E21-1 (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15°\text{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\text{bp}} &= m(100°\text{C}) + b, \\ (T_S)_{\text{mp}} &= m(0°\text{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\text{bp}} - (T_s)S_{\text{mp}} = 100\ \text{C}°m.$$

We are told this is 180 S°, so $m = 1.8$ S°/C°. Put this into the first equation and then find b, $b = 273.15°\text{C}m = 491.67°\text{S}$. The conversion is then

$$T_S = (1.8\ \text{S}°/\text{C}°)T_C + (491.67°\text{S}).$$

(b) The melting point for water is 491.67°S; the boiling point for water is 180 S° above this, or 671.67°S.

E21-2 $T_\text{F} = 9(-273.15\,\text{deg}\,\text{C})/5 + 32°\text{F} = -459.67°\text{F}$.

E21-3 (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15°\text{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\text{bp}} &= m(100°\text{C}) + b, \\ (T_S)_{\text{mp}} &= m(0°\text{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\text{bp}} - (T_s)S_{\text{mp}} = 100\ \text{C}°m.$$

We are told this is 100 Q°, so $m = 1.0$ Q°/C°. Put this into the first equation and then find b, $b = 273.15°\text{C} = 273.15°\text{Q}$. The conversion is then

$$T_S = T_C + (273.15°\text{S}).$$

(b) The melting point for water is 273.15°Q; the boiling point for water is 100 Q° above this, or 373.15°Q.

(c) Kelvin Scale.

E21-4 (a) $T = (9/5)(6000\,\text{K} - 273.15) + 32 = 10000°\text{F}$.

(b) $T = (5/9)(98.6°\text{F} - 32) = 37.0°\text{C}$.

(c) $T = (5/9)(-70°\text{F} - 32) = -57°\text{C}$.

(d) $T = (9/5)(-183°\text{C}) + 32 = -297°\text{F}$.

(e) It depends on what you think is hot. My mom thinks 79°F is too warm; that's $T = (5/9)(79°\text{F} - 32) = 26°\text{C}$.

E21-5 $T = (9/5)(310\,\text{K} - 273.15) + 32 = 98.3°\text{F}$, which is fine.

E21-6 (a) $T = 2(5/9)(T - 32)$, so $-T/10 = -32$, or $T = 320°\text{F}$.

(b) $2T = (5/9)(T - 32)$, so $13T/5 = -32$, or $T = -12.3°\text{F}$.

E21-7 If the temperature (in Kelvin) is directly proportional to the resistance then $T = kR$, where k is a constant of proportionality. We are given one point, $T = 273.16$ K when $R = 90.35\ \Omega$, but that is okay; we only have one unknown, k. Then $(273.16\ \text{K}) = k(90.35\ \Omega)$ or $k = 3.023\ \text{K}/\Omega$.

If the resistance is measured to be $R = 96.28\ \Omega$, we have a temperature of

$$T = kR = (3.023\ \text{K}/\Omega)(96.28\ \Omega) = 291.1\ \text{K}.$$

E21-8 $T = (510°\text{C})/(0.028\,\text{V})V$, so $T = (1.82\times10^{4}°\text{C/V})(0.0102\,\text{V}) = 186°\text{C}$.

E21-9 We must first find the equation which relates gain to temperature, and then find the gain at the specified temperature. If we let G be the gain we can write this linear relationship as

$$G = mT + b,$$

where m and b are constants to be determined. We have two known points:

$$\begin{aligned}(30.0) &= m(20.0°\ \text{C}) + b,\\ (35.2) &= m(55.0°\ \text{C}) + b.\end{aligned}$$

If we subtract the top equation from the bottom we get $5.2 = m(35.0°\ \text{C})$, or $m = 1.49\,\text{C}^{-1}$. Put this into either of the first two equations and

$$(30.0) = (0.149\,\text{C}^{-1})(20.0°\ \text{C}) + b,$$

which has a solution $b = 27.0$

Now to find the gain when $T = 28.0\,°\text{C}$:

$$G = mT + b = (0.149\,\text{C}^{-1})(28.0°\ \text{C}) + (27.0) = 31.2$$

E21-10 $p/p_{\text{tr}} = (373.15\,\text{K})/(273.16\,\text{K}) = 1.366$.

