}{(13 \text{ ft})} = \tan\theta,$$

or

$$t^2 = [(13 \text{ ft})\tan(55°) - (3 \text{ ft})][2/(32\,\text{m/s}^2)] = 0.973\,\text{s}^2,$$

or $t = 0.986\,\text{s}$. Then $v = (13 \text{ ft})/(\cos(55°)(0.986\,\text{s})) = 23$ ft/s.

E4-23 $v_x = x/t = (150 \text{ ft})/(4.50\,\text{s}) = 33.3$ ft/s. The time to the highest point is half the hang time, or 2.25 s. The vertical speed when the ball hits the ground is $v_y = -gt = -(32 \text{ ft/s}^2)(2.25\,\text{s}) =$ 72.0 ft/s. Then the initial vertical velocity is 72.0 ft/s. The magnitude of the initial velocity is $\sqrt{72^2 + 33^2}(\text{ft/s}) = 79$ ft/s. The direction is

$$\theta = \arctan(72/33) = 65°.$$

E4-24 (a) The magnitude of the initial velocity of the projectile is $v = 264$ ft/s. The projectile was in the air for a time t where

$$t = \frac{x}{v_x} = \frac{x}{v\cos\theta} = \frac{(2300 \text{ ft})}{(264 \text{ ft/s})\cos(-27°)} = 9.8\,\text{s}.$$

(b) The height of the plane was $-y$ where

$$-y = gt^2/2 - v_{y,0}t = (32 \text{ ft/s})(9.8\,\text{s})^2/2 - (264 \text{ ft/s})\sin(-27°)(9.8\,\text{s}) = 2700 \text{ ft}.$$

E4-25 Define the point the ball leaves the racquet as $\vec{\mathbf{r}} = 0$.

(a) The initial conditions are given as $v_{0x} = 23.6$ m/s and $v_{0y} = 0$. The time it takes for the ball to reach the horizontal location of the net is found from

$$\begin{aligned} x &= v_{0x}t, \\ (12 \text{ m}) &= (23.6 \text{ m/s})t, \\ 0.51 \text{ s} &= t, \end{aligned}$$

Find how far the ball has moved horizontally in this time:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (0)(0.51 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(0.51 \text{ s})^2 = -1.3 \text{ m}.$$

Did the ball clear the net? The ball started 2.37 m above the ground and “fell” through a distance of 1.3 m by the time it arrived at the net. So it is still 1.1 m above the ground and 0.2 m above the net.

(b) The initial conditions are now given by $v_{0x} = (23.6 \text{ m/s})(\cos[-5.0°]) = 23.5$ m/s and $v_{0y} = (23.6 \text{ m/s})(\sin[-5.0°]) = -2.1$ m/s. Now find the time to reach the net just as done in part (a):

$$t = x/v_{0x} = (12.0 \text{ m})/(23.5 \text{ m/s}) = 0.51 \text{ s}.$$

Find the vertical position of the ball when it arrives at the net:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (-2.1 \text{ m/s})(0.51 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(0.51 \text{ s})^2 = -2.3 \text{ m}.$$

Did the ball clear the net? Not this time; it started 2.37 m above the ground and then passed the net 2.3 m lower, or only 0.07 m above the ground.

E4-26 The initial speed of the ball is given by $v = \sqrt{gR} = \sqrt{(32\ \text{ft/s}^2)(350\ \text{ft})} = 106$ ft/s. The time of flight from the batter to the wall is

$$t = x/v_x = (320\ \text{ft})/[(106\ \text{ft/s})\cos(45^\circ)] = 4.3\,\text{s}.$$

The height of the ball at that time is given by $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, or

$$y = (4\ \text{ft}) + (106\ \text{ft/s})\sin(45^\circ)(4.3\,\text{s}) - (16\ \text{ft/s}^2)(4.3\,\text{s})^2 = 31\ \text{ft},$$

clearing the fence by 7 feet.

E4-27 The ball lands $x = (358\ \text{ft}) + (39\ \text{ft})\cos(28^\circ) = 392$ ft from the initial position. The ball lands $y = (39\ \text{ft})\sin(28^\circ) - (4.60\ \text{ft}) = 14$ ft above the initial position. The horizontal equation is $(392\ \text{ft}) = v\cos\theta t$; the vertical equation is $(14\ \text{ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{14\ \text{ft} + (g/2)t^2}{(392\ \text{ft})} = \tan\theta,$$

or

$$t^2 = [(392\ \text{ft})\tan(52^\circ) - (14\ \text{ft})][2/(32\,\text{m/s}^2)] = 30.5\,\text{s}^2,$$

or $t = 5.52$ s. Then $v = (392\ \text{ft})/(\cos(52^\circ)(5.52\,\text{s})) = 115$ ft/s.

E4-28 Since ball is traveling at 45° when it returns to the same level from which it was thrown for maximum range, then we can assume it actually traveled ≈ 1.6 m. farther than it would have had it been launched from the ground level. This won't make a big difference, but that means that $R = 60.0\,\text{m} - 1.6\,\text{m} = 58.4\,\text{m}$. If v_0 is initial speed of ball thrown directly up, the ball rises to the highest point in a time $t = v_0/g$, and that point is $y_{\text{max}} = gt^2/2 = v_0^2/(2g)$ above the launch point. But $v_0^2 = gR$, so $y_{\text{max}} = R/2 = (58.4\,\text{m})/2 = 29.2\,\text{m}$. To this we add the 1.60 m point of release to get 30.8 m.

E4-29 The net force on the pebble is zero, so $\sum F_y = 0$. There are only two forces on the pebble, the force of gravity W and the force of the water on the pebble F_{PW}. These point in opposite directions, so $0 = F_{PW} - W$. But $W = mg = (0.150\ \text{kg})(9.81\ \text{m/s}^2) = 1.47$ N. Since $F_{PW} = W$ in this problem, the force of the water on the pebble must also be 1.47 N.

