$$H = kA \frac{\Delta T}{\Delta x}$$

The rate at which heat flows out is given as a power per area (mW/m^2) , so the quantity given is really H/A. Then the temperature difference is

$$\Delta T = \frac{H}{A} \frac{\Delta x}{k} = (0.054 \,\mathrm{W/m^2}) \frac{(33,000 \,\mathrm{m})}{(2.5 \,\mathrm{W/m \cdot K})} = 710 \,\mathrm{K}$$

The heat flow is out, so that the temperature is higher at the base of the crust. The temperature there is then

$$710 + 10 = 720$$
 °C.

E23-2 We apply Eq. 23-1,

$$H = kA \frac{\Delta T}{\Delta x} = (0.74 \,\text{W/m} \cdot \text{K})(6.2 \,\text{m})(3.8 \,\text{m}) \frac{(44 \,\text{C}^{\circ})}{(0.32 \,\text{m})} = 2400 \,\text{W}.$$

E23-3 (a) $\Delta T/\Delta x = (136 \, \text{C}^{\circ})/(0.249 \, \text{m}) = 546 \, \text{C}^{\circ}/\text{m}.$

- (b) $H = kA\Delta T/\Delta x = (401 \text{ W/m} \cdot \text{K})(1.80 \text{ m}^2)(546 \text{ C}^{\circ}/\text{m}) = 3.94 \times 10^5 \text{ W}.$
- (c) $T_{\rm H} = (-12^{\circ}{\rm C} + 136\,{\rm C}^{\circ}) = 124^{\circ}{\rm C}$. Then

$$T = (124^{\circ}\text{C}) - (546 \,\text{C}^{\circ}/\text{m})(0.11 \,\text{m}) = 63.9^{\circ}\text{C}.$$

E23-4 (a) $H = (0.040 \,\mathrm{W/m \cdot K})(1.8 \,\mathrm{m}^2)(32 \,\mathrm{C}^{\circ})/(0.012 \,\mathrm{m}) = 190 \,\mathrm{W}.$

(b) Since k has increased by a factor of (0.60)/(0.04) = 15 then H should also increase by a factor of 15.

E23-5 There are three possible arrangements: a sheet of type 1 with a sheet of type 1; a sheet of type 2 with a sheet of type 2; and a sheet of type 1 with a sheet of type 2. We can look back on Sample Problem 23-1 to see how to start the problem; the heat flow will be

$$H_{12} = \frac{A\Delta T}{(L/k_1) + (L/k_2)}$$

for substances of different types; and

$$H_{11} = \frac{A\Delta T/L}{(L/k_1) + (L/k_1)} = \frac{1}{2} \frac{A\Delta T k_1}{L}$$

for a double layer if substance 1. There is a similar expression for a double layer of substance 2. For configuration (a) we then have

$$H_{11} + H_{22} = \frac{1}{2} \frac{A\Delta T k_1}{L} + \frac{1}{2} \frac{A\Delta T k_2}{L} = \frac{A\Delta T}{2L} (k_1 + k_2),$$

while for configuration (b) we have

$$H_{12} + H_{21} = 2 \frac{A\Delta T}{(L/k_1) + (L/k_2)} = \frac{2A\Delta T}{L} ((1/k_1) + (1/k_2))^{-1}.$$

We want to compare these, so expanding the relevant part of the second configuration

$$((1/k_1) + (1/k_2))^{-1} = ((k_1 + k_2)/(k_2k_2))^{-1} = \frac{k_1k_2}{k_1 + k_2}.$$

Then which is larger

$$(k_1 + k_2)/2$$
 or $\frac{2k_1k_2}{k_1 + k_2}$?

If $k_1 \gg k_2$ then the expression become

$$k_1/2$$
 and $2k_2$,

so the first expression is larger, and therefore configuration (b) has the lower heat flow. Notice that we get the same result if $k_1 \ll k_2$!

E23-6 There's a typo in the exercise.

 $H = A\Delta T/R$; since the heat flows through one slab and then through the other, we can write $(T_1 - T_x)/R_1 = (T_x - T_2)/R_2$. Rearranging,

$$T_x = (T_1 R_2 + T_2 R_1)/(R_1 + R_2).$$

E23-7 Use the results of Exercise 23-6. At the interface between ice and water $T_x = 0$ °C. Then $R_1T_2 + R_2T_1 = 0$, or $k_1T_1/L_1 + k_2T_2/L_2 = 0$. Not only that, $L_1 + L_2 = L$, so

$$k_1T_1L_2 + (L - L_2)k_2T_2 = 0,$$

so

$$L_2 = \frac{(1.42 \,\mathrm{m})(1.67 \,\mathrm{W/m \cdot K})(-5.20 \,^{\circ}\mathrm{C})}{(1.67 \,\mathrm{W/m \cdot K})(-5.20 \,^{\circ}\mathrm{C}) - (0.502 \,\mathrm{W/m \cdot K})(3.98 \,^{\circ}\mathrm{C})} = 1.15 \,\mathrm{m}.$$

E23-8 ΔT is the same in both cases. So is k. The top configuration has $H_{\rm t}=kA\Delta T/(2L)$. The bottom configuration has $H_{\rm b}=k(2A)\Delta T/L$. The ratio of $H_{\rm b}/H_{\rm t}=4$, so heat flows through the bottom configuration at 4 times the rate of the top. For the top configuration $H_{\rm t}=(10\,{\rm J})/(2\,{\rm min})=5\,{\rm J/min}$. Then $H_{\rm b}=20\,{\rm J/min}$. It will take

$$t = (30 \,\mathrm{J})/(20 \,\mathrm{J/min}) = 1.5 \,\mathrm{min}.$$

E23-9 (a) This exercise has a distraction: it asks about the heat flow through the window, but what you need to find first is the heat flow through the air near the window. We are given the temperature gradient both inside and outside the window. Inside,

$$\frac{\Delta T}{\Delta x} = \frac{(20^{\circ}\text{C}) - (5^{\circ}\text{C})}{(0.08\,\text{m})} = 190\,\text{C}^{\circ}/\text{m};$$

a similar expression exists for outside.

From Eq. 23-1 we find the heat flow through the air;

$$H = kA \frac{\Delta T}{\Delta x} = (0.026 \,\mathrm{W/m \cdot K})(0.6 \,\mathrm{m})^2 (190 \,\mathrm{C^{\circ}/m}) = 1.8 \,\mathrm{W}.$$

The value that we arrived at is the rate that heat flows through the air across an area the size of the window on either side of the window. This heat flow had to occur through the window as well, so

$$H = 1.8 \, \text{W}$$

answers the window question.

(b) Now that we know the rate that heat flows through the window, we are in a position to find the temperature difference across the window. Rearranging Eq. 32-1,

$$\Delta T = \frac{H\Delta x}{kA} = \frac{(1.8\,\mathrm{W})(0.005\,\mathrm{m})}{(1.0\,\mathrm{W/m\cdot K})(0.6\,\mathrm{m})^2} = 0.025\,\mathrm{C}^{\circ},$$

so we were well justified in our approximation that the temperature drop across the glass is very small.

E23-10 (a) $W = +214 \,\text{J}$, done on means positive.

