E22-1 (a) $n = (2.56 \text{ g})/(197 \text{ g/mol}) = 1.30\times10^{-2}\text{mol}$.
(b) $N = (6.02\times10^{23}\text{mol}^{-1})(1.30\times10^{-2}\text{mol}) = 7.83\times10^{21}$.

E22-2 (a) $N = pV/kT = (1.01\times10^5\text{Pa})(1.00\,\text{m}^3)/(1.38\times10^{-23}\text{J/K})(293\,\text{K}) = 2.50\times10^{25}$.
(b) $n = (2.50\times10^{25})/(6.02\times10^{23}\text{mol}^{-1}) = 41.5$ mol. Then

$$m = (41.5 \text{ mol})[75\%(28 \text{ g/mol}) + 25\%(32 \text{ g/mol})] = 1.20\,\text{kg}.$$

E22-3 (a) We first need to calculate the molar mass of ammonia. This is

$$M = M(\text{N}) + 3M(\text{H}) = (14.0 \text{ g/mol}) + 3(1.01 \text{ g/mol}) = 17.0 \text{ g/mol}$$

The number of moles of nitrogen present is

$$n = m/M_\text{r} = (315 \text{ g})/(17.0 \text{ g/mol}) = 18.5 \text{ mol}.$$

The volume of the tank is

$$V = nRT/p = (18.5 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(350\,\text{K})/(1.35\times10^6\text{Pa}) = 3.99\times10^{-2}\text{m}^3.$$

(b) After the tank is checked the number of moles of gas in the tank is

$$n = pV/(RT) = (8.68\times10^5\text{Pa})(3.99\times10^{-2}\text{m}^3)/[(8.31\,\text{J/mol}\cdot\text{K})(295\,\text{K})] = 14.1 \text{ mol}.$$

In that case, 4.4 mol must have escaped; that corresponds to a mass of

$$m = nM_\text{r} = (4.4 \text{ mol})(17.0 \text{ g/mol}) = 74.8 \text{ g}.$$

E22-4 (a) The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38\times10^{-23}\text{J/K})(285\,\text{K})/(1.01\times10^5\text{Pa}) = 3.89\times10^{-26}\text{m}^3.$$

The edge length is the cube root of this, or 3.39×10^{-9}m. The ratio is 11.3.
(b) The volume per particle is V/N_A, so

$$V/N_A = (18\times10^{-6}\text{m}^3)/(6.02\times10^{23}) = 2.99\times10^{-29}\text{m}^3.$$

The edge length is the cube root of this, or 3.10×10^{-10}m. The ratio is 1.03.

E22-5 The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38\times10^{-23}\text{J/K})(308\,\text{K})/(1.22)(1.01\times10^5\text{Pa}) = 3.45\times10^{-26}\text{m}^3.$$

The fraction actually occupied by the particle is

$$\frac{4\pi(0.710\times10^{-10}\text{m})^3/3)}{(3.45\times10^{-26}\text{m}^3)} = 4.34\times10^{-5}.$$

E22-6 The component of the momentum normal to the wall is

$$p_y = (3.3\times10^{-27}\text{kg})(1.0\times10^3\text{m/s})\cos(55°) = 1.89\times10^{-24}\text{kg}\cdot\text{m/s}.$$

The pressure exerted on the wall is

$$p = \frac{F}{A} = \frac{(1.6\times10^{23}/\text{s})2(1.89\times10^{-24}\text{kg}\cdot\text{m/s})}{(2.0\times10^{-4}\text{m}^2)} = 3.0\times10^3\text{Pa}.$$

E22-7 (a) From Eq. 22-9,

$$v_{\rm rms} = \sqrt{\frac{3p}{\rho}}.$$

Then

$$p = 1.23\times10^{-3}\ \text{atm}\left(\frac{1.01\times10^{5}\,\text{Pa}}{1\ \text{atm}}\right) = 124\,\text{Pa}$$

and

$$\rho = 1.32\times10^{-5}\ \text{g/cm}^3\left(\frac{1\text{kg}}{1000\ \text{g}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 1.32\times10^{-2}\ \text{kg/m}^3.$$