E21-11 100 cm Hg is 1000 torr. $P_{\text{He}} = (100\ \text{cm Hg})(373\,\text{K})/(273.16\,\text{K}) = 136.550$ cm Hg. Nitrogen records a temperature which is 0.2 K higher, so $P_{\text{N}} = (100\ \text{cm Hg})(373.2\,\text{K})/(273.16\,\text{K}) = 136.623$ cm Hg. The difference is 0.073 cm Hg.

E21-12 $\Delta L = (23\times10^{-6}/\text{C}°)(33\,\text{m})(15\text{C}°) = 1.1\times10^{-2}\text{m}$.

E21-13 $\Delta L = (3.2\times10^{-6}/\text{C}°)(200\ \text{in})(60\text{C}°) = 3.8\times10^{-2}\text{in}$.

E21-14 $L' = (2.725\text{cm})[1 + (23\times10^{-6}/\text{C}^\circ)(128\text{C}^\circ)] = 2.733$ cm.

E21-15 We want to focus on the temperature change, not the absolute temperature. In this case, $\Delta T = T_\text{f} - T_\text{i} = (42^\circ\text{C}) - (-5.0^\circ\text{C}) = 47\,\text{C}^\circ$.

Then

$$\Delta L = (11 \times 10^{-6}\,\text{C}^{-1})(12.0\,\text{m})(47\,\text{C}^\circ) = 6.2 \times 10^{-3}\,\text{m}.$$

E21-16 $\Delta A = 2\alpha A\Delta T$, so

$$\Delta A = 2(9\times10^{-6}/\text{C}^\circ)(2.0\,\text{m})(3.0\,\text{m})(30\text{C}^\circ) = 3.2\times10^{-3}\text{m}^2.$$

E21-17 (a) We'll apply Eq. 21-10. The surface area of a cube is six times the area of one face, which is the edge length squared. So $A = 6(0.332\,\text{m})^2 = 0.661\,\text{m}^2$. The temperature change is $\Delta T = (75.0^\circ\text{C}) - (20.0^\circ\text{C}) = 55.0\,\text{C}^\circ$. Then the increase in surface area is

$$\Delta A = 2\alpha A\Delta T = 2(19 \times 10^{-6}\,\text{C}^{-1})(0.661\,\text{m}^2)(55.0\,\text{C}^\circ) = 1.38 \times 10^{-3}\,\text{m}^2$$

(b) We'll now apply Eq. 21-11. The volume of the cube is the edge length cubed, so

$$V = (0.332\,\text{m})^3 = 0.0366\,\text{m}^3.$$

and then from Eq. 21-11,

$$\Delta V = 2\alpha V\Delta T = 3(19 \times 10^{-6}\,\text{C}^{-1})(0.0366\,\text{m}^3)(55.0\,\text{C}^\circ) = 1.15 \times 10^{-4}\,\text{m}^3,$$

is the change in volume of the cube.

E21-18 $V' = V(1 + 3\alpha\Delta T)$, so

$$V' = (530\ \text{cm}^3)[1 + 3(29\times10^{-6}/\text{C}^\circ)(-172\,\text{C}^\circ)] = 522\ \text{cm}^3.$$

E21-19 (a) The slope is approximately $1.6\times10^{-4}/\text{C}^\circ$.

(b) The slope is zero.

E21-20 $\Delta r = (\beta/3)r\Delta T$, so

$$\Delta r = [(3.2\times10^{-5}/\text{K})/3](6.37\times10^6\text{m})(2700\,\text{K}) = 1.8\times10^5\text{m}.$$

E21-21 We'll assume that the steel ruler measures length correctly at room temperature. Then the 20.05 cm measurement of the rod is correct. But both the rod and the ruler will expand in the oven, so the 20.11 cm measurement of the rod is *not* the actual length of the rod in the oven. What is the actual length of the rod in the oven? We can only answer that after figuring out how the 20.11 cm mark on the ruler moves when the ruler expands.

Let $L = 20.11$ cm correspond to the ruler mark at room temperature. Then

$$\Delta L = \alpha_\text{steel} L\Delta T = (11 \times 10^{-6}\,\text{C}^{-1})(20.11\ \text{cm})(250\,\text{C}^\circ) = 5.5 \times 10^{-2}\ \text{cm}$$

is the shift in position of the mark as the ruler is raised to the higher temperature. Then the change in length of the rod is *not* (20.11 cm) – (20.05 cm) = 0.06 cm, because the 20.11 cm mark is shifted out. We need to add 0.055 cm to this; the rod changed length by 0.115 cm.