E4-30 Terminal speed is when drag force equal weight, or $mg = bv_{\text{T}}{}^2$. Then $v_{\text{T}} = \sqrt{mg/b}$.

E4-31 Eq. 4-22 is

$$v_y(t) = v_{\text{T}}\left(1 - e^{-bt/m}\right),$$

where we have used Eq. 4-24 to substitute for the terminal speed. We want to solve this equation for time when $v_y(t) = v_{\text{T}}/2$, so

$$\begin{aligned}
\tfrac{1}{2}v_{\text{T}} &= v_{\text{T}}\left(1 - e^{-bt/m}\right),\\
\tfrac{1}{2} &= \left(1 - e^{-bt/m}\right),\\
e^{-bt/m} &= \tfrac{1}{2}\\
bt/m &= -\ln(1/2)\\
t &= \tfrac{m}{b}\ln 2
\end{aligned}$$

E4-32 The terminal speed is $7\,\text{m/s}$ for a raindrop with $r = 0.15$ cm. The mass of this drop is $m = 4\pi\rho r^3/3$, so

$$b = \frac{mg}{v_\text{T}} = \frac{4\pi(1.0\times10^{-3}\text{kg/cm}^3)(0.15\text{ cm})^3(9.81\,\text{m/s}^2)}{3(7\,\text{m/s})} = 2.0\times10^{-5}\text{kg/s}.$$

E4-33 (a) The speed of the train is $v = 9.58\,\text{m/s}$. The drag force on one car is $f = 243(9.58)\,\text{N} = 2330\,\text{N}$. The total drag force is $23(2330\,\text{N}) = 5.36\times10^4\text{N}$. The net force exerted on the cars (treated as a single entity) is $F = ma = 23(48.6\times10^3\text{kg})(0.182\,\text{m/s}^2) = 2.03\times10^5\text{N}$. The pull of the locomotive is then $P = 2.03\times10^5\text{N} + 5.36\times10^4\text{N} = 2.57\times10^5\text{N}$.

(b) If the locomotive is pulling the cars at constant speed up an incline then it must exert a force on the cars equal to the sum of the drag force and the parallel component of the weight. The drag force is the same in each case, so the parallel component of the weight is $W_{||} = W\sin\theta = 2.03\times10^5\text{N} = ma$, where a is the acceleration from part (a). Then

$$\theta = \arcsin(a/g) = \arcsin[(0.182\,\text{m/s}^2)/(9.81\,\text{m/s}^2)] = 1.06^\circ.$$

E4-34 (a) $a = v^2/r = (2.18\times10^6\text{m/s})^2/(5.29\times10^{-11}\text{m}) = 8.98\times10^{22}\text{m/s}^2$.

(b) $F = ma = (9.11\times10^{-31}\text{kg})(8.98\times10^{22}\text{m/s}^2) = 8.18\times10^{-8}\text{N}$, toward the center.

E4-35 (a) $v = \sqrt{ra_\text{c}} = \sqrt{(5.2\text{ m})(6.8)(9.8\text{ m/s}^2)} = 19$ m/s.

(b) Use the fact that one revolution corresponds to a length of $2\pi r$:

$$19\frac{\text{m}}{\text{s}}\left(\frac{1\text{ rev}}{2\pi(5.2\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 35\frac{\text{rev}}{\text{min}}.$$

E4-36 (a) $v = 2\pi r/T = 2\pi(15\,\text{m})/(12\,\text{s}) = 7.85\,\text{m/s}$. Then $a = v^2/r = (7.85\,\text{m/s})^2/(15\,\text{m}) = 4.11\,\text{m/s}^2$, directed toward center, which is down.

(b) Same arithmetic as in (a); direction is still toward center, which is now up.

(c) The magnitude of the net force in both (a) and (b) is $F = ma = (75\,\text{kg})(4.11\,\text{m/s}^2) = 310\,\text{N}$. The weight of the person is the same in both parts: $W = mg = (75\,\text{kg})(9.81\,\text{m/s}^2) = 740\,\text{N}$. At the top the net force is down, the weight is down, so the Ferris wheel is pushing up with a force of $P = W - F = (740\,\text{N}) - (310\,\text{N}) = 430\,\text{N}$. At the bottom the net force is up, the weight is down, so the Ferris wheel is pushing up with a force of $P = W + F = (740\,\text{N}) + (310\,\text{N}) = 1050\,\text{N}$.

E4-37 (a) $v = 2\pi r/T = 2\pi(20\times10^3\text{m})/(1.0\,\text{s}) = 1.26\times10^5\text{m/s}$.

(b) $a = v^2/r = (1.26\times10^5\text{m/s})^2/(20\times10^3\text{m}) = 7.9\times10^5\text{m/s}^2$.

E4-38 (a) $v = 2\pi r/T = 2\pi(6.37\times10^6\text{m})/(86400\,\text{s}) = 463\,\text{m/s}$. $a = v^2/r = (463\,\text{m/s})^2/(6.37\times10^6\text{m}) = 0.034\,\text{m/s}^2$.

(b) The net force on the object is $F = ma = (25.0\,\text{kg})(0.034\,\text{m/s}^2) = 0.85\,\text{N}$. There are two forces on the object: a force up from the scale (S), and the weight down, $W = mg = (25.0\,\text{kg})(9.80\,\text{m/s}^2) = 245\,\text{N}$. Then $S = F + W = 245\,\text{N} + 0.85\,\text{N} = 246\,\text{N}$.

E4-39 Let $\Delta x = 15$ m be the length; $t_w = 90$ s, the time to walk the stalled Escalator; $t_s = 60$ s, the time to ride the moving Escalator; and t_m, the time to walk up the moving Escalator.