Finally,

$$v_{\rm rms} = \sqrt{\frac{3(1240\,\text{Pa})}{(1.32\times10^{-2}\ \text{kg/m}^3)}} = 531\,\text{m/s}.$$

(b) The molar density of the gas is just n/V; but this can be found quickly from the ideal gas law as

$$\frac{n}{V} = \frac{p}{RT} = \frac{(1240\,\text{Pa})}{(8.31\,\text{J/mol}\cdot\text{K})(317\,\text{K})} = 4.71\times10^{-1}\ \text{mol/m}^3.$$

(c) We were given the density, which is mass per volume, so we could find the molar mass from

$$\frac{\rho}{n/V} = \frac{(1.32\times10^{-2}\ \text{kg/m}^3)}{(4.71\times10^{-1}\ \text{mol/m}^3)} = 28.0\ \text{g/mol}.$$

But what gas is it? It could contain any atom lighter than silicon; trial and error is the way to go. Some of my guesses include C_2H_4 (ethene), CO (carbon monoxide), and N_2. There's no way to tell which is correct at this point, in fact, the gas could be a mixture of all three.

E22-8 The density is $\rho = m/V = nM_r/V$, or

$$\rho = (0.350\ \text{mol})(0.0280\ \text{kg/mol})/\pi(0.125\,\text{m}/2)^2(0.560\,\text{m}) = 1.43\,\text{kg/m}^3.$$

The rms speed is

$$v_{\rm rms} = \sqrt{\frac{3(2.05)(1.01\times10^5\text{Pa})}{(1.43\,\text{kg/m}^3)}} = 659\,\text{m/s}.$$

E22-9 (a) $N/V = p/kT = (1.01\times10^5\text{Pa})/(1.38\times10^{-23}\text{J/K})(273\,\text{K}) = 2.68\times10^{25}/\text{m}^3$.

(b) Note that Eq. 22-11 is *wrong*; for the explanation read the last two paragraphs in the first column on page 502. We need an extra factor of $\sqrt{2}$, so $\pi d^2 = V/\sqrt{2}N\lambda$, so

$$d = \sqrt{1/\sqrt{2}\pi(2.68\times10^{25}/\text{m}^3)(285\times10^{-9}\text{m})} = 1.72\times10^{-10}\text{m}.$$

E22-10 (a) $\lambda = V/\sqrt{2}N\pi d^2$, so

$$\lambda = \frac{1}{\sqrt{2}(1.0\times10^6/\text{m}^3)\pi(2.0\times10^{-10}\text{m})^2} = 5.6\times10^{12}\text{m}.$$

(b) Particles effectively follow ballistic trajectories.

E22-11 We have $v = f\lambda$, where λ is the wavelength (which we will set equal to the mean free path), and v is the speed of sound. The mean free path is, from Eq. 22-13,

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

so

$$f = \frac{\sqrt{2}\pi d^2 pv}{kT} = \frac{\sqrt{2}\pi(315\times10^{-12}\text{m})^2(1.02 \times 1.01\times10^5\text{Pa})(343\,\text{m/s})}{(1.38\times10^{-23}\text{J/K})(291\,\text{K})} = 3.88\times10^9\text{Hz}.$$

E22-12 (a) $p = (1.10\times10^{-6}$ mm Hg$)(133\,\text{Pa/mm Hg}) = 1.46\times10^{-4}$Pa. The particle density is

$$N/V = (1.46\times10^{-4}\text{Pa})/(1.38\times10^{-23}\text{J/K})(295\,\text{K}) = 3.59\times10^{16}/\text{m}^3.$$

(b) The mean free path is

$$\lambda = 1/\sqrt{(2)}(3.59\times10^{16}/\text{m}^3)\pi(2.20\times10^{-10}\text{m})^2 = 130\,\text{m}.$$

E22-13 Note that $v_{\text{av}} \propto \sqrt{T}$, while $\lambda \propto T$. Then the collision rate is proportional to $1/\sqrt{T}$. Then

$$T = (300\,\text{K})\frac{(5.1\times10^9/\text{s})^2}{(6.0\times10^9/\text{s})^2} = 216\,\text{K}.$$

E22-14 (a) $v_{\text{av}} = (65\text{ km/s})/(10) = 6.5$ km/s.
(b) $v_{\text{rms}} = \sqrt{(505\text{ km/s})/(10)} = 7.1$ km/s.