The coefficient of thermal expansion for the rod is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{(0.115\ \text{cm})}{(20.05\ \text{cm})(250\,\text{C}^\circ)} = 23 \times 10^{-6}\,\text{C}^{-1}.$$

E21-22 $A = ab$, $A' = (a + \Delta a)(b + \Delta b) = ab + a\Delta b + b\Delta a + \Delta a\Delta b$, so

$$\begin{aligned} \Delta A &= a\Delta b + b\Delta a + \Delta a\Delta b, \\ &= A(\Delta b/b + \Delta a/a + \Delta a\Delta b/ab), \\ &\approx A(\alpha\Delta T + \alpha\Delta T), \\ &= 2\alpha A\Delta T. \end{aligned}$$

E21-23 Solve this problem by assuming the solid is in the form of a cube.

If the length of one side of a cube is originally L_0, then the volume is originally $V_0 = L_0^3$. After heating, the volume of the cube will be $V = L^3$, where $L = L_0 + \Delta L$.

Then

$$\begin{aligned} V &= L^3, \\ &= (L_0 + \Delta L)^3, \\ &= (L_0 + \alpha L_0\Delta T)^3, \\ &= L_0^3(1 + \alpha\Delta T)^3. \end{aligned}$$

As long as the quantity $\alpha\Delta T$ is much less than one we can expand the last line in a binomial expansion as

$$V \approx V_0(1 + 3\alpha\Delta T + \cdots),$$

so the change in volume is $\Delta V \approx 3\alpha V_0\Delta T$.

E21-24 (a) $\Delta A/A = 2(0.18\%) = (0.36\%)$.

(b) $\Delta L/L = 0.18\%$.

(c) $\Delta V/V = 3(0.18\%) = (0.54\%)$.

(d) Zero.

(e) $\alpha = (0.0018)/(100\,\mathrm{C}^\circ) = 1.8\times10^{-5}/\mathrm{C}^\circ$.

E21-25 $\rho' - \rho = m/V' - m/V = m/(V + \Delta V) - m/V \approx -m\Delta V/V^2$. Then

$$\Delta\rho = -(m/V)(\Delta V/V) = -\rho\beta\Delta T.$$

E21-26 Use the results of Exercise 21-25.

(a) $\Delta V/V = 3\Delta L/L = 3(0.092\%) = 0.276\%$. The change in density is

$$\Delta\rho/\rho = -\Delta V/V = -(0.276\%) = -0.28$$

(b) $\alpha = \beta/3 = (0.28\%)/3(40\,\mathrm{C}^\circ) = 2.3\times10^{-5}/\mathrm{C}^\circ$. Must be aluminum.

E21-27 The diameter of the rod as a function of temperature is

$$d_\mathrm{s} = d_{\mathrm{s},0}(1 + \alpha_\mathrm{s}\Delta T),$$

The diameter of the ring as a function of temperature is

$$d_\mathrm{b} = d_{\mathrm{b},0}(1 + \alpha_\mathrm{b}\Delta T).$$

We are interested in the temperature when the diameters are equal,

$$\begin{aligned}
d_{\mathrm{s},0}(1+\alpha_{\mathrm{s}}\Delta T) &= d_{\mathrm{b},0}(1+\alpha_{\mathrm{b}}\Delta T),\\
\alpha_{\mathrm{s}} d_{\mathrm{s},0}\Delta T - \alpha_{\mathrm{b}} d_{\mathrm{b},0}\Delta T &= d_{\mathrm{b},0} - d_{\mathrm{s},0},\\
\Delta T &= \frac{d_{\mathrm{b},0} - d_{\mathrm{s},0}}{\alpha_{\mathrm{s}} d_{\mathrm{s},0} - \alpha_{\mathrm{b}} d_{\mathrm{b},0}},\\
\Delta T &= \frac{(2.992\text{ cm}) - (3.000\text{ cm})}{(11\times 10^{-6}/\mathrm{C}^\circ)(3.000\text{ cm}) - (19\times 10^{-6}/\mathrm{C}^\circ)(2.992\text{ cm})},\\
&= 335\,\mathrm{C}^\circ.
\end{aligned}$$

The final temperature is then $T_{\mathrm{f}} = (25^\circ) + 335\,\mathrm{C}^\circ = 360^\circ$.