The walking speed of the person relative to a fixed Escalator is $v_{we} = \Delta x/t_w$; the speed of the Escalator relative to the ground is $v_{eg} = \Delta x/t_s$; and the speed of the walking person relative to the ground on a moving Escalator is $v_{wg} = \Delta x/t_m$. But these three speeds are related by $v_{wg} = v_{we}+v_{eg}$. Combine all the above:

$$\begin{aligned} v_{wg} &= v_{we} + v_{eg}, \\ \frac{\Delta x}{t_m} &= \frac{\Delta x}{t_w} + \frac{\Delta x}{t_s}, \\ \frac{1}{t_m} &= \frac{1}{t_w} + \frac{1}{t_s}. \end{aligned}$$

Putting in the numbers, $t_m = 36$ s.

E4-40 Let v_w be the walking speed, v_s be the sidewalk speed, and $v_\mathrm{m} = v_\mathrm{w} + v_\mathrm{s}$ be Mary's speed while walking on the moving sidewalk. All three cover the same distance x, so $v_i = x/t_i$, where i is one of w, s, or m. Put this into the Mary equation, and

$$1/t_\mathrm{m} = 1/t_\mathrm{w} + 1/t_\mathrm{s} = 1/(150\,\mathrm{s}) + 1/(70\,\mathrm{s}) = 1/48\,\mathrm{s}.$$

E4-41 If it takes longer to fly westward then the speed of the plane (relative to the ground) westward must be less than the speed of the plane eastward. We conclude that the jet-stream must be blowing east. The speed of the plane relative to the ground is $v_\mathrm{e} = v_\mathrm{p} + v_\mathrm{j}$ when going east and $v_\mathrm{w} = v_\mathrm{p} - v_\mathrm{j}$ when going west. In either case the distance is the same, so $x = v_i t_i$, where i is e or w. Since $t_\mathrm{w} - t_\mathrm{e}$ is given, we can write

$$t_\mathrm{w} - t_\mathrm{e} = \frac{x}{v_\mathrm{p} - v_\mathrm{j}} - \frac{x}{v_\mathrm{p} + v_\mathrm{j}} = x\frac{2v_\mathrm{j}}{{v_\mathrm{p}}^2 - {v_\mathrm{j}}^2}.$$

Solve the quadratic if you want, but since $v_\mathrm{j} \ll v_\mathrm{p}$ we can neglect it in the denominator and

$$v_\mathrm{j} = {v_\mathrm{p}}^2(0.83 \text{ h})/(2x) = (600 \text{ mi/h})^2(0.417 \text{ h})/(2700 \text{ mi}) = 56 \text{ mi/hr}.$$

E4-42 The horizontal component of the rain drop's velocity is 28 m/s. Since $v_x = v\sin\theta$, $v = (28\,\mathrm{m/s})/\sin(64^\circ) = 31\,\mathrm{m/s}$.

E4-43 (a) The position of the bolt relative to the elevator is y_{be}, the position of the bolt relative to the shaft is y_{bs}, and the position of the elevator relative to the shaft is y_{es}. Zero all three positions at $t = 0$; at this time $v_{0,bs} = v_{0,es} = 8.0$ ft/s.

The three equations describing the positions are

$$\begin{aligned} y_{bs} &= v_{0,bs}t - \frac{1}{2}gt^2, \\ y_{es} &= v_{0,es}t + \frac{1}{2}at^2, \\ y_{be} + r_{es} &= r_{bs}, \end{aligned}$$

where $a = 4.0$ m/s^2 is the upward acceleration of the elevator. Rearrange the last equation and solve for y_{be}; get $y_{be} = -\frac{1}{2}(g+a)t^2$, where advantage was taken of the fact that the initial velocities are the same.

Then

$$t = \sqrt{-2y_{be}/(g+a)} = \sqrt{-2(-9.0 \text{ ft})/(32 \text{ ft/s}^2 + 4 \text{ ft/s}^2)} = 0.71\,\mathrm{s}$$

(b) Use the expression for y_{bs} to find how the bolt moved relative to the shaft:

$$y_{bs} = v_{0,bs}t - \frac{1}{2}gt^2 = (8.0 \text{ ft})(0.71 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(0.71 \text{ s})^2 = -2.4 \text{ ft}.$$

E4-44 The speed of the plane relative to the ground is $v_{\text{pg}} = (810\text{ km})/(1.9\text{ h}) = 426\text{ km/h}$. The velocity components of the plane relative to the air are $v_N = (480\text{ km/h})\cos(21°) = 448\text{ km/h}$ and $v_E = (480\text{ km/h})\sin(21°) = 172\text{ km/h}$. The wind must be blowing with a component of 172 km/h to the west and a component of $448 - 426 = 22$ km/h to the south.

E4-45 (a) Let $\hat{\mathbf{i}}$ point east and $\hat{\mathbf{j}}$ point north. The velocity of the torpedo is $\vec{\mathbf{v}} = (50\text{ km/h})\hat{\mathbf{i}}\sin\theta + (50\text{ km/h})\hat{\mathbf{j}}\cos\theta$. The initial coordinates of the battleship are then $\vec{\mathbf{r}}_0 = (4.0\text{ km})\hat{\mathbf{i}}\sin(20°) + (4.0\text{ km})\hat{\mathbf{j}}\cos(20°) = (1.37\text{ km})\hat{\mathbf{i}} + (3.76\text{ km})\hat{\mathbf{j}}$. The final position of the battleship is $\vec{\mathbf{r}} = (1.37\text{ km} + 24\text{ km/h}t)\hat{\mathbf{i}} + (3.76\text{ km})\hat{\mathbf{j}}$, where t is the time of impact. The final position of the torpedo is the same, so

$$[(50\text{ km/h})\hat{\mathbf{i}}\sin\theta + (50\text{ km/h})\hat{\mathbf{j}}\cos\theta]t = (1.37\text{ km} + 24\text{ km/h}t)\hat{\mathbf{i}} + (3.76\text{ km})\hat{\mathbf{j}},$$

or

$$[(50\text{ km/h})\sin\theta]t - 24\text{ km/h}t = 1.37\text{ km}$$

and

$$[(50\text{ km/h})\cos\theta]t = 3.76\text{ km}.$$

Dividing the top equation by the bottom and rearranging,

$$50\sin\theta - 24 = 18.2\cos\theta.$$

Use any trick you want to solve this. I used Maple and found $\theta = 46.8°$.