E22-15 (a) The average is

$$\frac{4(200\,\text{s}) + 2(500\,\text{m/s}) + 4(600\,\text{m/s})}{4+2+4} = 420\,\text{m/s}.$$

The mean-square value is

$$\frac{4(200\,\text{s})^2 + 2(500\,\text{m/s})^2 + 4(600\,\text{m/s})^2}{4+2+4} = 2.1 \times 10^5\text{ m}^2/\text{s}^2.$$

The root-mean-square value is the square root of this, or 458 m/s.

(b) I'll be lazy. Nine particles are not moving, and the tenth has a speed of 10 m/s. Then the average speed is 1 m/s, and the root-mean-square speed is 3.16 m/s. Look, v_{rms} is larger than v_{av}!

(c) Can v_{rms}=v_{av}? Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\text{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\text{rms}} > 0$.

Only if *all* of the particles have the same speed will v_{rms}=v_{av}.

E22-16 Use Eq. 22-20:

$$v_{\text{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\text{J/K})(329\,\text{K})}{(2.33\times10^{-26}\text{kg} + 3 \times 1.67\times10^{-27}\text{kg})}} = 694\,\text{m/s}.$$

E22-17 Use Eq. 22-20:

$$v_{\text{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\text{J/K})(2.7\,\text{K})}{(2 \times 1.67\times10^{-27}\text{kg})}} = 180\,\text{m/s}.$$

E22-18 Eq. 22-14 is in the form $N = Av^2 e^{-Bv^2}$. Taking the derivative,

$$\frac{dN}{dv} = 2Ave^{-Bv^2} - 2ABv^3 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 1/B = 2kT/m.$$

E22-19 We want to integrate

$$\begin{aligned} v_{\rm av} &= \frac{1}{N}\int_0^\infty N(v) v\, dv, \\ &= \frac{1}{N}\int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} v\, dv, \\ &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^2 e^{-mv^2/2kT} v\, dv. \end{aligned}$$

The easiest way to attack this is first with a change of variables— let $x = mv^2/2kT$, then $kT\,dx = mv\,dv$. The limits of integration don't change, since $\sqrt{\infty} = \infty$. Then

$$\begin{aligned} v_{\rm av} &= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty \frac{2kT}{m} x e^{-x} \frac{kT}{m} dx, \\ &= 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty x e^{-\alpha x} dx \end{aligned}$$

The factor of α that was introduced in the last line is a Feynman trick; we'll set it equal to one when we are finished, so it won't change the result.

Feynman's trick looks like

$$\frac{d}{d\alpha}\int e^{-\alpha x} dx = \int \frac{\partial}{\partial \alpha} e^{-\alpha x} dx = \int (-x) e^{-\alpha x} dx.$$

Applying this to our original problem,

$$\begin{aligned} v_{\rm av} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty x e^{-\alpha x} dx, \\ &= -\frac{d}{d\alpha} 2\left(\frac{2kT}{\pi m}\right)^{1/2} \int_0^\infty e^{-\alpha x} dx, \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{d}{d\alpha}\left(\left.\frac{-1}{\alpha} e^{-\alpha x}\right|_0^\infty \right), \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{d}{d\alpha}\left(\frac{1}{\alpha}\right), \\ &= -2\left(\frac{2kT}{\pi m}\right)^{1/2} \frac{-1}{\alpha^2}. \end{aligned}$$