- (b) $Q = -293 \,\mathrm{J}$, extracted from means negative.
- (c) $\Delta E_{\text{int}} = Q + W = (-293 \text{ J}) + (+214 \text{ J}) = -79.0 \text{ J}.$
- **E23-11** (a) ΔE_{int} along any path between these two points is

$$\Delta E_{\text{int}} = Q + W = (50 \,\text{J}) + (-20 \,\text{J}) = 30 \,\text{J}.$$

Then along ibf W = (30 J) - (36 J) = -6 J.

- (b) Q = (-30 J) (+13 J) = -43 J.
- (c) $E_{\text{int},f} = E_{\text{int},i} + \Delta E_{\text{int}} = (10 \text{ J}) + (30 \text{ J}) = 40 \text{ J}.$
- (d) $\Delta E_{\text{int}ib} = (22 \text{ J}) (10 \text{ J}) = 12 \text{ J}$; while $\Delta E_{\text{int}bf} = (40 \text{ J}) (22 \text{ J}) = 18 \text{ J}$. There is no work done on the path bf, so

$$Q_{bf} = \Delta E_{\text{int}bf} - W_{bf} = (18 \text{ J}) - (0) = 18 \text{ J},$$

and
$$Q_{ib} = Q_{ibf} - Q_{bf} = (36 \text{ J}) - (18 \text{ J}) = 18 \text{ J}.$$

E23-12
$$Q = mL = (0.10)(2.1 \times 10^8 \text{ kg})(333 \times 10^3 \text{ J/kg}) = 7.0 \times 10^{12} \text{ J}.$$

E23-13 We don't need to know the outside temperature because the amount of heat energy required is explicitly stated: 5.22 GJ. We just need to know how much water is required to transfer this amount of heat energy. Use Eq. 23-11, and then

$$m = \frac{Q}{c\Delta T} = \frac{(5.22 \times 10^9 \text{ J})}{(4190 \text{ J/kg} \cdot \text{K})(50.0^{\circ}\text{C} - 22.0^{\circ}\text{C})} = 4.45 \times 10^4 \text{ kg}.$$

This is the mass of the water, we want to know the volume, so we'll use the density, and then

$$V = \frac{m}{\rho} = \frac{(4.45 \times 10^4 \,\mathrm{kg})}{(998 \,\mathrm{kg/m^3})} = 44.5 \,\mathrm{m}^3.$$

E23-14 The heat energy required is $Q = mc\Delta T$. The time required is t = Q/P. Then

$$t = \frac{(0.136 \,\mathrm{kg})(4190 \,\mathrm{J/kg \cdot K})(100^{\circ}\mathrm{C} - 23.5^{\circ}\mathrm{C})}{(220 \,\mathrm{W})} = 198 \,\mathrm{s}.$$

E23-15 Q = mL, so $m = (50.4 \times 10^3 \text{J})/(333 \times 10^3 \text{J/kg}) = 0.151 \text{ kg}$ is the mount which freezes. Then (0.258 kg) - (0.151 kg) = 0.107 kg is the amount which does not freeze.

E23-16 (a) $W = mg\Delta y$; if |Q| = |W|, then

$$\Delta T = \frac{mg\Delta y}{mc} = \frac{(9.81 \,\mathrm{m/s^2})(49.4 \,\mathrm{m})}{(4190 \,\mathrm{J/kg \cdot K})} = 0.112 \,\mathrm{C}^{\circ}.$$

E23-17 There are three "things" in this problem: the copper bowl (b), the water (w), and the copper cylinder (c). The total internal energy changes must add up to zero, so

$$\Delta E_{\text{int,b}} + \Delta E_{\text{int,w}} + \Delta E_{\text{int,c}} = 0.$$

As in Sample Problem 23-3, no work is done on any object, so

$$Q_{\rm b} + Q_{\rm w} + Q_{\rm c} = 0.$$

The heat transfers for these three objects are

$$\begin{split} Q_{\rm b} &= m_{\rm b} c_{\rm b}(T_{\rm f,b} - T_{\rm i,b}), \\ Q_{\rm w} &= m_{\rm w} c_{\rm w}(T_{\rm f,w} - T_{\rm i,w}) + L_{\rm v} m_2, \\ Q_{\rm c} &= m_{\rm c} c_{\rm c}(T_{\rm f,c} - T_{\rm i,c}). \end{split}$$

For the most part, this looks exactly like the presentation in Sample Problem 23-3; but there is an extra term in the second line. This term reflects the extra heat required to vaporize $m_2 = 4.70$ g of water at 100° C into steam 100° C.

Some of the initial temperatures are specified in the exercise: $T_{\rm i,b} = T_{\rm i,w} = 21.0^{\circ} \rm C$ and $T_{\rm f,b} = T_{\rm f,w} = T_{\rm f,c} = 100^{\circ} \rm C$.

(a) The heat transferred to the water, then, is

$$Q_{\rm w} = (0.223 \,{\rm kg})(4190 \,{\rm J/kg \cdot K}) ((100^{\circ}{\rm C}) - (21.0^{\circ}{\rm C})) ,$$

 $+ (2.26 \times 10^{6} \,{\rm J/kg})(4.70 \times 10^{-3} \,{\rm kg}),$
 $= 8.44 \times 10^{4} \,{\rm J}.$

This answer differs from the back of the book. I think that they (or was it me) used the latent heat of fusion when they should have used the latent heat of vaporization!

(b) The heat transferred to the bowl, then, is

$$Q_{\rm w} = (0.146 \,{\rm kg})(387 \,{\rm J/kg \cdot K}) ((100^{\circ}{\rm C}) - (21.0^{\circ}{\rm C})) = 4.46 \times 10^{3} \,{\rm J}.$$

(c) The heat transferred from the cylinder was transferred into the water and bowl, so

$$Q_{\rm c} = -Q_{\rm b} - Q_{\rm w} = -(4.46 \times 10^3 \,\mathrm{J}) - (8.44 \times 10^4 \,\mathrm{J}) = -8.89 \times 10^4 \,\mathrm{J}.$$

The initial temperature of the cylinder is then given by

$$T_{\rm i,c} = T_{\rm f,c} - \frac{Q_{\rm c}}{m_c c_c} = (100^{\circ} \text{C}) - \frac{(-8.89 \times 10^4 \text{ J})}{(0.314 \text{ kg})(387 \text{ J/kg} \cdot \text{K})} = 832^{\circ} \text{C}.$$

E23-18 The temperature of the silver must be raised to the melting point and then the heated silver needs to be melted. The heat required is

$$Q = mL + mc\Delta T = (0.130 \text{ kg})[(105 \times 10^3 \text{J/kg}) + (236 \text{J/kg} \cdot \text{K})(1235 \text{ K} - 289 \text{ K})] = 4.27 \times 10^4 \text{J}.$$

E23-19 (a) Use
$$Q = mc\Delta T$$
, $m = \rho V$, and $t = Q/P$. Then

$$t = \frac{[m_{\rm a}c_{\rm a} + \rho_{\rm w}V_{\rm w}c_{\rm w}]\Delta T}{P},$$

$$= \frac{[(0.56\,{\rm kg})(900\,{\rm J/kg\cdot K}) + (998\,{\rm kg/m^3})(0.64\times10^{-3}{\rm m^3})(4190\,{\rm J/kg\cdot K})](100^{\circ}{\rm C} - 12^{\circ}{\rm C})}{(2400\,{\rm W})} = 117\,{\rm s}.$$

(b) Use Q = mL, $m = \rho V$, and t = Q/P. Then

$$t = \frac{\rho_{\rm w} V_{\rm w} L_{\rm w}}{P} = \frac{(998 \,\text{kg/m}^3)(0.640 \times 10^{-3} \,\text{m}^3)(2256 \times 10^3 \,\text{J/kg})}{(2400 \,\text{W})} = 600 \,\text{s}$$

is the additional time required.