(b) The time to impact is then $t = 3.76\text{ km}/[(50\text{ km/h})\cos(46.8°)] = 0.110$ h, or 6.6 minutes.

P4-1 Let $\vec{\mathbf{r}}_A$ be the position of particle of particle A, and $\vec{\mathbf{r}}_B$ be the position of particle B. The equations for the motion of the two particles are then

$$\begin{aligned}
\vec{\mathbf{r}}_A &= \vec{\mathbf{r}}_{0,A} + \vec{\mathbf{v}}t, \\
&= d\hat{\mathbf{j}} + vt\hat{\mathbf{i}}; \\
\vec{\mathbf{r}}_B &= \frac{1}{2}\vec{\mathbf{a}}t^2, \\
&= \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2.
\end{aligned}$$

A collision will occur if there is a time when $\vec{\mathbf{r}}_A = \vec{\mathbf{r}}_B$. Then

$$d\hat{\mathbf{j}} + vt\hat{\mathbf{i}} = \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2,$$

but this is really *two* equations: $d = \frac{1}{2}at^2\cos\theta$ and $vt = \frac{1}{2}at^2\sin\theta$.

Solve the second one for t and get $t = 2v/(a\sin\theta)$. Substitute that into the first equation, and then rearrange,

$$\begin{aligned}
d &= \frac{1}{2}at^2\cos\theta, \\
d &= \frac{1}{2}a\left(\frac{2v}{a\sin\theta}\right)^2\cos\theta, \\
\sin^2\theta &= \frac{2v}{ad}\cos\theta, \\
1 - \cos^2\theta &= \frac{2v^2}{ad}\cos\theta, \\
0 &= \cos^2\theta + \frac{2v^2}{ad}\cos\theta - 1.
\end{aligned}$$

This last expression is quadratic in $\cos\theta$. It simplifies the solution if we define $b = 2v/(ad) = 2(3.0\ \text{m/s})^2/([0.4\ \text{m/s}^2][30\ \text{m}]) = 1.5$, then

$$\cos\theta = \frac{-b \pm \sqrt{b^2+4}}{2} = -0.75 \pm 1.25.$$

Then $\cos\theta = 0.5$ and $\theta = 60°$.

P4-2 (a) The acceleration of the ball is $\vec{\mathbf{a}} = (1.20\,\text{m/s}^2)\hat{\mathbf{i}} - (9.81\,\text{m/s}^2)\hat{\mathbf{j}}$. Since $\vec{\mathbf{a}}$ is constant the trajectory is given by $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2$, since $\vec{\mathbf{v}}_0 = 0$ and we choose $\vec{\mathbf{r}}_0 = 0$. This is a straight line trajectory, with a direction given by $\vec{\mathbf{a}}$. Then

$$\theta = \arctan(9.81/1.20) = 83.0°.$$

and $R = (39.0\,\text{m})/\tan(83.0°) = 4.79\,\text{m}$. It will be useful to find $H = (39.0\,\text{m})/\sin(83.0°) = 39.3\,\text{m}$.

(b) The magnitude of the acceleration of the ball is $a = \sqrt{9.81^2 + 1.20^2}(\text{m/s}^2) = 9.88\,\text{m/s}^2$. The time for the ball to travel down the hypotenuse of the figure is then $t = \sqrt{2(39.3\,\text{m})/(9.88\,\text{m/s}^2)} = 2.82\,\text{s}$.

(c) The magnitude of the speed of the ball at the bottom will then be

$$v = at = (9.88\,\text{m/s}^2)(2.82\,\text{s}) = 27.9\,\text{m/s}.$$

P4-3 (a) The rocket thrust is $\vec{\mathbf{T}} = (61.2\ \text{kN})\cos(58.0°)\hat{\mathbf{i}} + (61.2\ \text{kN})\sin(58.0°)\hat{\mathbf{i}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}}$. The net force on the rocket is the $\vec{\mathbf{F}} = \vec{\mathbf{T}} + \vec{\mathbf{W}}$, or

$$\vec{\mathbf{F}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}} - (3030\,\text{kg})(9.81\,\text{m/s}^2)\hat{\mathbf{j}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 22.2\ \text{kN}\hat{\mathbf{j}}.$$

The acceleration (until rocket cut-off) is this net force divided by the mass, or

$$\vec{\mathbf{a}} = 10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}}.$$

The position at rocket cut-off is given by

$$\begin{aligned}\vec{\mathbf{r}} &= \vec{\mathbf{a}}t^2/2 = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s})^2/2,\\ &= 1.23\times10^4\text{m}\hat{\mathbf{i}} + 8.44\times10^3\text{m}\hat{\mathbf{j}}.\end{aligned}$$

The altitude at rocket cut-off is then 8.44 km.

(b) The velocity at rocket cut-off is

$$\vec{\mathbf{v}} = \vec{\mathbf{a}}t = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s}) = 514\,\text{m/s}\hat{\mathbf{i}} + 352\,\text{m/s}\hat{\mathbf{j}},$$

this becomes the initial velocity for the "free fall" part of the journey. The rocket will hit the ground after t seconds, where t is the solution to

$$0 = -(9.81\,\text{m/s}^2)t^2/2 + (352\,\text{m/s})t + 8.44\times10^3\,\text{m}.$$

The solution is $t = 90.7\,\text{s}$. The rocket lands a horizontal distance of $x = v_x t = (514\,\text{m/s})(90.7\,\text{s}) = 4.66\times10^4$ m beyond the rocket cut-off; the total horizontal distance covered by the rocket is 46.6 km+ 12.3 km = 58.9 km.