We promised, however, that we would set $\alpha = 1$ in the end, so this last line is

$$\begin{aligned} v_{\rm av} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2}, \\ &= \sqrt{\frac{8kT}{\pi m}}. \end{aligned}$$

E22-20 We want to integrate

$$\begin{aligned}(v^2)_{\rm av} &= \frac{1}{N}\int_0^\infty N(v)v^2\,dv,\\ &= \frac{1}{N}\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}v^2\,dv,\\ &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}v^2\,dv.\end{aligned}$$

The easiest way to attack this is first with a change of variables— let $x^2 = mv^2/2kT$, then $\sqrt{2kT/m}dx = dv$. The limits of integration don't change. Then

$$\begin{aligned}(v^2)_{\rm av} &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty \left(\frac{2kT}{m}\right)^{5/2} x^4 e^{-x^2}\,dx,\\ &= \frac{8kT}{\sqrt{\pi}\,m}\int_0^\infty x^4 e^{-x^2}\,dx\end{aligned}$$

Look up the integral; although you can solve it by first applying a Feynman trick (see solution to Exercise 22-21) and then squaring the integral and changing to polar coordinates. I looked it up. I found $3\sqrt{\pi}/8$, so

$$(v^2)_{\rm av} = \frac{8kT}{\sqrt{\pi}\,m}3\sqrt{\pi}/8 = 3kT/m.$$

E22-21 Apply Eq. 22-20:

$$v_{\rm rms} = \sqrt{3(1.38\times10^{-23}{\rm J/K})(287\,{\rm K})/(5.2\times10^{-17}{\rm kg})} = 1.5\times10^{-2}{\rm m/s}.$$

E22-22 Since $v_{\rm rms} \propto \sqrt{T/m}$, we have

$$T = (299\,{\rm K})(4/2) = 598\,{\rm K},$$

or 325°C.

E22-23 (a) The escape speed is found on page 310; $v = 11.2\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.0\times10^4{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.6\times10^5{\rm K}.$$

(b) The escape speed is found on page 310; $v = 2.38\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 460{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 7300{\rm K}.$$

(c) There should be more oxygen than hydrogen.

E22-24 (a) $v_{\rm av} = (70\ {\rm km/s})/(22) = 3.18\ {\rm km/s}$.
(b) $v_{\rm rms} = \sqrt{(250\ {\rm km}^2/{\rm s}^2)/(22)} = 3.37\ {\rm km/s}$.
(c) 3.0 km/s.

E22-25 According to the equation directly beneath Fig. 22-8,

$$\omega = v\phi/L = (212\,\text{m/s})(0.0841\text{ rad})/(0.204\,\text{m}) = 87.3\text{ rad/s}.$$

E22-26 If $v_\text{p} = v_\text{rms}$ then $2T_2 = 3T_1$, or $T_2/T_1 = 3/2$.

E22-27 Read the last paragraph on the first column of page 505. The distribution of speeds is proportional to

$$v^3 e^{-mv^2/2kT} = v^3 e^{-Bv^2},$$

taking the derivative dN/dv and setting equal to zero yields

$$\frac{dN}{dv} = 3v^2 e^{-Bv^2} - 2Bv^4 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 3/2B = 3kT/m.$$

E22-28 (a) $v = \sqrt{3(8.31\,\text{J/mol}\cdot\text{K})(4220\,\text{K})/(0.07261\,\text{kg/mol})} = 1200\,\text{m/s}$.

(b) Half of the sum of the diameters, or 273 pm.

