

E24-1 For isothermal processes the entropy expression is almost trivial, $\Delta S = Q/T$, where if Q is positive (heat flow into system) the entropy increases.

$$\text{Then } Q = T\Delta S = (405 \text{ K})(46.2 \text{ J/K}) = 1.87 \times 10^4 \text{ J.}$$

E24-2 Entropy is a state variable and is path independent, so

$$\begin{aligned} \text{(a) } \Delta S_{ab,2} &= \Delta S_{ab,1} = +0.60 \text{ J/K,} \\ \text{(b) } \Delta S_{ba,2} &= -\Delta S_{ab,2} = -0.60 \text{ J/K,} \end{aligned}$$

E24-3 (a) Heat only enters along the top path, so

$$Q_{\text{in}} = T\Delta S = (400 \text{ K})(0.6 \text{ J/K} - 0.1 \text{ J/K}) = 200 \text{ J.}$$

(b) Heat leaves only bottom path, so

$$Q_{\text{out}} = T\Delta S = (250 \text{ K})(0.1 \text{ J/K} - 0.6 \text{ J/K}) = -125 \text{ J.}$$

Since $Q + W = 0$ for a cyclic path,

$$W = -Q = -[(200 \text{ J}) + (-125 \text{ J})] = -75 \text{ J.}$$

E24-4 (a) The work done for isothermal expansion is given by Eq. 23-18,

$$W = -(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(410 \text{ K}) \ln \frac{3.45V_1}{V_1} = -1.69 \times 10^4 \text{ J.}$$

(b) For isothermal process, $Q = -W$, then

$$\Delta S = Q/T = (1.69 \times 10^4 \text{ J})/(410 \text{ K}) = 41.2 \text{ J/K.}$$

(c) Entropy change is zero for reversible adiabatic processes.

E24-5 (a) We want to find the heat absorbed, so

$$Q = mc\Delta T = (1.22 \text{ kg})(387 \text{ J/mol} \cdot \text{K})((105^\circ \text{C}) - (25.0^\circ \text{C})) = 3.77 \times 10^4 \text{ J.}$$

(b) We want to find the entropy change, so, according to Eq. 24-1,

$$\begin{aligned} \Delta S &= \int_{T_i}^{T_f} \frac{dQ}{T}, \\ &= \int_{T_i}^{T_f} \frac{mc dT}{T}, \\ &= mc \ln \frac{T_f}{T_i}. \end{aligned}$$

The entropy change of the copper block is then

$$\Delta S = mc \ln \frac{T_f}{T_i} = (1.22 \text{ kg})(387 \text{ J/mol} \cdot \text{K}) \ln \frac{(378 \text{ K})}{(298 \text{ K})} = 112 \text{ J/K.}$$

E24-6 $\Delta S = Q/T = mL/T$, so

$$\Delta S = (0.001 \text{ kg})(-333 \times 10^3 \text{ J/kg})/(263 \text{ K}) = -1.27 \text{ J/K.}$$

E24-7 Use the first equation on page 551.

$$n = \frac{\Delta S}{R \ln(V_f/V_i)} = \frac{(24 \text{ J/K})}{(8.31 \text{ J/mol} \cdot \text{K}) \ln(3.4/1.3)} = 3.00 \text{ mol.}$$

E24-8 $\Delta S = Q/T_c - Q/T_h$.

(a) $\Delta S = (260 \text{ J})(1/100 \text{ K} - 1/400 \text{ K}) = 1.95 \text{ J/K}$.

(b) $\Delta S = (260 \text{ J})(1/200 \text{ K} - 1/400 \text{ K}) = 0.65 \text{ J/K}$.

(c) $\Delta S = (260 \text{ J})(1/300 \text{ K} - 1/400 \text{ K}) = 0.217 \text{ J/K}$.

(d) $\Delta S = (260 \text{ J})(1/360 \text{ K} - 1/400 \text{ K}) = 0.0722 \text{ J/K}$.