P4-4 (a) The horizontal speed of the ball is $v_x = 135$ ft/s. It takes

$$t = x/v_x = (30.0 \text{ ft})/(135 \text{ ft/s}) = 0.222\,\text{s}$$

to travel the 30 feet horizontally, whether the first 30 feet, the last 30 feet, or 30 feet somewhere in the middle.

(b) The ball "falls" $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.222\,\text{s})^2/2 = -0.789$ ft while traveling the first 30 feet.

(c) The ball "falls" a total of $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.444\,\text{s})^2/2 = -3.15$ ft while traveling the first 60 feet, so during the last 30 feet it must have fallen $(-3.15 \text{ ft}) - (-0.789 \text{ ft}) = -2.36$ ft.

(d) The distance fallen because of acceleration is not linear in time; the distance moved horizontally is linear in time.

P4-5 (a) The initial velocity of the ball has components

$$v_{x,0} = (25.3\,\text{m/s})\cos(42.0^\circ) = 18.8\,\text{m/s}$$

and

$$v_{y,0} = (25.3\,\text{m/s})\sin(42.0^\circ) = 16.9\,\text{m/s}.$$

The ball is in the air for $t = x/v_x = (21.8\,\text{m})/(18.8\,\text{m/s}) = 1.16\,\text{s}$ before it hits the wall.

(b) $y = -gt^2/2 + v_{y,0}t = -(4.91\,\text{m/s}^2)(1.16\,\text{s})^2 + (16.9\,\text{m/s})(1.16\,\text{s}) = 13.0\,\text{m}$.

(c) $v_x = v_{x,0} = 18.8\,\text{m/s}$. $v_y = -gt + v_{y,0} = -(9.81\,\text{m/s}^2)(1.16\,\text{s}) + (16.9\,\text{m/s}) = 5.52\,\text{m/s}$.

(d) Since $v_y > 0$ the ball is still heading up.

P4-6 (a) The initial vertical velocity is $v_{y,0} = v_0 \sin\phi_0$. The time to the highest point is $t = v_{y,0}/g$. The highest point is $H = gt^2/2$. Combining,

$$H = g(v_0 \sin\phi_0/g)^2/2 = v_0^2 \sin^2\phi_0/(2g).$$

The range is $R = (v_0^2/g)\sin 2\phi_0 = 2(v_0^2/g)\sin\phi_0\cos\phi_0$. Since $\tan\theta = H/(R/2)$, we have

$$\tan\theta = \frac{2H}{R} = \frac{v_0^2 \sin^2\phi_0/g}{2(v_0^2/g)\sin\phi_0\cos\phi_0} = \frac{1}{2}\tan\phi_0.$$

(b) $\theta = \arctan(0.5\tan 45^\circ) = 26.6^\circ$.

P4-7 The components of the initial velocity are given by $v_{0x} = v_0\cos\theta = 56$ ft/s and $v_{0y} = v_0\sin\theta = 106$ ft/s where we used $v_0 = 120$ ft/s and $\theta = 62^\circ$.

(a) To find h we need only find out the vertical position of the stone when $t = 5.5$ s.

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(5.5 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(5.5 \text{ s})^2 = 99 \text{ ft}.$$

(b) Look at this as a vector problem:

$$\begin{aligned}
\vec{\mathbf{v}} &= \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t,\\
&= \left(v_{0x}\hat{\mathbf{i}} + v_{0y}\hat{\mathbf{j}}\right) - g\hat{\mathbf{j}}t,\\
&= v_{0x}\hat{\mathbf{i}} + (v_{0y} - gt)\,\hat{\mathbf{j}},\\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + \left((106 \text{ ft/s} - (32 \text{ ft/s}^2)(5.5 \text{ s})\right)\hat{\mathbf{j}},\\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + (-70.0 \text{ ft/s})\hat{\mathbf{j}}.
\end{aligned}$$

The magnitude of this vector gives the speed when $t = 5.5$ s; $v = \sqrt{56^2 + (-70)^2}$ ft/s= 90 ft/s.

(c) Highest point occurs when $v_y = 0$. Solving Eq. 4-9(b) for time; $v_y = 0 = v_{0y} - gt = (106 \text{ ft/s}) - (32 \text{ ft/s}^2)t$; $t = 3.31$ s. Use this time in Eq. 4-10(b),

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(3.31 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(3.31 \text{ s})^2 = 176 \text{ ft}.$$

P4-8 (a) Since $R = (v_0^2/g)\sin 2\phi_0$, it is sufficient to prove that $\sin 2\phi_0$ is the same for both $\sin 2(45^\circ + \alpha)$ and $\sin 2(45^\circ - \alpha)$.

$$\sin 2(45^\circ \pm \alpha) = \sin(90^\circ \pm 2\alpha) = \cos(\pm 2\alpha) = \cos(2\alpha).$$

Since the $\pm$ dropped out, the two quantities are equal.

(b) $\phi_0 = (1/2)\arcsin(Rg/v_0^2) = (1/2)\arcsin((20.0\,\text{m})(9.81\,\text{m/s}^2)/(30.0\,\text{m/s})^2) = 6.3^\circ$. The other choice is $90^\circ - 6.3^\circ = 83.7^\circ$.

P4-9 To score the ball must pass the horizontal distance of 50 m with an altitude of no less than 3.44 m. The initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$ where $v_0 = 25$ m/s, and θ is the unknown.

The time to the goal post is $t = x/v_{0x} = x/(v_0 \cos\theta)$.