E23-20 The heat given off by the steam will be

$$Q_{\rm s} = m_{\rm s} L_{\rm v} + m_{\rm s} c_{\rm w} (50 \, {\rm C}^{\circ}).$$

The hear taken in by the ice will be

$$Q_{\rm i} = m_{\rm i} L_{\rm f} + m_{\rm i} c_{\rm w} (50 \, {\rm C}^{\circ}).$$

Equating,

$$m_{\rm s} = mrm_i \frac{L_{\rm f} + c_{\rm w}(50 \,{\rm C}^{\circ})}{L_{\rm v} + c_{\rm w}(50 \,{\rm C}^{\circ})},$$

$$= (0.150 \,{\rm kg}) \frac{(333 \times 10^3 \,{\rm J/kg}) + (4190 \,{\rm J/kg \cdot K})(50 \,{\rm C}^{\circ})}{(2256 \times 10^3 \,{\rm J/kg}) + (4190 \,{\rm J/kg \cdot K})(50 \,{\rm C}^{\circ})} = 0.033 \,{\rm kg}.$$

E23-21 The linear dimensions of the ring and sphere change with the temperature change according to

$$\Delta d_{\rm r} = \alpha_{\rm r} d_{\rm r} (T_{\rm f,r} - T_{\rm i,r}),$$

$$\Delta d_{\rm s} = \alpha_{\rm s} d_{\rm s} (T_{\rm f,s} - T_{\rm i,s}).$$

When the ring and sphere are at the same (final) temperature the ring and the sphere have the same diameter. This means that

$$d_{\rm r} + \Delta d_{\rm r} = d_{\rm s} + \Delta d_{\rm s}$$

when $T_{\rm f,s} = T_{\rm f,r}$. We'll solve these expansion equations first, and then go back to the heat equations.

$$d_{\rm r} + \Delta d_{\rm r} = d_{\rm s} + \Delta d_{\rm s},$$

$$d_{\rm r} (1 + \alpha_{\rm r} (T_{\rm f.r} - T_{\rm i.r})) = d_{\rm s} (1 + \alpha_{\rm s} (T_{\rm f.s} - T_{\rm i.s})),$$

which can be rearranged to give

$$\alpha_{\rm r} d_{\rm r} T_{\rm f,r} - \alpha_{\rm s} d_{\rm s} T_{\rm f,s} = d_{\rm s} \left(1 - \alpha_{\rm s} T_{\rm i,s} \right) - d_{\rm r} \left(1 - \alpha_{\rm r} T_{\rm i,r} \right),$$

but since the final temperatures are the same,

$$T_{\rm f} = \frac{d_{\rm s} \left(1 - \alpha_{\rm s} T_{\rm i,s}\right) - d_{\rm r} \left(1 - \alpha_{\rm r} T_{\rm i,r}\right)}{\alpha_{\rm r} d_{\rm r} - \alpha_{\rm s} d_{\rm s}}$$

Putting in the numbers,

$$\begin{split} T_{\mathrm{f}} &= \\ & \frac{(2.54533 \mathrm{cm})[1 - (23 \times 10^{-6}/\mathrm{C}^{\circ})(100^{\circ}\mathrm{C})] - (2.54000 \mathrm{cm})[1 - (17 \times 10^{-6}/\mathrm{C}^{\circ})(0^{\circ}\mathrm{C})]}{(2.54000 \mathrm{cm})(17 \times 10^{-6}/\mathrm{C}^{\circ}) - (2.54533 \mathrm{cm})(23 \times 10^{-6}/\mathrm{C}^{\circ})}, \\ &= 34.1^{\circ}\mathrm{C}. \end{split}$$

No work is done, so we only have the issue of heat flow, then

$$Q_{\rm r} + Q_{\rm s} = 0.$$

Where "r" refers to the copper ring and "s" refers to the aluminum sphere. The heat equations are

$$Q_{\rm r} = m_{\rm r} c_{\rm r} (T_{\rm f} - T_{\rm i,r}),$$

 $Q_{\rm s} = m_{\rm s} c_{\rm s} (T_{\rm f} - T_{\rm i,s}).$

Equating and rearranging,

$$m_{\rm s} = \frac{m_{\rm r}c_{\rm r}(T_{\rm i,r} - T_{\rm f})}{c_{\rm s}(T_{\rm f} - T_{\rm i,s})}$$

or

$$m_{\rm s} = \frac{(21.6 \text{ g})(387 \text{ J/kg} \cdot \text{K})(0^{\circ}\text{C} - 34.1^{\circ}\text{C})}{(900 \text{ J/kg} \cdot \text{K})(34.1^{\circ}\text{C} - 100^{\circ}\text{C})} = 4.81 \text{ g}.$$

E23-22 The problem is compounded because we don't know if the final state is only water, only ice, or a mixture of the two.

Consider first the water. Cooling it to 0°C would require the removal of

$$Q_{\rm w} = (0.200 \,\mathrm{kg})(4190 \,\mathrm{J/kg \cdot K})(0^{\circ}\mathrm{C} - 25^{\circ}\mathrm{C}) = -2.095 \times 10^{4}\mathrm{J}.$$

Consider now the ice. Warming the ice to would require the addition of

$$Q_{\rm i} = (0.100 \,\mathrm{kg})(2220 \,\mathrm{J/kg \cdot K})(0^{\circ}\mathrm{C} + 15^{\circ}\mathrm{C}) = 3.33 \times 10^{3}\mathrm{J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_{\rm im} = (0.100 \,\mathrm{kg})(333 \times 10^3 \,\mathrm{J/kg}) = 3.33 \times 10^4 \,\mathrm{J}.$$

This is far more than will be liberated by the cooling water, so the final temperature is 0°C, and consists of a mixture of ice and water.

(b) Consider now the ice. Warming the ice to would require the addition of

$$Q_i = (0.050 \text{ kg})(2220 \text{ J/kg} \cdot \text{K})(0^{\circ}\text{C} + 15^{\circ}\text{C}) = 1.665 \times 10^3 \text{J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_{\rm im} = (0.050 \,\mathrm{kg})(333 \times 10^3 \,\mathrm{J/kg}) = 1.665 \times 10^4 \,\mathrm{J}.$$

This is still not enough to cool the water to freezing. Hence, we need to solve

$$Q_{\rm i} + Q_{\rm im} + m_{\rm i} c_{\rm w} (T - 0^{\circ} \text{C}) + m_{\rm w} c_{\rm w} (T - 25^{\circ} \text{C}) = 0,$$

which has solution

$$T = \frac{(4190 \,\mathrm{J/kg \cdot K})(0.200 \,\mathrm{kg})(25^{\circ}\mathrm{C}) - (1.665 \times 10^{3}\mathrm{J}) + (1.665 \times 10^{4}\mathrm{J})}{(4190 \,\mathrm{J/kg \cdot K})(0.250 \,\mathrm{kg})} = 2.5^{\circ}\mathrm{C}.$$

E23-23 (a) $c = (320 \text{ J})/(0.0371 \text{ kg})(42.0^{\circ}\text{C} - 26.1^{\circ}\text{C}) = 542 \text{ J/kg} \cdot \text{K}.$

- (b) n = m/M = (37.1 g)/(51.4 g/mol) = 0.722 mol.
- (c) $c = (542 \text{ J/kg} \cdot \text{K})(51.4 \times 10^{-3} \text{ kg/mol}) = 27.9 \text{ J/mol} \cdot \text{K}.$

E23-24 (1) $W = -p\Delta V = (15 \,\text{Pa})(4 \,\text{m}^3) = -60 \,\text{J}$ for the horizontal path; no work is done during the vertical path; the net work done on the gas is $-60 \,\text{J}$.