E21-28 (a) $\Delta L = \Delta L_1 + \Delta L_2 = (L_1\alpha_1 + L_2\alpha_2)\Delta T$. The effective value for α is then

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L}.$$

(b) Since $L_2 = L - L_1$ we can write

$$\begin{aligned}
\alpha_1 L_1 + \alpha_2(L - L_1) &= \alpha L,\\
L_1 &= L\frac{\alpha - \alpha_2}{\alpha_1 - \alpha_2},\\
&= (0.524\,\mathrm{m})\frac{(13\times 10^{-6}) - (11\times 10^{-6})}{(19\times 10^{-6}) - (11\times 10^{-6})} = 0.131\,\mathrm{m}.
\end{aligned}$$

The brass length is then 13.1 cm and the steel is 39.3 cm.

E21-29 At 100°C the glass and mercury each have a volume V_0. After cooling, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_{\mathrm{g}} - \beta_{\mathrm{m}})\Delta T.$$

Since $m = \rho V$, the mass of mercury that needs to be added can be found by multiplying though by the density of mercury. Then

$$\Delta m = (0.891\,\mathrm{kg})[3(9.0\times 10^{-6}/\mathrm{C}^\circ) - (1.8\times 10^{-4}/\mathrm{C}^\circ)](-135\mathrm{C}^\circ) = 0.0184\,\mathrm{kg}.$$

This is the additional amount required, so the total is now 909 g.

E21-30 (a) The rotational inertia is given by $I = \int r^2 dm$; changing the temperature requires $r \to r' = r + \Delta r = r(1 + \alpha\Delta T)$. Then

$$I' = \int (1+\alpha\Delta T)^2 r^2 dm \approx (1 + 2\alpha\Delta T)\int r^2 dm,$$

so $\Delta I = 2\alpha I\Delta T$.

(b) Since $L = I\omega$, then $0 = \omega\Delta I + I\Delta\omega$. Rearranging, $\Delta\omega/\omega = -\Delta I/I = -2\alpha\Delta T$. Then

$$\Delta\omega = -2(19\times 10^{-6}/\mathrm{C}^\circ)(230\text{ rev/s})(170\,\mathrm{C}^\circ) = -1.5\text{ rev/s}.$$

E21-31 This problem is related to objects which expand when heated, but we never actually need to calculate any temperature changes. We will, however, be interested in the change in rotational inertia. Rotational inertia is directly proportional to the square of the (appropriate) linear dimension, so

$$I_{\rm f}/I_{\rm i} = (r_{\rm f}/r_{\rm i})^2.$$

(a) If the bearings are frictionless then there are no external torques, so the angular momentum is constant.

(b) If the angular momentum is constant, then

$$\begin{aligned} L_{\rm i} &= L_{\rm f}, \\ I_{\rm i}\omega_{\rm i} &= I_{\rm f}\omega_{\rm f}. \end{aligned}$$

We are interested in the percent change in the angular velocity, which is

$$\frac{\omega_{\rm f}-\omega_{\rm i}}{\omega_{\rm i}} = \frac{\omega_{\rm f}}{\omega_{\rm i}} - 1 = \frac{I_{\rm i}}{I_{\rm f}} - 1 = \left(\frac{r_{\rm i}}{r_{\rm f}}\right)^2 - 1 = \left(\frac{1}{1.0018}\right)^2 - 1 = -0.36\%.$$

(c) The rotational kinetic energy is proportional to $I\omega^2 = (I\omega)\omega = L\omega$, but L is constant, so

$$\frac{K_{\rm f}-K_{\rm i}}{K_{\rm i}} = \frac{\omega_{\rm f}-\omega_{\rm i}}{\omega_{\rm i}} = -0.36\%.$$

E21-32 (a) The period of a physical pendulum is given by Eq. 17-28. There are two variables in the equation that depend on length. I, which is proportional to a length squared, and d, which is proportional to a length. This means that the period have an overall dependence on length proportional to $\sqrt{r}$. Taking the derivative,

$$\Delta P \approx dP = \frac{1}{2}\frac{P}{r}dr \approx \frac{1}{2}P\alpha\Delta T.$$