(c) The mean free path of the germanium in the argon is

$$\lambda = 1/\sqrt{2}(4.13\times10^{25}/\text{m}^3)\pi(273\times10^{-12}\text{m})^2 = 7.31\times10^{-8}\text{m}.$$

The collision rate is

$$(1200\,\text{m/s})/(7.31\times10^{-8}\text{m}) = 1.64\times10^{10}/\text{s}.$$

E22-29 The fraction of particles that interests us is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\int_{0.01kT}^{0.03kT} E^{1/2}e^{-E/kT}\,dE.$$

Change variables according to $E/kT = x$, so that $dE = kT\,dx$. The integral is then

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2}e^{-x}\,dx.$$

Since the value of x is so small compared to 1 throughout the range of integration, we can expand according to

$$e^{-x} \approx 1 - x \text{ for } x \ll 1.$$

The integral then simplifies to

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2}(1-x)\,dx = \frac{2}{\sqrt{\pi}}\left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_{0.01}^{0.03} = 3.09\times10^{-3}.$$

E22-30 Write $N(E) = N(E_\text{p} + \epsilon)$. Then

$$N(E_\text{p} + \epsilon) \approx N(E_\text{p}) + \epsilon\,\frac{dN(E)}{dE}\bigg|_{E_\text{p}} + ...$$

But the very definition of E_p implies that the first derivative is zero. Then the fraction of [particles with energies in the range $E_\text{p} \pm 0.02kT$ is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}(kT/2)^{1/2}e^{-1/2}(0.02kT),$$

or $0.04\sqrt{1/2e\pi} = 9.68\times10^{-3}$.

E22-31 The volume correction is on page 508; we need first to find d. If we assume that the particles in water are arranged in a cubic lattice (a bad guess, but we'll use it anyway), then 18 grams of water has a volume of $18\times10^{-6}\text{m}^3$, and

$$d^3 = \frac{(18\times10^{-6}\,\text{m}^3)}{(6.02\times10^{23})} = 3.0\times10^{-29}\text{m}^3$$

is the volume allocated to each water molecule. In this case $d = 3.1\times10^{-10}\text{m}$. Then

$$b = \frac{1}{2}(6.02\times10^{23})(\frac{4}{3}\pi(3.1\times10^{-10}\text{m})^3) = 3.8\,\text{m}^3/\text{mol}.$$

E22-32 $d^3 = 3b/2\pi N_A$, or

$$d = \sqrt[3]{\frac{3(32\times10^{-6}\text{m}^3/\text{mol})}{2\pi(6.02\times10^{23}/\text{mol})}} = 2.9\times10^{-10}\text{m}.$$

E22-33 a has units of energy volume per square mole, which is the same as energy per mole times volume per mole.

P22-1 Solve $(1-x)(1.429) + x(1.250) = 1.293$ for x. The result is $x = 0.7598$.

P22-2

P22-3 The only thing that matters is the total number of moles of gas (2.5) and the number of moles of the second gas (0.5). Since 1/5 of the total number of moles of gas is associated with the second gas, then 1/5 of the total pressure is associated with the second gas.

P22-4 Use Eq. 22-11 with the appropriate $\sqrt{2}$ inserted.

$$\lambda = \frac{(1.0\times10^{-3}\text{m}^3)}{\sqrt{2}(35)\pi(1.0\times10^{-2}\text{m})^2} = 6.4\times10^{-2}\text{m}.$$

P22-5 (a) Since $\lambda \propto 1/d^2$, we have

$$\frac{d_\text{a}}{d_\text{n}} = \sqrt{\frac{\lambda_\text{n}}{\lambda_\text{a}}} = \sqrt{\frac{(27.5\times10^{-8}\text{m})}{(9.90\times10^{-8}\text{m})}} = 1.67.$$

(b) Since $\lambda \propto 1/p$, we have

$$\lambda_2 = \lambda_1\frac{p_1}{p_2} = (9.90\times10^{-8}\text{m})\frac{(75.0\text{ cm Hg})}{(15.0\text{ cm Hg})} = 49.5\times10^{-8}\text{m}.$$

(c) Since $\lambda \propto T$, we have

$$\lambda_2 = \lambda_1\frac{T_2}{T_1} = (9.90\times10^{-8}\text{m})\frac{(233\,\text{K})}{(293\,\text{K})} = 7.87\times10^{-8}\,\text{m}.$$

P22-6 We can assume the molecule will collide with something. Then

$$1 = \int_0^\infty A e^{-cr} dr = A/c,$$

so $A = c$. If the molecule has a mean free path of λ, then

$$\lambda = \int_0^\infty r c e^{-cr} dr = 1/c,$$

so $A = c = 1/\lambda$.