E24-9 (a) If the rod is in a steady state we wouldn't expect the entropy of the rod to change. Heat energy is flowing out of the hot reservoir into the rod, but this process happens at a fixed temperature, so the entropy change in the hot reservoir is

$$\Delta S_H = \frac{Q_H}{T_H} = \frac{(-1200 \text{ J})}{(403 \text{ K})} = -2.98 \text{ J/K}.$$

The heat energy flows into the cold reservoir, so

$$\Delta S_C = \frac{Q_H}{T_H} = \frac{(1200 \text{ J})}{(297 \text{ K})} = 4.04 \text{ J/K}.$$

The total change in entropy of the system is the sum of these two terms

$$\Delta S = \Delta S_H + \Delta S_C = 1.06 \text{ J/K}.$$

(b) Since the rod is in a steady state, nothing is changing, not even the entropy.

E24-10 (a) $Q_c + Q_l = 0$, so

$$m_c c_c (T - T_c) + m_l c_l (T - T_l) = 0,$$

which can be solved for T to give

$$T = \frac{(0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K})(400 \text{ K}) + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K})(200 \text{ K})}{(0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K}) + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K})} = 320 \text{ K}.$$

(b) Zero.

(c) $\Delta S = mc \ln T_f/T_i$, so

$$\Delta S = (0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K}) \ln \frac{(320 \text{ K})}{(400 \text{ K})} + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K}) \ln \frac{(320 \text{ K})}{(200 \text{ K})} = 1.75 \text{ J/K}.$$

E24-11 The total mass of ice and water is 2.04 kg. If eventually the ice and water have the same mass, then the final state will have 1.02 kg of each. This means that 1.78 kg – 1.02 kg = 0.76 kg of water changed into ice.

(a) The change of water at 0°C to ice at 0°C is isothermal, so the entropy change is

$$\Delta S = \frac{Q}{T} = \frac{-mL}{T} = \frac{(0.76 \text{ kg})(333 \times 10^3 \text{ J/kg})}{(273 \text{ K})} = -927 \text{ J/K}.$$

(b) The entropy change is now +927 J/K.

E24-12 (a) $Q_a + Q_w = 0$, so

$$m_a c_a (T - T_a) + m_w c_w (T - T_w) = 0,$$

which can be solved for T to give

$$T = \frac{(0.196 \text{ kg})(900 \text{ J/kg} \cdot \text{K})(380 \text{ K}) + (0.0523 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(292 \text{ K})}{(0.196 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) + (0.0523 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})} = 331 \text{ K}.$$

That's the same as 58°C .

(b) $\Delta S = mc \ln T_f/T_i$, so

$$\Delta S_a = (0.196 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) \ln \frac{(331 \text{ K})}{(380 \text{ K})} = -24.4 \text{ J/K}.$$

(c) For the water,

$$(0.0523 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \frac{(331 \text{ K})}{(292 \text{ K})} = 27.5 \text{ J/K}.$$

(d) $\Delta S = (27.5 \text{ J/K}) + (-24.4 \text{ J/K}) = 3.1 \text{ J/K}$.

E24-13 (a) $e = 1 - (36.2 \text{ J}/52.4 \text{ J}) = 0.309$.

(b) $W = Q_h - Q_c = (52.4 \text{ J}) - (36.2 \text{ J}) = 16.2 \text{ J}$.

E24-14 (a) $Q_h = (8.18 \text{ kJ})/(0.25) = 32.7 \text{ kJ}$, $Q_c = Q_h - W = (32.7 \text{ kJ}) - (8.18 \text{ kJ}) = 24.5 \text{ kJ}$.

(b) $Q_h = (8.18 \text{ kJ})/(0.31) = 26.4 \text{ kJ}$, $Q_c = Q_h - W = (26.4 \text{ kJ}) - (8.18 \text{ kJ}) = 18.2 \text{ kJ}$.

E24-15 One hour's worth of coal, when burned, will provide energy equal to

$$(382 \times 10^3 \text{ kg})(28.0 \times 10^6 \text{ J/kg}) = 1.07 \times 10^{13} \text{ J}.$$

In this hour, however, the plant only generates

$$(755 \times 10^6 \text{ W})(3600 \text{ s}) = 2.72 \times 10^{12} \text{ J}.$$

The efficiency is then

$$e = (2.72 \times 10^{12} \text{ J})/(1.07 \times 10^{13} \text{ J}) = 25.4\%.$$