(b) $\Delta P/P = (0.7\times10^{-6}{\rm C}^\circ)(10{\rm C}^\circ)/2 = 3.5\times10^{-6}$. After 30 days the clock will be slow by

$$\Delta t = (30 \times 24 \times 60 \times 60\,{\rm s})(3.5\times10^{-6}) = 9.07\,{\rm s}.$$

E21-33 Refer to the Exercise 21-32.

$$\Delta P = (3600\,{\rm s})(19\times10^{-6}{\rm C}^\circ)(-20{\rm C}^\circ)/2 = 0.68\,{\rm s}.$$

E21-34 At 22°C the aluminum cup and glycerin each have a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_{\rm a} - \beta_{\rm g})\Delta T.$$

The amount that spills out is then

$$\Delta V = (110\ {\rm cm}^3)[3(23\times10^{-6}/{\rm C}^\circ) - (5.1\times10^{-4}/{\rm C}^\circ)](6{\rm C}^\circ) = -0.29\ {\rm cm}^3.$$

E21-35 At 20.0°C the glass tube is filled with liquid to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_{\rm g} - \beta_{\rm l})\Delta T.$$

The cross sectional area of the tube changes according to

$$\Delta A = A_0 2\alpha_{\rm g}\Delta T.$$

Consequently, the height of the liquid changes according to

$$\begin{aligned}\Delta V &= (h_0+\Delta h)(A_0+\Delta A) - h_0 A,\\ &\approx h_0\Delta A + A_0\Delta h,\\ \Delta V/V_0 &= \Delta A/A_0 + \Delta h/h_0.\end{aligned}$$

Then

$$\Delta h = (1.28\,\text{m}/2)[(1.1\times10^{-5}/\text{C}^\circ) - (4.2\times10^{-5}/\text{C}^\circ)](13\,\text{C}^\circ) = 2.6\times10^{-4}\text{m}.$$

E21-36 (a) $\beta = (dV/dT)/V$. If $pV = nRT$, then $p\,dV = nR\,dT$, so

$$\beta = (nR/p)/V = nR/pV = 1/T.$$

(b) Kelvins.
(c) $\beta \approx 1/(300/\text{K}) = 3.3\times10^{-3}/\text{K}$.

E21-37 (a) $V = (1 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(273\,\text{K})/(1.01\times10^5\text{Pa}) = 2.25\times10^{-2}\text{m}^3$.
(b) $(6.02\times10^{23}\text{mol}^{-1})/(2.25\times10^4/\text{cm}^3) = 2.68\times10^{19}$.

E21-38 $n/V = p/kT$, so

$$n/V = (1.01\times10^{-13}\text{Pa})/(1.38\times10^{-23}\text{J/K})(295\,\text{K}) = 25 \text{ part/cm}^3.$$

E21-39 (a) Using Eq. 21-17,

$$n = \frac{pV}{RT} = \frac{(108\times10^3\text{Pa})(2.47\,\text{m}^3)}{(8.31\,\text{J/mol·K})([12+273]\,\text{K})} = 113 \text{ mol}.$$

(b) Use the same expression again,

$$V = \frac{nRT}{p} = \frac{(113 \text{ mol})(8.31\,\text{J/mol·K})([31+273]\,\text{K})}{(316\times10^3\text{Pa})} = 0.903\,\text{m}^3.$$

E21-40 (a) $n = pV/RT = (1.01\times10^5\text{Pa})(1.13\times10^{-3}\text{m}^3)/(8.31\,\text{J/mol·K})(315\,\text{K}) = 4.36\times10^{-2}\text{mol}$.
(b) $T_\text{f} = T_\text{i}p_\text{f}V_\text{f}/p_\text{i}V_\text{i}$, so

$$T_\text{f} = \frac{(315\,\text{K})(1.06\times10^5\text{Pa})(1.530\times10^{-3}\text{m}^3)}{(1.01\times10^5\text{Pa})(1.130\times10^{-3}\text{m}^3)} = 448\,\text{K}.$$

E21-41 $p_\text{i} = (14.7+24.2) \text{ lb/in}^2 = 38.9 \text{ lb/in}^2$. $p_\text{f} = p_\text{i}T_\text{f}V_\text{i}/T_\text{i}V_\text{f}$, so

$$p_\text{f} = \frac{(38.9 \text{ lb/in}^2)(299\text{K})(988 \text{ in}^3)}{(270\,\text{K})(1020 \text{ in}^3)} = 41.7 \text{ lb/in}^2.$$

The gauge pressure is then $(41.7-14.7) \text{ lb/in}^2 = 27.0 \text{ lb/in}^2$.