P22-7 What is important here is the temperature; since the temperatures are the same then the average kinetic energies per particle are the same. Then

$$\frac{1}{2} m_1 (v_{\text{rms},1})^2 = \frac{1}{2} m_2 (v_{\text{rms},2})^2.$$

We are given in the problem that $v_{\text{av},2} = 2 v_{\text{rms},1}$. According to Eqs. 22-18 and 22-20 we have

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\pi}{8}} \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3\pi}{8}} v_{\text{av}}.$$

Combining this with the kinetic energy expression above,

$$\frac{m_1}{m_2} = \left(\frac{v_{\text{rms},2}}{v_{\text{rms},1}} \right)^2 = \left(2\sqrt{\frac{3\pi}{8}} \right)^2 = 4.71.$$

P22-8 (a) Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\text{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\text{rms}} > 0$.

(b) Only if *all* of the particles have the same speed will v_{rms}=v_{av}.

P22-9 (a) We need to first find the number of particles by integrating

$$\begin{aligned} N &= \int_0^\infty N(v)\, dv, \\ &= \int_0^{v_0} C v^2\, dv + \int_{v_0}^\infty (0)\, dv = C \int_0^{v_0} v^2\, dv = \frac{C}{3} v_0^3. \end{aligned}$$

Invert, then $C = 3N/v_0^3$.

(b) The average velocity is found from

$$v_{\text{av}} = \frac{1}{N} \int_0^\infty N(v) v\, dv.$$

Using our result from above,

$$\begin{aligned} v_{\text{av}} &= \frac{1}{N} \int_0^{v_0} \left(\frac{3N}{v_0^3} v^2 \right) v\, dv, \\ &= \frac{3}{v_0^3} \int_0^{v_0} v^3\, dv = \frac{3}{v_0^3} \frac{v_0^4}{4} = \frac{3}{4} v_0. \end{aligned}$$

As expected, the average speed is less than the maximum speed. We can make a prediction about the root mean square speed; it will be larger than the average speed (see Exercise 22-15 above) but smaller than the maximum speed.

(c) The root-mean-square velocity is found from

$$v_{\text{rms}}^2 = \frac{1}{N}\int_0^\infty N(v)v^2\,dv.$$

Using our results from above,

$$\begin{aligned} v_{\text{rms}}^2 &= \frac{1}{N}\int_0^{v_0}\left(\frac{3N}{v_0^3}v^2\right)v^2\,dv, \\ &= \frac{3}{v_0^3}\int_0^{v_0} v^4\,dv = \frac{3}{v_0^3}\frac{v_0^5}{5} = \frac{3}{5}v_0^2. \end{aligned}$$

Then, taking the square root,

$$v_{\text{rms}}^2 = \sqrt{\frac{3}{5}}\,v_0$$

Is $\sqrt{3/5} > 3/4$? It had better be.

P22-10

P22-11

P22-12

P22-13

P22-14

P22-15 The mass of air displaced by 2180 m^3 is $m = (1.22\,\text{kg/m}^3)(2180\,\text{m}^3) = 2660\,\text{kg}$. The mass of the balloon and basket is 249 kg and we want to lift 272 kg; this leaves a remainder of 2140 kg for the mass of the air inside the balloon. This corresponds to $(2140\,\text{kg})/(0.0289\,\text{kg/mol}) = 7.4\times10^4\text{mol}$.

The temperature of the gas inside the balloon is then

$$T = (pV)/(nR) = [(1.01\times10^5\text{Pa})(2180\,\text{m}^3)]/[(7.4\times10^4\text{mol})(8.31\,\text{J/mol}\cdot\text{K}) = 358\,\text{K}.$$

That's 85°C.

P22-16

P22-17