E24-16 We use the convention that all quantities are positive, regardless of direction. $W_A = 5W_B$; $Q_{i,A} = 3Q_{i,B}$; and $Q_{o,A} = 2Q_{o,B}$. But $W_A = Q_{i,A} - Q_{o,A}$, so

$$5W_B = 3Q_{i,B} - 2Q_{o,B},$$

or, applying $W_B = Q_{i,B} - Q_{o,B}$,

$$\begin{aligned} 5W_B &= 3Q_{i,B} - 2(Q_{i,B} - W_B), \\ 3W_B &= Q_{i,B}, \\ W_B/Q_{i,B} &= 1/3 = e_B. \end{aligned}$$

Then

$$e_A = \frac{W_A}{Q_{i,A}} = \frac{5W_B}{3Q_{i,B}} = \frac{5}{3} \frac{1}{3} = \frac{5}{9}.$$

E24-17 (a) During an isothermal process $W = -Q = -2090 \text{ J}$. The negative indicates that the gas did work on the environment.

(b) The efficiency is $e = 1 - (297 \text{ K})/(412 \text{ K}) = 0.279$. Then

$$Q_o = Q_i(1 - e) = (2090 \text{ J})[1 - (0.279)] = 1510 \text{ J}.$$

Since this is rejected heat it should actually be negative.

(c) During an isothermal process $W = -Q = 1510 \text{ J}$. Positive indicates that the gas did work on the environment.

E24-18 $1 - e = T_c/T_h$, or $T_c = T_h(1 - e)$. The difference is

$$\Delta T = T_h - T_c = T_h e,$$

so $T_h = (75 \text{ C}^\circ)/(0.22) = 341 \text{ K}$, and

$$T_c = (341 \text{ K})[(1 - (0.22))] = 266 \text{ K}.$$

E24-19 The BC and DA processes are both adiabatic; so if we could find an expression for work done during an adiabatic process we might be almost done. But what is an adiabatic process? It is a process for which $Q = 0$, so according to the first law

$$\Delta E_{\text{int}} = W.$$

But for an ideal gas

$$\Delta E_{\text{int}} = nC_V \Delta T,$$

as was pointed out in Table 23-5. So we have

$$|W| = nC_V |\Delta T|$$

and since the adiabatic paths BC and DA operate between the same two isotherms, we can conclude that the magnitude of the work is the same for both paths.

E24-20 (a) To save typing, assume that all quantities are positive. Then

$$e_1 = 1 - T_2/T_1,$$

$W_1 = e_1 Q_1$, and $Q_2 = Q_1 - W_1$. Not only that, but

$$e_2 = 1 - T_3/T_2,$$

and $W_2 = e_2 Q_2$. Combining,

$$e = \frac{W_1 + W_2}{Q_1} = \frac{e_1 Q_1 + e_2(Q_1 - W_1)}{Q_1} = e_1 + e_2(1 - e_1) = e_1 + e_2 - e_1 e_2,$$

or

$$e = 1 - \frac{T_2}{T_1} + 1 - \frac{T_3}{T_2} - 1 + \frac{T_2}{T_1} + \frac{T_3}{T_2} - \frac{T_3}{T_1} = 1 - \frac{T_3}{T_1}.$$

(b) $e = 1 - (311 \text{ K})/(742 \text{ K}) = 0.581$.

E24-21 (a) $p_2 = (16.0 \text{ atm})(1/5.6)^{(1.33)} = 1.62 \text{ atm}$.

(b) $T_2 = T_1(1/5.6)^{(1.33)-1} = (0.567)T_1$, so

$$e = 1 - (0.567) = 0.433.$$

E24-22 (a) The area of the cycle is $\Delta V \Delta p = p_0 V_0$, so the work done by the gas is

$$W = (1.01 \times 10^5 \text{ Pa})(0.0225 \text{ m}^3) = 2270 \text{ J}.$$

(b) Let the temperature at a be T_a . Then

$$T_b = T_a(V_b/V_a)(p_b/p_a) = 2T_a.$$

Let the temperature at c be T_c . Then

$$T_c = T_a(V_c/V_a)(p_c/p_a) = 4T_a.$$

Consequently, $\Delta T_{ab} = T_a$ and $\Delta T_{bc} = 2T_a$. Putting this information into the constant volume and constant pressure heat expressions,

$$Q_{ab} = \frac{3}{2}nR\Delta T_{ab} = \frac{3}{2}nRT_a = \frac{3}{2}p_a V_a,$$

and

$$Q_{bc} = \frac{5}{2}nR\Delta T_{bc} = \frac{5}{2}nR2T_a = 5p_a V_a,$$

so that $Q_{ac} = \frac{13}{2}p_0 V_0$, or

$$Q_{ac} = \frac{13}{2}(1.01 \times 10^5 \text{ Pa})(0.0225 \text{ m}^3) = 1.48 \times 10^4 \text{ J}.$$