- (2) It is easiest to consider work as the (negative of) the area under the curve; then $W = -(15 \,\mathrm{Pa} + 5 \,\mathrm{Pa})(4 \,\mathrm{m}^3)/2 = -40 \,\mathrm{J}$.
- (3) No work is done during the vertical path; $W = -p\Delta V = (5 \,\mathrm{Pa})(4 \,\mathrm{m}^3) = -20 \,\mathrm{J}$ for the horizontal path; the net work done on the gas is $-20 \,\mathrm{J}$.

E23-25 Net work done on the gas is given by Eq. 23-15,

$$W = -\int p \, dV.$$

But integrals are just the area under the curve; and that's the easy way to solve this problem. In the case of closed paths, it becomes the area inside the curve, with a clockwise sense giving a positive value for the integral.

The magnitude of the area is the same for either path, since it is a rectangle divided in half by a square. The area of the rectangle is

$$(15 \times 10^3 \text{Pa})(6 \text{ m}^3) = 90 \times 10^3 \text{J},$$

so the area of path 1 (counterclockwise) is -45 kJ; this means the work done on the gas is -(-45 kJ) or 45 kJ. The work done on the gas for path 2 is the negative of this because the path is clockwise.

E23-26 During the isothermal expansion,

$$W_1 = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{p_1}{p_2}.$$

During cooling at constant pressure,

$$W_2 = -p_2 \Delta V = -p_2 (V_1 - V_2) = -p_2 V_1 (1 - p_1/p_2) = V_1 (p_1 - p_2).$$

The work done is the sum, or

$$-(204 \times 10^{3} Pa)(0.142 m^{3}) \ln \frac{(204 \times 10^{3} Pa)}{(101 \times 10^{3} Pa)} + (0.142 m^{3})(103 Pa) = -5.74 \times 10^{3} J.$$

E23-27 During the isothermal expansion,

$$W = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{V_2}{V_1},$$

so

$$W = -(1.32)(1.01 \times 10^5 \text{Pa})(0.0224 \text{ m}^3) \ln \frac{(0.0153 \text{ m}^3)}{(0.0224 \text{ m}^3)} = 1.14 \times 10^3 \text{J}.$$

E23-28 (a) pV^{γ} is a constant, so

$$p_2 = p_1 (V_1/V_2)^{\gamma} = (1.00 \text{ atm})[(1 \text{ l})/(0.5 \text{ l})]^{1.32} = 2.50 \text{ atm};$$

 $T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (273 \text{ K}) \frac{(2.50 \text{ atm})}{(1.00 \text{ atm})} \frac{(0.5 \text{ l})}{(1 \text{ l})} = 341 \text{ K}.$$

(b) $V_3 = V_2(p_2/p_1)(T_3/T_2)$, so

$$V_3 = (0.5 \text{ l}) \frac{(273 \text{ K})}{(341 \text{ K})} = 0.40 \text{ l}.$$

(c) During the adiabatic process.

$$W_{12} = \frac{(1.01 \times 10^5 \mathrm{Pa/atm})(1 \times 10^{-3} \mathrm{m}^3 / \mathrm{l})}{(1.32) - 1} [(2.5 \mathrm{~atm})(0.5 \mathrm{~l}) - (1.0 \mathrm{~atm})(1 \mathrm{~l})] = 78.9 \, \mathrm{J}.$$

During the cooling process.

$$W_{23} = -p\Delta V = -(1.01\times10^5 \mathrm{Pa/atm})(2.50~\mathrm{atm})(1\times10^{-3}\mathrm{m^3/l})[(0.4~\mathrm{l}) - (0.5~\mathrm{l})] = 25.2~\mathrm{J}.$$

The net work done is $W_{123} = 78.9 \,\mathrm{J} + 25.2 \,\mathrm{J} = 104.1 \,\mathrm{J}$.

E23-29 (a) According to Eq. 23-20,

$$p_{\rm f} = \frac{p_{\rm i} V_{\rm i}^{\gamma}}{V_{\rm f}^{\gamma}} = \frac{(1.17 \text{ atm})(4.33 \text{ L})^{(1.40)}}{(1.06 \text{ L})^{(1.40)}} = 8.39 \text{ atm.}$$

(b) The final temperature can be found from the ideal gas law.

$$T_{\rm f} = T_{\rm i} \frac{p_{\rm f} V_{\rm f}}{p_{\rm i} V_{\rm i}} = (310 \, {\rm K}) \frac{(8.39 \, {\rm atm})(1.06 \, {\rm L})}{(1.17 \, {\rm atm})(4.33 \, {\rm L})} = 544 \, {\rm K}.$$

(c) The work done (for an adiabatic process) is given by Eq. 23-22,

$$W = \frac{1}{(1.40) - 1} \left[(8.39 \times 1.01 \times 10^{5} \text{Pa}) (1.06 \times 10^{-3} \text{m}^{3}) - (1.17 \times 1.01 \times 10^{5} \text{Pa}) (4.33 \times 10^{-3} \text{m}^{3}) \right],$$

= 966 J.

E23-30 Air is mostly diatomic (N_2 and O_2), so use $\gamma = 1.4$.

(a) pV^{γ} is a constant, so

$$V_2 = V_1 \sqrt[7]{p_1/p_2} = V_1 \sqrt[1.4]{(1.0 \text{ atm})/(2.3 \text{ atm})} = 0.552V_1.$$

 $T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (291 \text{ K}) \frac{(2.3 \text{ atm})}{(1.0 \text{ atm})} \frac{(0.552V_1)}{V_1} = 369 \text{ K},$$

or 96° C.