The vertical motion is given by

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2, \\ &= x\frac{\sin\theta}{\cos\theta} - \frac{gx^2}{2v_0^2}\frac{1}{\cos^2\theta}. \end{aligned}$$

In this last expression y needs to be greater than 3.44 m. In this last expression use

$$\frac{1}{\cos^2\theta} - 1 + 1 = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta} + 1 = \frac{1-\cos^2\theta}{\cos^2\theta} + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1 = \tan^2\theta + 1.$$

This gives for our y expression

$$y = x\tan\theta - \frac{gx^2}{2v_0}\left(\tan^2\theta + 1\right),$$

which can be combined with numbers and constraints to give

$$\begin{aligned} (3.44 \text{ m}) &\leq (50 \text{ m})\tan\theta - \frac{(9.8 \text{ m/s}^2)(50 \text{ m})^2}{2(25 \text{ m/s})^2}\left(\tan^2\theta + 1\right), \\ 3.44 &\leq 50\tan\theta - 20\left(\tan^2\theta + 1\right), \\ 0 &\leq -20\tan^2\theta + 50\tan\theta - 23 \end{aligned}$$

Solve, and $\tan\theta = 1.25 \pm 0.65$, so the allowed kicking angles are between $\theta = 31^\circ$ and $\theta = 62^\circ$.

P4-10 (a) The height of the projectile at the highest point is $H = L\sin\theta$. The amount of time before the projectile hits the ground is $t = \sqrt{2H/g} = \sqrt{2L\sin\theta/g}$. The horizontal distance covered by the projectile in this time is $x = v_x t = v\sqrt{2L\sin\theta/g}$. The horizontal distance to the projectile when it is at the highest point is $x' = L\cos\theta$. The projectile lands at

$$D = x - x' = v\sqrt{2L\sin\theta/g} - L\cos\theta.$$

(b) The projectile will pass overhead if $D > 0$.

P4-11 $v^2 = v_x^2 + v_y^2$. For a projectile v_x is constant, so we need only evaluate $d^2(v_y^2)/dt^2$. The first derivative is $2v_y\,dv_y/dt = -2v_y g$. The derivative of this (the second derivative) is $-2g\,dv_y/dt = 2g^2$.

P4-12 $|\vec{\mathbf{r}}|$ is a maximum when r^2 is a maximum. $r^2 = x^2 + y^2$, or

$$\begin{aligned} r^2 &= (v_{x,0}t)^2 + (-gt^2/2 + v_{y,0}t)^2, \\ &= (v_0 t\cos\phi_0)^2 + \left(v_0 t\sin\phi_0 - gt^2/2\right)^2, \\ &= v_0^2 t^2 - v_0 g t^3 \sin\phi_0 + g^2 t^4/4. \end{aligned}$$

We want to look for the condition which will allow dr^2/dt to vanish. Since

$$dr^2/dt = 2v_0^2 t - 3v_0 g t^2 \sin\phi_0 + g^2 t^3$$

we can focus on the quadratic discriminant, $b^2 - 4ac$, which is

$$9v_0^2 g^2 \sin^2\phi_0 - 8v_0^2 g^2,$$

a quantity which will only be greater than zero if $9\sin^2\phi_0 > 8$. The critical angle is then

$$\phi_c = \arcsin(\sqrt{8/9}) = 70.5°.$$

P4-13 There is a downward force on the balloon of 10.8 kN from gravity and an upward force of 10.3 kN from the buoyant force of the air. The resultant of these two forces is 500 N down, but since the balloon is descending at constant speed so the net force on the balloon must be zero. This is possible because there is a drag force on the balloon of $D = bv^2$, this force is directed upward. The magnitude must be 500 N, so the constant b is

$$b = \frac{(500\,\text{N})}{(1.88\,\text{m/s})^2} = 141\,\text{kg/m}.$$

If the crew drops 26.5 kg of ballast they are "lightening" the balloon by

$$(26.5\,\text{kg})(9.81\,\text{m/s}^2) = 260\,\text{N}.$$

This reduced the weight, but not the buoyant force, so the drag force at constant speed will now be $500\,\text{N} - 260\,\text{N} = 240\,\text{N}$.

The new constant downward speed will be

$$v = \sqrt{D/b} = \sqrt{(240\,\text{N})/(141\,\text{kg/m})} = 1.30\,\text{m/s}.$$

P4-14 The constant b is

$$b = (500\,\text{N})/(1.88\,\text{m/s}) = 266\,\text{N}\cdot\text{s/m}.$$

The drag force after "lightening" the load will still be 240 N. The new downward speed will be

$$v = D/b = (240\,\text{N})/(266\,\text{N}\cdot\text{s/m}) = 0.902\,\text{m/s}.$$

P4-15 (a) Initially $v_0 = 0$, so $D = 0$, the only force is the weight, so $a = -g$.

(b) After some time the acceleration is zero, then $W = D$, or $b{v_\text{T}}^2 = mg$, or $v_\text{T} = \sqrt{mg/b}$.

(c) When $v = v_\text{T}/2$ the drag force is $D = b{v_\text{T}}^2/4 = mg/4$, so the net force is $F = D - W = -3mg/4$. The acceleration is the $a = -3g/4$.

P4-16 (a) The net force on the barge is $F = -D = -bv$, this results in a differential equation $m\,dv/dt = -bv$, which can be written as

$$\begin{aligned} dv/v &= -(b/m)dt, \\ \int dv/v &= -(b/m)\int dt, \\ \ln(v_\mathrm{f}/v_\mathrm{i}) &= -bt/m. \end{aligned}$$

Then $t = (m/b)\ln(v_\mathrm{i}/v_\mathrm{f})$.

(b) $t = [(970\,\mathrm{kg})/(68\,\mathrm{N\cdot s/m})]\ln(32/8.3) = 19\,\mathrm{s}$.