(c) $e = (2270 \text{ J})/(14800 \text{ J}) = 0.153$.

(d) $e_c = 1 - (T_a/4T_a) = 0.75$.

E24-23 According to Eq. 24-15,

$$K = \frac{T_L}{T_H - T_L} = \frac{(261 \text{ K})}{(299 \text{ K}) - (261 \text{ K})} = 6.87$$

Now we solve the question out of order.

(b) The work required to run the freezer is

$$|W| = |Q_L|/K = (185 \text{ kJ})/(5.70) = 32.5 \text{ kJ}.$$

(a) The freezer will discharge heat into the room equal to

$$|Q_L| + |W| = (185 \text{ kJ}) + (32.5 \text{ kJ}) = 218 \text{ kJ}.$$

E24-24 (a) $K = |Q_L|/|W| = (568 \text{ J})/(153 \text{ J}) = 3.71$.

(b) $|Q_H| = |Q_L| + |W| = (568 \text{ J}) + (153 \text{ J}) = 721 \text{ J}$.

E24-25 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$.

(a) $|W| = (10.0 \text{ J})(300 \text{ K}/280 \text{ K} - 1) = 0.714 \text{ J}$.

(b) $|W| = (10.0 \text{ J})(300 \text{ K}/200 \text{ K} - 1) = 5.00 \text{ J}$.

(c) $|W| = (10.0 \text{ J})(300 \text{ K}/100 \text{ K} - 1) = 20.0 \text{ J}$.

(d) $|W| = (10.0 \text{ J})(300 \text{ K}/50 \text{ K} - 1) = 50.0 \text{ J}$.

E24-26 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. Then

$$|Q_H| = |Q_L| + |W| = |Q_L|(T_H/T_L) = (0.150 \text{ J})(296 \text{ K}/4.0 \text{ K}) = 11 \text{ J}.$$

E24-27 We will start with the assumption that the air conditioner is a Carnot refrigerator. $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. For fun, I'll convert temperature to the absolute Rankine scale! Then

$$|Q_L| = (1.0 \text{ J})/(555^\circ\text{R}/530^\circ\text{R} - 1) = 21 \text{ J}.$$

E24-28 The best coefficient of performance is

$$K_c = (276 \text{ K})/(308 \text{ K} - 276 \text{ K}) = 8.62.$$

The inventor claims they have a machine with

$$K = (20 \text{ kW} - 1.9\text{kW})/(1.9 \text{ kW}) = 9.53.$$

Can't be done.

E24-29 (a) $e = 1 - (258 \text{ K}/322 \text{ K}) = 0.199$. $|W| = (568 \text{ J})(0.199) = 113 \text{ J}$.

(b) $K = (258 \text{ K})/(322 \text{ K} - 258 \text{ K}) = 4.03$. $|W| = (1230 \text{ J})/(4.03) = 305 \text{ J}$.

E24-30 The temperatures are distractors!

$$|W| = |Q_H| - |Q_L| = |Q_H| - K|W|,$$

so

$$|W| = |Q_H|/(1 + K) = (7.6 \text{ MJ})/(1 + 3.8) = 1.58 \text{ MJ}.$$

Then $P = (1.58 \text{ MJ})/(3600 \text{ s}) = 440 \text{ W}$.

E24-31 $K = (260 \text{ K})/(298 \text{ K} - 260 \text{ K}) = 6.8$.