(b) The work required for delivering 1 liter of compressed air is

$$W_{12} = \frac{(1.01 \times 10^5 \,\mathrm{Pa/atm})(1 \times 10^{-3} \,\mathrm{m}^3/\mathrm{l})}{(1.40) - 1} [(2.3 \,\mathrm{atm})(1.0 \,\mathrm{l}) - (1.0 \,\mathrm{atm})(1.0 \,\mathrm{l}/0.552)] = 123 \,\mathrm{J}.$$

The number of liters per second that can be delivered is then

$$\Delta V/\Delta t = (230 \,\text{W})/(123 \,\text{J/l}) = 1.87 \,\text{l}.$$

E23-31 $E_{\text{int,rot}} = nRT = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(298 \text{ K}) = 2480 \text{ J}.$

E23-32 $E_{\text{int,rot}} = \frac{3}{2}nRT = (1.5)(1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(523 \text{ K}) = 6520 \text{ J}.$

E23-33 (a) Invert Eq. 32-20,

$$\gamma = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} = \frac{\ln(122 \text{ kPa/1450 kPa})}{\ln(1.36 \text{ m}^3/10.7 \text{ m}^3)} = 1.20.$$

(b) The final temperature is found from the ideal gas law,

$$T_{\rm f} = T_{\rm i} \frac{p_{\rm f} V_{\rm f}}{p_{\rm i} V_{\rm i}} = (250 \,\mathrm{K}) \frac{(1450 \times 10^3 \,\mathrm{Pa})(1.36 \,\mathrm{m}^3)}{(122 \times 10^3 \,\mathrm{Pa})(10.7 \,\mathrm{m}^3)} = 378 \,\mathrm{K},$$

which is the same as 105°C.

(c) Ideal gas law, again:

$$n = [pV]/[RT] = [(1450 \times 10^3 \text{Pa})(1.36 \text{ m}^3)]/[(8.31 \text{ J/mol} \cdot \text{K})(378 \text{ K})] = 628 \text{ mol}.$$

(d) From Eq. 23-24,

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(250 \text{ K}) = 1.96 \times 10^6 \text{J}$$

before the compression and

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(378 \text{ K}) = 2.96 \times 10^6 \text{ J}$$

after the compression.

(e) The ratio of the rms speeds will be proportional to the square root of the ratio of the internal energies,

$$\sqrt{(1.96 \times 10^6 \text{J})/(2.96 \times 10^6 \text{J})} = 0.813;$$

we can do this because the number of particles is the same before and after, hence the ratio of the energies per particle is the same as the ratio of the total energies.

E23-34 We can assume neon is an ideal gas. Then $\Delta T = 2\Delta E_{\rm int}/3nR$, or

$$\Delta T = \frac{2(1.34 \times 10^{12} \text{eV})(1.6 \times 10^{-19} \text{J/eV})}{3(0.120 \text{ mol})(8.31 \text{J/mol} \cdot \text{K})} = 1.43 \times 10^{-7} \text{J}.$$

E23-35 At constant pressure, doubling the volume is the same as doubling the temperature. Then

$$Q = nC_p\Delta T = (1.35 \text{ mol})\frac{7}{2}(8.31 \text{ J/mol} \cdot \text{K})(568 \text{ K} - 284 \text{ K}) = 1.12 \times 10^4 \text{J}.$$

E23-36 (a) n = m/M = (12 g)/(28 g/mol) = 0.429 mol.

(b) This is a constant volume process, so

$$Q = nC_V \Delta T = (0.429 \text{ mol}) \frac{5}{2} (8.31 \text{ J/mol} \cdot \text{K}) (125^{\circ}\text{C} - 25^{\circ}\text{C}) = 891 \text{J}.$$

E23-37 (a) From Eq. 23-37,

$$Q = nc_p \Delta T = (4.34 \text{ mol})(29.1 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 7880 \text{ J}.$$

(b) From Eq. 23-28,

$$E_{\text{int}} = \frac{5}{2} nR\Delta T = \frac{5}{2} (4.34 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 5630 \text{ J}.$$

(c) From Eq. 23-23,

$$K_{\text{trans}} = \frac{3}{2} nR\Delta T = \frac{5}{2} (4.34 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 3380 \text{ J}.$$

E23-38 $c_V = \frac{3}{2}(8.31 \,\mathrm{J/mol\cdot K})/(4.00 \,\mathrm{g/mol}) = 3120 \,\mathrm{J/kg\cdot K}.$

E23-39 Each species will experience the same temperature change, so

$$Q = Q_1 + Q_2 + Q_3,$$

= $n_1 C_1 \Delta T + n_2 C_2 \Delta T + n_3 C_3 \Delta T$,

Dividing this by $n = n_1 + n_2 + n_3$ and ΔT will return the specific heat capacity of the mixture, so

$$C = \frac{n_1 C_1 + n_2 C_2 + n_3 C_3}{n_1 + n_2 + n_3}.$$

E23-40 $W_{AB}=0$, since it is a constant volume process, consequently, $W=W_{AB}+W_{ABC}=-15\,\mathrm{J}$. But around a closed path Q=-W, so $Q=15\,\mathrm{J}$. Then

$$Q_{CA} = Q - Q_{AB} - Q_{BC} = (15 \text{ J}) - (20 \text{ J}) - (0 \text{ J}) = -5 \text{ J}.$$

Note that this heat is removed from the system!

E23-41 According to Eq. 23-25 (which is specific to ideal gases),

$$\Delta E_{\rm int} = \frac{3}{2} nR \Delta T,$$

and for an isothermal process $\Delta T = 0$, so for an ideal gas $\Delta E_{\rm int} = 0$. Consequently, Q + W = 0 for an ideal gas which undergoes an isothermal process.

But we know W for an isotherm, Eq. 23-18 shows

$$W = -nRT \ln \frac{V_{\rm f}}{V_{\rm f}}$$

Then finally

$$Q = -W = nRT \ln \frac{V_{\rm f}}{V_{\rm i}}$$

E23-42 Q is greatest for constant pressure processes and least for adiabatic. W is greatest (in magnitude, it is negative for increasing volume processes) for constant pressure processes and least for adiabatic. $\Delta E_{\rm int}$ is greatest for constant pressure (for which it is positive), and least for adiabatic (for which is is negative).

E23-43 (a) For a monatomic gas, $\gamma = 1.667$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma - 1} = (292 \,\mathrm{K})[(1)(1/10)]^{1.667 - 1} = 1360 \,\mathrm{K}.$$

(b) For a diatomic gas, $\gamma = 1.4$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma - 1} = (292 \,\mathrm{K})[(1)(1/10)]^{1.4 - 1} = 733 \,\mathrm{K}.$$

E23-44 This problem cannot be solved without making some assumptions about the type of process occurring on the two curved portions.

E23-45 If the pressure and volume are both doubled along a straight line then the process can be described by

$$p = \frac{p_1}{V_1}V$$

The final point involves the doubling of both the pressure and the volume, so according to the ideal gas law, pV = nRT, the final temperature T_2 will be four times the initial temperature T_1 .

Now for the exercises.