P4-17 (a) The acceleration is the time derivative of the velocity,

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{mg}{b}\left(1-e^{-bt/m}\right)\right) = \frac{mg}{b}\frac{b}{m}e^{-bt/m},$$

which can be simplified as $a_y = ge^{-bt/m}$. For large t this expression approaches 0; for small t the exponent can be expanded to give

$$a_y \approx g\left(1-\frac{bt}{m}\right) = g - v_\mathrm{T}t,$$

where in the last line we made use of Eq. 4-24.

(b) The position is the integral of the velocity,

$$\begin{aligned} \int_0^t v_y\,dt &= \int_0^t \left(\frac{mg}{b}\left(1-e^{-bt/m}\right)\right)dt, \\ \int_0^t \frac{dy}{dt}\,dt &= \frac{mg}{b}\left(t-(-m/b)e^{-bt/m}\right)\Big|_0^t, \\ \int_0^y dy &= v_\mathrm{T}\left(t+\frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g}-1\right)\right), \\ y &= v_\mathrm{T}\left(t+\frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g}-1\right)\right). \end{aligned}$$

P4-18 (a) We have $v_y = v_\mathrm{T}(1-e^{-bt/m})$ from Eq. 4-22; this can be substituted into the last line of the solution for P4-17 to give

$$y_{95} = v_\mathrm{T}\left(t-\frac{v_y}{g}\right).$$

We can also rearrange Eq. 4-22 to get $t = -(m/b)\ln(1-v_y/v_\mathrm{T})$, so

$$y_{95} = {v_\mathrm{T}}^2/g\left(-\ln(1-v_y/v_\mathrm{T})-\frac{v_y}{v_\mathrm{T}}\right).$$

But $v_y/v_\mathrm{T} = 0.95$, so

$$y_{95} = {v_\mathrm{T}}^2/g\,(-\ln(0.05)-0.95) = {v_\mathrm{T}}^2/g\,(\ln 20 - 19/20)\,.$$

(b) $y_{95} = (42\,\mathrm{m/s})^2/(9.81\,\mathrm{m/s^2})(2.05) = 370\,\mathrm{m}$.

P4-19 (a) Convert units first. $v = 86.1\,\mathrm{m/s}$, $a = 0.05(9.81\,\mathrm{m/s^2}) = 0.491\,\mathrm{m/s^2}$. The minimum radius is $r = v^2/a = (86.1\,\mathrm{m/s})/(0.491\,\mathrm{m/s^2}) = 15$ km.

(b) $v = \sqrt{ar} = \sqrt{(0.491\,\mathrm{m/s^2})(940\,\mathrm{m})} = 21.5\,\mathrm{m/s}$. That's 77 km/hr.

P4-20 (a) The position is given by $\vec{\mathbf{r}} = R\sin\omega t\,\hat{\mathbf{i}} + R(1-\cos\omega t)\hat{\mathbf{j}}$, where $\omega = 2\pi/(20\,\mathrm{s}) = 0.314\,\mathrm{s}^{-1}$ and $R = 3.0\,\mathrm{m}$. When $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{r}} = (3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 7.5\,\mathrm{s}$ $\vec{\mathbf{r}} = (2.1\,\mathrm{m})\hat{\mathbf{i}} + (5.1\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 10\,\mathrm{s}$ $\vec{\mathbf{r}} = (6.0\,\mathrm{m})\hat{\mathbf{j}}$. These vectors have magnitude 4.3 m, 5.5 m and 6.0 m, respectively. The vectors have direction 45°, 68° and 90° respectively.

(b) $\Delta\vec{\mathbf{r}} = (-3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$, which has magnitude 4.3 m and direction 135°.

(c) $v_{\mathrm{av}} = \Delta r/\Delta t = (4.3\,\mathrm{m})/(5.0\,\mathrm{s}) = 0.86\,\mathrm{m/s}$. The direction is the same as $\Delta\vec{\mathbf{r}}$.

(d) The velocity is given by $\vec{\mathbf{v}} = R\omega\cos\omega t\,\hat{\mathbf{i}} + R\omega\sin\omega t\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{v}} = (0.94\,\mathrm{m/s})\hat{\mathbf{j}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{v}} = (-0.94\,\mathrm{m/s})\hat{\mathbf{i}}$.

(e) The acceleration is given by $\vec{\mathbf{a}} = -R\omega^2\sin\omega t\,\hat{\mathbf{i}} + R\omega^2\cos\omega t\,\hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{i}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{j}}$.

P4-21 Start from where the stone lands; in order to get there the stone fell through a vertical distance of 1.9 m while moving 11 m horizontally. Then

$$y = -\frac{1}{2}gt^2 \text{ which can be written as } t = \sqrt{\frac{-2y}{g}}.$$

Putting in the numbers, $t = 0.62\,\mathrm{s}$ is the time of flight from the moment the string breaks. From this time find the horizontal velocity,

$$v_x = \frac{x}{t} = \frac{(11\,\mathrm{m})}{(0.62\,\mathrm{s})} = 18\,\mathrm{m/s}.$$

Then the centripetal acceleration is

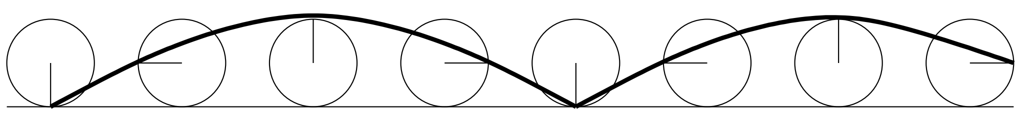
$$a_c = \frac{v^2}{r} = \frac{(18\,\mathrm{m/s})^2}{(1.4\,\mathrm{m})} = 230\,\mathrm{m/s^2}.$$

P4-22 (a) The path traced out by her feet has circumference $c_1 = 2\pi r\cos 50°$, where r is the radius of the earth; the path traced out by her head has circumference $c_2 = 2\pi(r+h)\cos 50°$, where h is her height. The difference is $\Delta c = 2\pi h\cos 50° = 2\pi(1.6\,\mathrm{m})\cos 50° = 6.46\,\mathrm{m}$.