E24-32 $K = (0.85)(270\text{K})/(299\text{K}-270\text{K}) = 7.91$. In 15 minutes the motor can do $(210 \text{ W})(900 \text{ s}) = 1.89 \times 10^5 \text{ J}$ of work. Then

$$|Q_L| = K|W| = (7.91)(1.89 \times 10^5 \text{ J}) = 1.50 \times 10^6 \text{ J}.$$

E24-33 The Carnot engine has an efficiency

$$\epsilon = 1 - \frac{T_2}{T_1} = \frac{|W|}{|Q_1|}.$$

The Carnot refrigerator has a coefficient of performance

$$K = \frac{T_4}{T_3 - T_4} = \frac{|Q_4|}{|W|}.$$

Lastly, $|Q_4| = |Q_3| - |W|$. We just need to combine these three expressions into one. Starting with the first, and solving for $|W|$,

$$|W| = |Q_1| \frac{T_1 - T_2}{T_1}.$$

Then we combine the last two expressions, and

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3| - |W|}{|W|} = \frac{|Q_3|}{|W|} - 1.$$

Finally, combine them all,

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3|}{|Q_1|} \frac{T_1}{T_1 - T_2} - 1.$$

Now, we rearrange,

$$\begin{aligned} \frac{|Q_3|}{|Q_1|} &= \left(\frac{T_4}{T_3 - T_4} + 1 \right) \frac{T_1 - T_2}{T_1}, \\ &= \left(\frac{T_3}{T_3 - T_4} \right) \frac{T_1 - T_2}{T_1}, \\ &= (1 - T_2/T_1)/(1 - T_4/T_3). \end{aligned}$$

E24-34 (a) Integrate:

$$\ln N! \approx \int_1^N \ln x \, dx = N \ln N - N + 1 \approx N \ln N - N.$$

(b) 91, 752, and about 615,000. You will need to use the Stirling approximation extended to a double inequality to do the last two:

$$\sqrt{2\pi n} n^{n+1/2} e^{-n+1/(12n+1)} < n! < \sqrt{2\pi n} n^{n+1/2} e^{-n+1/(12n)}.$$

E24-35 (a) For this problem we don't care how the particles are arranged inside a section, we only care how they are divided up between the two sides.

Consequently, there is only one way to arrange the particles: you put them all on one side, and you have no other choices. So the multiplicity in this case is one, or $w_1 = 1$.

(b) Once the particles are allowed to mix we have more work in computing the multiplicity. Using Eq. 24-19, we have

$$w_2 = \frac{N!}{(N/2)!(N/2)!} = \frac{N!}{((N/2)!)^2}$$

(c) The entropy of a state of multiplicity w is given by Eq. 24-20,

$$S = k \ln w$$

For part (a), with a multiplicity of 1, $S_1 = 0$. Now for part (b),

$$S_2 = k \ln \left(\frac{N!}{((N/2)!)^2} \right) = k \ln N! - 2k \ln(N/2)!$$

and we need to expand each of those terms with Stirling's approximation.

Combining,

$$\begin{aligned} S_2 &= k(N \ln N - N) - 2k((N/2) \ln(N/2) - (N/2)), \\ &= kN \ln N - kN - kN \ln(N/2) + kN, \\ &= kN \ln 2 \end{aligned}$$

Finally, $\Delta S = S_2 - S_1 = kN \ln 2$.

(d) The answer should be the same; it is a free expansion problem in both cases!

P24-1 We want to evaluate

$$\begin{aligned}\Delta S &= \int_{T_i}^{T_f} \frac{nC_V dT}{T}, \\ &= \int_{T_i}^{T_f} \frac{nAT^3 dT}{T}, \\ &= \int_{T_i}^{T_f} nAT^2 dT, \\ &= \frac{nA}{3} (T_f^3 - T_i^3).\end{aligned}$$

Into this last expression, which is true for many substances at sufficiently low temperatures, we substitute the given numbers.

$$\Delta S = \frac{(4.8 \text{ mol})(3.15 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4)}{3} ((10 \text{ K})^3 - (5.0 \text{ K})^3) = 4.41 \times 10^{-2} \text{ J/K}.$$

P24-2

P24-3 (a) Work is only done along path ab , where $W_{ab} = -p\Delta V = -3p_0\Delta V_0$. So $W_{abc} = -3p_0V_0$.