(a) The work done on the gas is

$$W = -\int_{1}^{2} p \, dV = -\int_{1}^{2} \frac{p_{1}}{V_{1}} V \, dV = -\frac{p_{1}}{V_{1}} \left(\frac{V_{2}^{2}}{2} - \frac{V_{1}^{2}}{2} \right)$$

We want to express our answer in terms of T_1 . First we take advantage of the fact that $V_2 = 2V_1$, then

$$W = -\frac{p_1}{V_1} \left(\frac{4V_1^2}{2} - \frac{V_1^2}{2} \right) = -\frac{3}{2} p_1 V_1 = -\frac{3}{2} nRT_1$$

(b) The nice thing about $\Delta E_{\rm int}$ is that it is path independent, we care only of the initial and final points. From Eq. 23-25,

$$\Delta E_{\text{int}} = \frac{3}{2} nR \Delta T = \frac{3}{2} nR (T_2 - T_1) = \frac{9}{2} nR T_1$$

(c) Finally we are in a position to find Q by applying the first law,

$$Q = \Delta E_{\text{int}} - W = \frac{9}{2}nRT_1 + \frac{3}{2}nRT_1 = 6nRT_1.$$

(d) If we define specific heat as heat divided by temperature change, then

$$c = \frac{Q}{n\Delta T} = \frac{6RT_1}{4T_1 - T_1} = 2R.$$

E23-46 The work done is the area enclosed by the path. If the pressure is measured in units of 10MPa, then the shape is a semi-circle, and the area is

$$W = (\pi/2)(1.5)^2(10\text{MPa})(1\times10^{-3}\text{m}^3) = 3.53\times10^4\text{J}.$$

The heat is given by $Q = -W = -3.53 \times 10^4 \,\mathrm{J}.$

E23-47 (a) Internal energy changes according to $\Delta E_{\rm int} = Q + W$, so

$$\Delta E_{\text{int}} = (20.9 \,\text{J}) - (1.01 \times 10^5 \,\text{Pa})(113 \times 10^{-6} \,\text{m}^3 - 63 \times 10^{-6} \,\text{m}^3) = 15.9 \,\text{J}.$$

(b) $T_1 = p_1 V_1 / nR$ and $T_2 = p_2 V_2 / nR$, but p is constant, so $\Delta T = p \Delta V / nR$. Then

$$C_P = \frac{Q}{n\Delta T} = \frac{QR}{p\Delta V} = \frac{(20.9\,\mathrm{J})(8.31\,\mathrm{J/mol\cdot K})}{(1.01\times10^5\mathrm{Pa})(113\times10^{-6}\mathrm{m}^3 - 63\times10^{-6}\mathrm{m}^3)} = 34.4\,\mathrm{J/mol\cdot K}.$$

(c)
$$C_V = C_P - R = (34.4 \text{ J/mol} \cdot \text{K}) - (8.31 \text{ J/mol} \cdot \text{K}) = 26.1 \text{ J/mol} \cdot \text{K}.$$

E23-48 Constant Volume

- (a) $Q = 3(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 4080 \text{ J}.$
- (b) W = 0.
- (c) $\Delta Erm_{int} = 3(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 4080 \text{ J}.$

Constant Pressure

- (a) $Q = 4(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 5450 \text{ J}.$
- (b) $W = -p\Delta V = -nR\Delta T = -(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = -1360 \text{ J}.$
- (c) $\Delta Erm_{int} = 3(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 4080 \text{ J}.$

Adiabatic

- (a) Q = 0.
- (b) $W = (p_f V_f p_i V_i)/(\gamma 1) = nR\Delta T/(\gamma 1) = 3(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 4080 \text{ J}.$
- (c) $\Delta Erm_{int} = 3(3.15 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(52.0 \text{ K}) = 4080 \text{ J}.$

P23-1 (a) The temperature difference is

$$(5 \,\mathrm{C}^{\circ}/9 \,\mathrm{F}^{\circ})(72^{\circ}\mathrm{F} - 20^{\circ}\mathrm{F}) = 51.1 \,\mathrm{C}^{\circ}.$$

The rate of heat loss is

$$H = (1.0 \,\mathrm{W/m \cdot K})(1.4 \,\mathrm{m}^2)(51.1 \,\mathrm{C}^\circ)/(3.0 \times 10^{-3} \,\mathrm{m}) = 2.4 \times 10^4 \,\mathrm{W}.$$

(b) Start by finding the R values.

$$R_{\rm g} = (3.0 \times 10^{-3} \,\mathrm{m})/(1.0 \,\mathrm{W/m \cdot K}) = 3.0 \times 10^{-3} \,\mathrm{m}^2 \cdot \mathrm{K/W},$$

 $R_{\rm a} = (7.5 \times 10^{-2} \,\mathrm{m})/(0.026 \,\mathrm{W/m \cdot K}) = 2.88 \,\mathrm{m}^2 \cdot \mathrm{K/W}.$

$$R_{\rm a} = (7.5 \times 10^{-2} \text{m})/(0.026 \text{W/m} \cdot \text{K}) = 2.88 \text{m}^2 \cdot \text{K/W}$$

Then use Eq. 23-5,

$$H = \frac{(1.4 \,\mathrm{m}^2)(51.1 \,\mathrm{C}^\circ)}{2(3.0 \times 10^{-3} \,\mathrm{m}^2 \cdot \mathrm{K/W}) + (2.88 \,\mathrm{m}^2 \cdot \mathrm{K/W})} = 25 \,\mathrm{W}.$$

Get double pane windows!

P23-2 (a) $H = (428 \text{ W/m} \cdot \text{K})(4.76 \times 10^{-4} \text{m}^2)(100 \text{ C}^\circ)/(1.17 \text{ m}) = 17.4 \text{ W}.$ (b) $\Delta m/\Delta t = H/L = (17.4 \text{ W})/(333 \times 10^3 \text{J/kg}) = 5.23 \times 10^{-5} \text{kg/s}$, which is the same as 188 g/h.

P23-3 Follow the example in Sample Problem 23-2. We start with Eq. 23-1:

$$H = kA\frac{dT}{dr},$$

$$H = k(4\pi r^2)\frac{dT}{dr},$$

$$\int_{r_1}^{r_2} H\frac{dr}{4\pi r^2} = \int_{T_1}^{T_2} kdT,$$

$$\frac{H}{4pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = k(T_1 - T_2),$$

$$H\left(\frac{r_2 - r_1}{r_1 r_2}\right) = 4\pi k(T_1 - T_2),$$

$$H = \frac{4\pi k(T_1 - T_2)r_1r_2}{r_2 - r_1}.$$

P23-4 (a) $H = (54 \times 10^{-3} \text{W/m}^2) 4\pi (6.37 \times 10^6 \text{m})^2 = 2.8 \times 10^{13} \text{W}.$

(b) Using the results of Problem 23-3,

$$\Delta T = \frac{(2.8 \times 10^{13} \mathrm{W})(6.37 \times 10^6 \mathrm{m} - 3.47 \times 10^6 \mathrm{m})}{4\pi (4.2 \, \mathrm{W/m \cdot K})(6.37 \times 10^6 \mathrm{m})(3.47 \times 10^6 \mathrm{m})} = 7.0 \times 10^4 \mathrm{C}^{\circ}.$$

Since $T_2 = 0$ °C, we expect $T_1 = 7.0 \times 10^4$ C°.

P23-5 Since H = -kA dT/dx, then H dx = -aT dT. H is a constant, so integrate both side according to

$$\begin{split} \int H \, dx &= - \int a T \, dT, \\ HL &= -a \frac{1}{2} (T_2^2 - T_1^2), \\ H &= \frac{a A}{2 L} (T_1^2 - T_2^2). \end{split}$$

P23-6 Assume the water is all at 0°C. The heat flow through the ice is then $H = kA\Delta T/x$; the rate of ice formation is $\Delta m/\Delta t = H/L$. But $\Delta m = \rho A\Delta x$, so

$$\frac{\Delta x}{\Delta t} = \frac{H}{\rho A L} = \frac{k \Delta T}{\rho L x},$$

$$\frac{(1.7 \,\mathrm{W/m \cdot K})(10 \,\mathrm{C}^{\circ})}{(920 \,\mathrm{kg/m^3})(333 \times 10^3 \,\mathrm{J/kg})(0.05 \,\mathrm{m})} = 1.11 \times 10^{-6} \,\mathrm{m/s}.$$

That's the same as 0.40 cm/h.