(b) $a = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$. Then $\Delta a = 4\pi^2\Delta r/T^2$. Note that $\Delta r = h\cos\theta$! Then

$$\Delta a = 4\pi^2(1.6\,\mathrm{m})\cos 50°/(86400\,\mathrm{s})^2 = 5.44\times10^{-9}\mathrm{m/s^2}.$$

P4-23 (a) A cycloid looks something like this:



(b) The position of the particle is given by

$$\vec{\mathbf{r}} = (R\sin\omega t + \omega Rt)\hat{\mathbf{i}} + (R\cos\omega t + R)\hat{\mathbf{j}}.$$

The maximum value of y occurs whenever $\cos\omega t = 1$. The minimum value of y occurs whenever $\cos\omega t = -1$. At either of those times $\sin\omega t = 0$.

The velocity is the derivative of the displacement vector,

$$\vec{\mathbf{v}} = (R\omega\cos\omega t + \omega R)\hat{\mathbf{i}} + (-R\omega\sin\omega t)\hat{\mathbf{j}}.$$

When y is a maximum the velocity simplifies to

$$\vec{\mathbf{v}} = (2\omega R)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

When y is a minimum the velocity simplifies to

$$\vec{\mathbf{v}} = (0)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

The acceleration is the derivative of the velocity vector,

$$\vec{\mathbf{a}} = (-R\omega^2 \sin \omega t)\hat{\mathbf{i}} + (-R\omega^2 \cos \omega t)\hat{\mathbf{j}}.$$

When y is a maximum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (-R\omega^2)\hat{\mathbf{j}}.$$

When y is a minimum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (R\omega^2)\hat{\mathbf{j}}.$$

P4-24 (a) The speed of the car is $v_c = 15.3\,\text{m/s}$. The snow appears to fall with an angle $\theta = \arctan(15.3/7.8) = 63°$.

(b) The apparent speed is $\sqrt{(15.3)^2 + (7.8)^2}(\text{m/s}) = 17.2\,\text{m/s}$.

P4-25 (a) The decimal angles are 89.994250° and 89.994278°. The earth moves in the orbit around the sun with a speed of $v = 2.98\times10^4\text{m/s}$ (Appendix C). The speed of light is then between $c = (2.98\times10^4\text{m/s})\tan(89.994250°) = 2.97\times10^8\text{m/s}$ and $c = (2.98\times10^4\text{m/s})\tan(89.994278°) = 2.98\times10^8\text{m/s}$. This method is *highly* sensitive to rounding. Calculating the orbital speed from the radius and period of the Earth's orbit will likely result in different answers!

P4-26 (a) Total distance is $2l$, so $t_0 = 2l/v$.

(b) Assume wind blows east. Time to travel out is $t_1 = l/(v+u)$, time to travel back is $t_2 = l/(v-u)$. Total time is sum, or

$$t_E = \frac{l}{v+u} + \frac{l}{v-u} = \frac{2lv}{v^2-u^2} = \frac{t_0}{1-u^2/v^2}.$$

If wind blows west the times reverse, but the result is otherwise the same.

(c) Assume wind blows north. The airplane will still have a speed of v relative to the wind, but it will need to fly with a heading away from east. The speed of the plane relative to the ground will be $\sqrt{v^2-u^2}$. This will be the speed even when it flies west, so

$$t_N = \frac{2l}{\sqrt{v^2-u^2}} = \frac{t_0}{\sqrt{1-u^2/v^2}}.$$

(d) If $u > v$ the wind sweeps the plane along in one general direction only; it can never fly back. Sort of like a black hole event horizon.

P4-27 The velocity of the police car with respect to the ground is $\vec{\mathbf{v}}_{pg} = -76\text{km/h}\hat{\mathbf{i}}$. The velocity of the motorist with respect the ground is $\vec{\mathbf{v}}_{mg} = -62\text{ km/h}\hat{\mathbf{j}}$.

The velocity of the motorist with respect to the police car is given by solving

$$\vec{\mathbf{v}}_{mg} = \vec{\mathbf{v}}_{mp} + \vec{\mathbf{v}}_{pg},$$

so $\vec{\mathbf{v}}_{mp} = 76\text{km/h}\hat{\mathbf{i}} - 62\text{ km/h}\hat{\mathbf{j}}$. This velocity has magnitude

$$v_{mp} = \sqrt{(76\text{km/h})^2 + (-62\text{ km/h})^2} = 98\text{ km/h}.$$

The direction is

$$\theta = \arctan(-62 \text{ km/h})/(76\text{km/h}) = -39^\circ,$$

but that is relative to $\hat{\mathbf{i}}$. We want to know the direction relative to the line of sight. The line of sight is

$$\alpha = \arctan(57\,\text{m})/(41\,\text{m}) = -54^\circ$$

relative to $\hat{\mathbf{i}}$, so the answer must be 15°.

P4-28 (a) The velocity of the plane with respect to the air is $\vec{\mathbf{v}}_{pa}$; the velocity of the air with respect to the ground is $\vec{\mathbf{v}}_{ag}$, the velocity of the plane with respect to the ground is $\vec{\mathbf{v}}_{pg}$. Then $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{ag}$. This can be represented by a triangle; since the sides are given we can find the angle between $\vec{\mathbf{v}}_{ag}$ and $\vec{\mathbf{v}}_{pg}$ (points north) using the cosine law

$$\theta = \arccos\left(\frac{(135)^2 - (135)^2 - (70)^2}{-2(135)(70)}\right) = 75^\circ.$$

(b) The direction of $\vec{\mathbf{v}}_{pa}$ can also be found using the cosine law,

$$\theta = \arccos\left(\frac{(70)^2 - (135)^2 - (135)^2}{-2(135)(135)}\right) = 30^\circ.$$