(b) $\Delta E_{intbc} = \frac{3}{2}nR\Delta T_{bc}$, with a little algebra,

$$\Delta E_{intbc} = \frac{3}{2}(nRT_c - nRT_b) = \frac{3}{2}(p_cV_c - p_bV_b) = \frac{3}{2}(8 - 4)p_0V_0 = 6p_0V_0.$$

$\Delta S_{bc} = \frac{3}{2}nR \ln(T_c/T_b)$, with a little algebra,

$$\Delta S_{bc} = \frac{3}{2}nR \ln(p_c/p_b) = \frac{3}{2}nR \ln 2.$$

(c) Both are zero for a cyclic process.

P24-4 (a) For an isothermal process,

$$p_2 = p_1(V_1/V_2) = p_1/3.$$

For an adiabatic process,

$$p_3 = p_1(V_1/V_2)^\gamma = p_1(1/3)^{1.4} = 0.215p_1.$$

For a constant volume process,

$$T_3 = T_2(p_3/p_2) = T_1(0.215/0.333) = 0.646T_1.$$

(b) The easiest ones first: $\Delta E_{int12} = 0$, $W_{23} = 0$, $Q_{31} = 0$, $\Delta S_{31} = 0$. The next easier ones:

$$\Delta E_{int23} = \frac{5}{2}nR\Delta T_{23} = \frac{5}{2}nR(0.646T_1 - T_1) = -0.885p_1V_1,$$

$$Q_{23} = \Delta E_{int23} - W_{23} = -0.885p_1V_1,$$

$$\Delta E_{int31} = -\Delta E_{int23} - \Delta E_{int12} = 0.885p_1V_1,$$

$$W_{31} = \Delta E_{int31} - Q_{31} = 0.885p_1V_1.$$

Finally, some harder ones:

$$W_{12} = -nRT_1 \ln(V_2/V_1) = -p_1 V_1 \ln(3) = -1.10p_1 V_1,$$

$$Q_{12} = \Delta E_{\text{int}12} - W_{12} = 1.10p_1 V_1.$$

And now, the hardest:

$$\Delta S_{12} = Q_{12}/T_1 = 1.10nR,$$

$$\Delta S_{23} = -\Delta S_{12} - \Delta S_{31} = -1.10nR.$$

P24-5 Note that $T_A = T_B = T_C/4 = T_D$.

Process I: ABC

$$(a) Q_{AB} = -W_{AB} = nRT_0 \ln(V_B/V_A) = p_0 V_0 \ln 2. \quad Q_{BC} = \frac{3}{2}nR(T_C - T_B) = \frac{3}{2}(p_C V_C - p_B V_B) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0.$$

$$(b) W_{AB} = -nRT_0 \ln(V_B/V_A) = -p_0 V_0 \ln 2. \quad W_{BC} = 0.$$

$$(c) E_{\text{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_C V_C - p_A V_A) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0.$$

$$(d) \Delta S_{AB} = nR \ln(V_B/V_A) = nR \ln 2; \quad \Delta S_{BC} = \frac{3}{2}nR \ln(T_C/T_B) = \frac{3}{2}nR \ln 4 = 3nR \ln 2. \quad \text{Then } \Delta S_{AC} = 4nR.$$

Process II: ADC

$$(a) Q_{AD} = -W_{AD} = nRT_0 \ln(V_D/V_A) = -p_0 V_0 \ln 2. \quad Q_{DC} = \frac{5}{2}nR(T_C - T_D) = \frac{5}{2}(p_C V_C - p_D V_D) = \frac{5}{2}(4p_0 V_0 - p_0 V_0) = 10p_0 V_0.$$

$$(b) W_{AB} = -nRT_0 \ln(V_D/V_A) = p_0 V_0 \ln 2. \quad W_{DC} = -p\Delta V = -p_0(2V_0 - V_0/2) = -\frac{3}{2}p_0 V_0.$$

$$(c) E_{\text{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_C V_C - p_A V_A) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0.$$

$$(d) \Delta S_{AD} = nR \ln(V_D/V_A) = -nR \ln 2; \quad \Delta S_{DC} = \frac{5}{2}nR \ln(T_C/T_D) = \frac{5}{2}nR \ln 4 = 5nR \ln 2. \quad \text{Then } \Delta S_{AC} = 4nR.$$