P23-7 (a) Start with the heat equation:

$$Q_{\rm t} + Q_{\rm i} + Q_{\rm w} = 0,$$

where Q_t is the heat from the tea, Q_i is the heat from the ice when it melts, and Q_w is the heat from the water (which used to be ice). Then

$$m_{\rm t}c_{\rm t}(T_{\rm f}-T_{\rm t,i})+m_{\rm i}L_{\rm f}+m_{\rm w}c_{\rm w}(T_{\rm f}-T_{\rm w,i})=0,$$

which, since we have assumed all of the ice melts and the masses are all equal, can be solved for T_f as

$$T_f = \frac{c_{\rm t} T_{\rm t,i} + c_{\rm w} T_{\rm w,i} - L_{\rm f}}{c_{\rm t} + c_{\rm w}},$$

$$= \frac{(4190 \text{J/kg} \cdot \text{K})(90^{\circ}\text{C}) + (4190 \text{J/kg} \cdot \text{K})(0^{\circ}\text{C}) - (333 \times 10^{3} \text{J/kg})}{(4190 \text{J/kg} \cdot \text{K}) + (4190 \text{J/kg} \cdot \text{K})},$$

$$= 5.3^{\circ}\text{C}.$$

(b) Once again, assume all of the ice melted. Then we can do the same steps, and we get

$$\begin{split} T_f &= \frac{c_{\rm t} T_{\rm t,i} + c_{\rm w} T_{\rm w,i} - L_{\rm f}}{c_{\rm t} + c_{\rm w}}, \\ &= \frac{(4190 {\rm J/kg \cdot K})(70^{\circ}{\rm C}) + (4190 {\rm J/kg \cdot K})(0^{\circ}{\rm C}) - (333 \times 10^{3} {\rm J/kg})}{(4190 {\rm J/kg \cdot K}) + (4190 {\rm J/kg \cdot K})}, \\ &= -4.7^{\circ}{\rm C}. \end{split}$$

So we must have guessed wrong when we assumed that all of the ice melted. The heat equation then simplifies to

$$m_{\rm t}c_{\rm t}(T_{\rm f}-T_{\rm t,i})+m_{\rm i}L_{\rm f}=0,$$

and then

$$m_{\rm i} = \frac{m_{\rm t}c_{\rm t}(T_{\rm t,i}-T_{\rm f})}{L_{\rm f}},$$

 $= \frac{(0.520\,{\rm kg})(4190{\rm J/kg\cdot K})(90^{\circ}{\rm C}-0^{\circ})}{(333\times10^{3}{\rm J/kg})},$
 $= 0.458\,{\rm kg}.$

P23-8 $c = Q/m\Delta T = H/(\Delta m/\Delta t)\Delta T$. But $\Delta m/\Delta t = \rho \Delta V/\Delta t$. Combining,

$$c = \frac{H}{(\Delta V/\Delta t)\rho\Delta T} = \frac{(250\,\mathrm{W})}{(8.2\times10^{-6}\mathrm{m}^3/\mathrm{s})(0.85\times10^3\mathrm{kg/m}^3)(15\,\mathrm{C}^\circ)} = 2.4\times10^3\mathrm{J/kg}\cdot\mathrm{K}.$$

P23-9 (a) $n = N_A/M$, so

$$\epsilon = \frac{(2256 \times 10^3 \mathrm{J/kg})}{(6.02 \times 10^{23}/\mathrm{mol})/(0.018~\mathrm{kg/mol})} = 6.75 \times 10^{-20} \mathrm{J}.$$

(b) $E_{\text{av}} = \frac{3}{2}kT$, so

$$\frac{\epsilon}{E_{\text{av}}} = \frac{2(6.75 \times 10^{-20} \text{J})}{3(1.38 \times 10^{-23} \text{J/K})(305 \text{ K})} = 10.7.$$

P23-10 $Q_{\rm w} + Q_{\rm t} = 0$, so

$$C_{\rm t}\Delta T_{\rm t} + m_{\rm w}c_{\rm w}(T_{\rm f} - T_{\rm i}) = 0,$$

or

$$T_{\rm i} = \frac{(0.3\,{\rm kg})(4190\,{\rm J/kg\cdot m})(44.4^{\circ}{\rm C}) + (46.1\,{\rm J/K})(44.4^{\circ}{\rm C} - 15.0^{\circ}{\rm C})}{(0.3\,{\rm kg})(4190\,{\rm J/kg\cdot m})} = 45.5^{\circ}{\rm C}.$$

P23-11 We can use Eq. 23-10, but we will need to approximate c first. If we assume that the line is straight then we use c = mT + b. I approximate m from

$$m = \frac{(14 \text{ J/mol} \cdot \text{K}) - (3 \text{ J/mol} \cdot \text{K})}{(500 \text{ K}) - (200 \text{ K})} = 3.67 \times 10^{-2} \text{ J/mol}.$$

Then I find b from those same data points,

$$b = (3 \text{ J/mol} \cdot \text{K}) - (3.67 \times 10^{-2} \text{ J/mol})(200 \text{ K}) = -4.34 \text{ J/mol} \cdot \text{K}.$$

Then from Eq. 23-10,

$$Q = n \int_{T_{i}}^{T_{f}} c \, dT,$$

$$= n \int_{T_{i}}^{T_{f}} (mT + b) \, dT,$$

$$= n \left[\frac{m}{2} T^{2} + bT \right]_{T_{i}}^{T_{f}},$$

$$= n \left(\frac{m}{2} (T_{f}^{2} - T_{i}^{2}) + b(T_{f} - T_{i}) \right),$$

$$= (0.45 \text{mol}) \left(\frac{(3.67 \times 10^{-2} \text{J/mol})}{2} ((500 \text{ K})^{2} - (200 \text{ K})^{2}) + (-4.34 \text{J/mol} \cdot \text{K})(500 \text{ K} - 200 \text{ K})),$$

$$= 1.15 \times 10^{3} \text{J}.$$

P23-12 $\delta Q = nC\delta T$, so

$$\begin{array}{lcl} Q & = & n \int C \, dT, \\ & = & n \left[(0.318 \, \mathrm{J/mol} \cdot \mathrm{K}^2) T^2 / 2 - (0.00109 \, \mathrm{J/mol} \cdot \mathrm{K}^3) T^3 / 3 - (0.628 \, \mathrm{J/mol} \cdot \mathrm{K}) T \right]_{50 \, \mathrm{K}}^{90 \, \mathrm{K}}, \\ & = & n (645.8 \, \mathrm{J/mol}). \end{array}$$

Finally,

$$Q = (645.8 \text{ J/mol})(316 \text{ g})/(107.87 \text{ g/mol}) = 189 \text{ J}.$$

P23-13 $TV^{\gamma-1}$ is a constant, so

$$T_2 = (292 \,\mathrm{K})(1/1.28)^{(1.40)-1} = 265 \,\mathrm{K}$$

P23-14 $W = -\int p \, dV$, so

$$W = -\int \left[\frac{nRT}{V - nb} - \frac{an^2}{V^2} \right] dV,$$

$$= -nRT \ln(V - nb) - \frac{an^2}{V} \Big|_i^f,$$

$$= -nRT \ln \frac{V_f - nb}{V_i - nb} - an^2 \left(\frac{1}{V_f} - \frac{1}{V_i} \right).$$

P23-15 When the tube is horizontal there are two regions filled with gas, one at $p_{1,i}$, $V_{1,i}$; the other at $p_{2,i}$, $V_{2,i}$. Originally $p_{1,i} = p_{2,i} = 1.01 \times 10^5 \text{Pa}$ and $V_{1,i} = V_{2,i} = (0.45 \text{ m})A$, where A is the cross sectional area of the tube.

When the tube is held so that region 1 is on top then the mercury has three forces on it: the force of gravity, mg; the force from the gas above pushing down $p_{2,f}A$; and the force from the gas below pushing up $p_{1,f}A$. The balanced force expression is

$$p_{1,f}A = p_{2,f}A + mg.$$

If we write $m = \rho l_{\rm m} A$ where $l_{\rm m} = 0.10$ m, then

$$p_{1,f} = p_{2,f} + \rho g l_{\rm m}$$
.

Finally, since the tube has uniform cross section, we can write V = Al everywhere.

(a) For an isothermal process $p_i l_i = p_f l_f$, where we have used V = Al, and then

$$p_{1,i} \frac{l_{1,i}}{l_{1,f}} - p_{2,i} \frac{l_{2,i}}{l_{2,f}} = \rho g l_{\text{m}}.$$

But we can factor out $p_{1,i} = p_{2,i}$ and $l_{1,i} = l_{2,i}$, and we can apply $l_{1,f} + l_{2,f} = 0.90$ m. Then

$$\frac{1}{l_{1,f}} - \frac{1}{0.90 \,\mathrm{m} - l_{1,f}} = \frac{\rho g l_{\mathrm{m}}}{p_{\mathrm{i}} l_{\mathrm{i}}}.$$

Put in some numbers and rearrange,

$$0.90 \,\mathrm{m} - 2 l_{1,\mathrm{f}} = (0.294 \,\mathrm{m}^{-1}) l_{1,\mathrm{f}} (0.90 \,\mathrm{m} - l_{1,\mathrm{f}}),$$

which can be written as an ordinary quadratic,

$$(0.294 \,\mathrm{m}^{-1})l_{1\,\mathrm{f}}^2 - (2.265)l_{1\,\mathrm{f}} + (0.90 \,\mathrm{m}) = 0$$

The solutions are $l_{1,\rm f}=7.284\,\rm m$ and $0.421\,\rm m$. Only one of these solutions is reasonable, so the mercury shifted down $0.450-0.421=0.029\,\rm m$.

(b) The math is a wee bit uglier here, but we can start with $p_i l_i^{\gamma} = p_f l_f^{\gamma}$, and this means that everywhere we had a $l_{1,f}$ in the previous derivation we need to replace it with $l_{1,f}^{\gamma}$. Then we have

$$\frac{1}{l_{1,f}^{\gamma}} - \frac{1}{(0.90 \,\mathrm{m} - l_{1,f})^{\gamma}} = \frac{\rho g l_{\mathrm{m}}}{p_{\mathrm{i}} l_{\mathrm{i}}^{\gamma}}.$$

This can be written as

$$(0.90 \,\mathrm{m} - l_{1\,\mathrm{f}})^{\gamma} - l_{1\,\mathrm{f}}^{\gamma} = (0.404 \,\mathrm{m}^{-\gamma}) l_{1\,\mathrm{f}}^{\gamma} (0.90 \,\mathrm{m} - l(0.1\,\mathrm{f})^{\gamma},$$

which looks nasty to me! I'll use Maple to get the answer, and find $l_{1,f} = 0.429$, so the mercury shifted down 0.450 - 0.429 = 0.021 m.

Which is more likely? Turn the tube fast, and the adiabatic approximation works. Eventually the system will return to room temperature, and then the isothermal approximation is valid.

P23-16 Internal energy for an ideal diatomic gas can be written as

$$E_{\rm int} = \frac{5}{2}nRT = \frac{5}{2}pV,$$

simply by applying the ideal gas law. The room, however, has a fixed pressure and volume, so the internal energy is independent of the temperature. As such, any energy supplied by the furnace leaves the room, either as heat or as expanding gas doing work on the outside.

P23-17 The speed of sound in the iodine gas is

$$v = f\lambda = (1000 \text{ Hz})(2 \times 0.0677 \text{ m}) = 135 \text{ m/s}.$$

Then

$$\gamma = \frac{Mv^2}{RT} = \frac{n(0.127 \,\text{kg/mol})(135 \,\text{m/s})^2}{(8.31 \,\text{J/mol} \cdot \text{K})(400 \,\text{K})} = n(0.696).$$

Since γ is greater than one, $n \geq 2$. If n = 2 then $\gamma = 1.39$, which is consistent; if n = 3 then $\gamma = 2.08$, which is not consistent.

Consequently, iodine gas is diatomic.

P23-18
$$W = -Q = mL = (333 \times 10^3 \text{ J/kg})(0.122 \text{ kg}) = 4.06 \times 10^4 \text{ J}.$$

```
P23-19 (a)
Process AB
Q = \frac{3}{2}(1.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K}) = 3740 \text{ J}.
W = 0.
\Delta E_{\text{int}} = Q + W = 3740 \text{ J}.
Process BC
Q = 0.
W = (p_{\text{f}}V_{\text{f}} - p_{\text{i}}V_{\text{i}})/(\gamma - 1) = nR\Delta T/(\gamma - 1) = (1.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(-145 \text{ K})/(1.67 - 1) = -1800 \text{ J}
\Delta E_{\text{int}} = Q + W = -1800 \text{ J}.
Process AB
Q = \frac{5}{2}(1.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(-155 \text{ K}) = -3220 \text{ J}.
W = -p\Delta V = -nR\Delta T = -(1.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(-155 \text{ K}) = 1290 \text{ J}.
\Delta E_{\text{int}} = Q + W = 1930 \text{ J}.
Cycle
Q = 520 \text{ J}; W = -510 \text{ J} \text{ (rounding error!)}; \Delta E_{\text{int}} = 10 \text{ J} \text{ (rounding error!)}
```

P23-20

P23-21 $p_{\rm f} = (16.0 \text{ atm})(50/250)^{1.40} = 1.68 \text{ atm}$. The work done by the gas during the expansion is

$$W = [p_iV_i - p_fV_f]/(\gamma - 1),$$

$$= \frac{(16.0 \text{ atm})(50 \times 10^{-6} \text{m}^3) - (1.68 \text{ atm})(250 \times 10^{-6} \text{m}^3)}{(1.40) - 1}(1.01 \times 10^5 \text{Pa/atm}),$$

$$= 96.0 \text{ J}.$$

This process happens 4000 times per minute, but the actual time to complete the process is half of the cycle, or 1/8000 of a minute. Then $P = (96 \,\mathrm{J})(8000)/(60 \,\mathrm{s}) = 12.8 \times 10^3 \,\mathrm{W}$.