P24-6 The heat required to melt the ice is

$$\begin{aligned} Q &= m(c_w \Delta T_{23} + L + c_i \Delta T_{12}), \\ &= (0.0126 \text{ kg})[(4190 \text{ J/kg} \cdot \text{K})(15 \text{ C}^\circ) + (333 \times 10^3 \text{ J/kg}) + (2220 \text{ J/kg} \cdot \text{K})(10 \text{ C}^\circ)], \\ &= 5270 \text{ J}. \end{aligned}$$

The change in entropy of the ice is

$$\begin{aligned} \Delta S_i &= m[c_w \ln(T_3/T_2) + L/T_2 + c_i \ln(T_2/T_1)], \\ &= (0.0126 \text{ kg})[(4190 \text{ J/kg} \cdot \text{K}) \ln(288/273) + (333 \times 10^3 \text{ J/kg})/(273 \text{ K}), \\ &\quad + (2220 \text{ J/kg} \cdot \text{K}) \ln(273/263)], \\ &= 19.24 \text{ J/K} \end{aligned}$$

The change in entropy of the lake is $\Delta S_l = (-5270 \text{ J})/(288 \text{ K}) = 18.29 \text{ J/K}$. The change in entropy of the system is 0.95 J/kg .

P24-7 (a) This is a problem where the total internal energy of the two objects doesn't change, but since no work is done during the process, we can start with the simpler expression $Q_1 + Q_2 = 0$. The heat transfers by the two objects are

$$\begin{aligned} Q_1 &= m_1 c_1 (T_1 - T_{1,i}), \\ Q_2 &= m_2 c_2 (T_2 - T_{2,i}). \end{aligned}$$

Note that we don't call the final temperature T_f here, because we *are not* assuming that the two objects are at equilibrium.

We combine these three equations,

$$\begin{aligned} m_2 c_2 (T_2 - T_{2,i}) &= -m_1 c_1 (T_1 - T_{1,i}), \\ m_2 c_2 T_2 &= m_2 c_2 T_{2,i} + m_1 c_1 (T_{1,i} - T_1), \\ T_2 &= T_{2,i} + \frac{m_1 c_1}{m_2 c_2} (T_{1,i} - T_1) \end{aligned}$$

As object 1 “cools down”, object 2 “heats up”, as expected.

(b) The entropy change of *one* object is given by

$$\Delta S = \int_{T_i}^{T_f} \frac{mc dT}{T} = mc \ln \frac{T_f}{T_i},$$

and the total entropy change for the system will be the sum of the changes for each object, so

$$\Delta S = m_1 c_1 \ln \frac{T_1}{T_{1,i}} + m_2 c_2 \ln \frac{T_2}{T_{2,i}}.$$

Into the this last equation we need to substitute the expression for T_2 in as a function of T_1 . There’s no new physics in doing this, just a mess of algebra.

(c) We want to evaluate $d(\Delta S)/dT_1$. To save on algebra we will work with the last expression, remembering that T_2 is a function, not a variable. Then

$$\frac{d(\Delta S)}{dT_1} = \frac{m_1 c_1}{T_1} + \frac{m_2 c_2}{T_2} \frac{dT_2}{dT_1}.$$

We’ve saved on algebra, but now we need to evaluate dT_2/dT_1 . Starting with the results from part (a),

$$\begin{aligned} \frac{dT_2}{dT_1} &= \frac{d}{dT_1} \left(T_{2,i} + \frac{m_1 c_1}{m_2 c_2} (T_{1,i} - T_1) \right), \\ &= -\frac{m_1 c_1}{m_2 c_2}. \end{aligned}$$

Now we collect the two results and write

$$\begin{aligned} \frac{d(\Delta S)}{dT_1} &= \frac{m_1 c_1}{T_1} + \frac{m_2 c_2}{T_2} \left(-\frac{m_1 c_1}{m_2 c_2} \right), \\ &= m_1 c_1 \left(\frac{1}{T_1} - \frac{1}{T_2} \right). \end{aligned}$$

We could consider writing T_2 out in all of its glory, but what would it gain us? Nothing. There is actually considerably more physics in the expression as written, because...

(d) ...we get a maximum for ΔS when $d(\Delta S)/dT_1 = 0$, and this can only occur when $T_1 = T_2$ according to the expression.

P24-8 $T_b = (10.4 \times 1.01 \times 10^5 \text{ Pa})(1.22 \text{ m}^3)/(2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K}) = 7.71 \times 10^4 \text{ K}$. Maybe not so realistic? T_a can be found after finding

$$p_c = p_b (V_b/V_c)^\gamma = (10.4 \times 1.01 \times 10^5 \text{ Pa})(1.22/9.13)^{1.67} = 3.64 \times 10^4 \text{ Pa},$$

Then

$$T_a = T_b (p_a/p_b) = (7.71 \times 10^4 \text{ K})(3.64 \times 10^4/1.05 \times 10^6) = 2.67 \times 10^3 \text{ K}.$$

Similarly,

$$T_c = T_a(V_c/V_a) = (2.67 \times 10^3 \text{K})(9.13/1.22) = 2.00 \times 10^4 \text{K}.$$

(a) Heat is added during process ab only;

$$Q_{ab} = \frac{3}{2}nR(T_b - T_a) = \frac{3}{2}(2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(7.71 \times 10^4 \text{K} - 2.67 \times 10^3 \text{K}) = 1.85 \times 10^6 \text{J}.$$

(b) Heat is removed during process ca only;

$$Q_{ca} = \frac{5}{2}nR(T_a - T_c) = \frac{5}{2}(2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(2.67 \times 10^3 \text{K} - 2.00 \times 10^4 \text{K}) = -0.721 \times 10^6 \text{J}.$$

(c) $W = |Q_{ab}| - |Q_{ca}| = (1.85 \times 10^6 \text{J}) - (0.721 \times 10^6 \text{J}) = 1.13 \times 10^6 \text{J}.$

(d) $e = W/Q_{ab} = (1.13 \times 10^6)/(1.85 \times 10^6) = 0.611.$

P24-9 The pV diagram for this process is Figure 23-21, except the cycle goes clockwise.

(a) Heat is input during the constant volume heating and the isothermal expansion. During heating,

$$Q_1 = \frac{3}{2}nR\Delta T = \frac{3}{2}(1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K} - 300 \text{K}) = 3740 \text{J};$$

During isothermal expansion,

$$Q_2 = -W_2 = nRT \ln(V_f/V_i) = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K}) \ln(2) = 3460 \text{J};$$

so $Q_{\text{in}} = 7200 \text{J}.$

(b) Work is only done during the second and third processes; we've already solved the second, $W_2 = -3460 \text{J};$

$$W_3 = -p\Delta V = p_a V_c - p_a V_a = nR(T_c - T_a) = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K} - 300 \text{K}) = 2490 \text{J};$$

So $W = -970 \text{J}.$

(c) $e = |W|/|Q_{\text{in}}| = (970 \text{J})/(7200 \text{J}) = 0.13.$

P24-10 (a) $T_b = T_a(p_b/p_a) = 3T_a;$

$$T_c = T_b(V_b/V_c)^{\gamma-1} = 3T_a(1/4)^{0.4} = 1.72T_a;$$

$$p_c = p_b(V_b/V_c)^{\gamma} = 3p_a(1/4)^{1.4} = 0.430p_a;$$

$$T_d = T_a(V_a/V_d)^{\gamma-1} = T_a(1/4)^{0.4} = 0.574T_a;$$

$$p_d = p_a(V_a/V_d)^{\gamma} = p_a(1/4)^{1.4} = 0.144p_a.$$

(b) Heat in occurs during process ab , so $Q_i = \frac{5}{2}nR\Delta T_{ab} = 5nRT_a;$ Heat out occurs during process cd , so $Q_o = \frac{5}{2}nR\Delta T_{cd} = 2.87nRT_a.$ Then

$$e = 1 - (2.87nRT_a/5nRT_a) = 0.426.$$

P24-11 (c) $(V_B/V_A) = (p_A/p_B) = (0/0.5) = 2.$ The work done on the gas during the isothermal compression is

$$W = -nRT \ln(V_B/V_A) = -(1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{K}) \ln(2) = -1730 \text{J}.$$

Since $\Delta E_{\text{int}} = 0$ along an isotherm $Q_h = 1730 \text{J}.$

The cycle has an efficiency of $e = 1 - (100/300) = 2/3.$ Then for the cycle,

$$W = eQ_h = (2/3)(1730 \text{J}) = 1150 \text{J}.$$