E21-42 Since $p = F/A$ and $F = mg$, a reasonable estimate for the mass of the atmosphere is

$$m = pA/g = (1.01\times10^5\text{Pa})4\pi(6.37\times10^6\text{m})^2/(9.81\,\text{m/s}^2) = 5.25\times10^{18}\text{kg}.$$

E21-43 $p = p_0 + \rho g h$, where h is the depth. Then $P_\mathrm{f} = 1.01\times10^5\mathrm{Pa}$ and

$$p_\mathrm{i} = (1.01\times10^5\mathrm{Pa}) + (998\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(41.5\,\mathrm{m}) = 5.07\times10^5\mathrm{Pa}.$$

$V_\mathrm{f} = V_\mathrm{i} p_\mathrm{i} T_\mathrm{f}/p_\mathrm{f} T_\mathrm{i}$, so

$$V_\mathrm{f} = \frac{(19.4\ \mathrm{cm^3})(5.07\times10^5\mathrm{Pa.})(296\,\mathrm{K})}{(1.01\times10^5\mathrm{Pa})(277\,\mathrm{K})} = 104\ \mathrm{cm^3}.$$

E21-44 The new pressure in the pipe is

$$p_\mathrm{f} = p_\mathrm{i} V_\mathrm{i}/V_\mathrm{f} = (1.01\times10^5\mathrm{Pa})(2) = 2.02\times10^5\mathrm{Pa}.$$

The water pressure at some depth y is given by $p = p_0 + \rho g y$, so

$$y = \frac{(2.02\times10^5\mathrm{Pa}) - (1.01\times10^5\mathrm{Pa})}{(998\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})} = 10.3\,\mathrm{m}.$$

Then the water/air interface inside the tube is at a depth of 10.3 m; so $h = (10.3\,\mathrm{m}) + (25.0\,\mathrm{m})/2 = 22.8\,\mathrm{m}$.

P21-1 (a) The dimensions of A must be $[\mathrm{time}]^{-1}$, as can be seen with a quick inspection of the equation. We would expect that A would depend on the surface area at the very least; however, that means that it must also depend on some other factor to fix the dimensionality of A.

(b) Rearrange and integrate,

$$\begin{aligned} \int_{\Delta T_0}^{T} \frac{d\Delta T}{\Delta T} &= -\int_0^t A\,dt, \\ \ln(\Delta T/\Delta T_0) &= -At, \\ \Delta T &= \Delta T_0 e^{-At}. \end{aligned}$$

P21-2 First find A.

$$A = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\mathrm{C^\circ})/(25\,\mathrm{C^\circ})]}{(45\ \mathrm{min})} = 3.30\times10^{-3}/\mathrm{min}.$$

Then find time to new temperature difference.

$$t = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\mathrm{C^\circ})/(21\,\mathrm{C^\circ})]}{(3.30\times10^{-3}/\mathrm{min})} = 97.8\mathrm{min}$$

This happens $97.8 - 45 = 53$ minutes later.

P21-3 If we neglect the expansion of the tube then we can assume the cross sectional area of the tube is constant. Since $V = Ah$, we can assume that $\Delta V = A\Delta h$. Then since $\Delta V = \beta V_0 \Delta T$, we can write $\Delta h = \beta h_0 \Delta T$.

P21-4 For either container we can write $p_i V_i = n_i R T_i$. We are told that V_i and n_i are constants. Then $\Delta p = AT_1 - BT_2$, where A and B are constants. When $T_1 = T_2$ $\Delta p = 0$, so $A = B$. When $T_1 = T_\mathrm{tr}$ and $T_2 = T_\mathrm{b}$ we have

$$(120\ \mathrm{mm\ Hg}) = A(373\,\mathrm{K} - 273.16\,\mathrm{K}),$$

so $A = 1.202$ mm Hg/K. Then

$$T = \frac{(90\ \mathrm{mm\ Hg}) + (1.202\ \mathrm{mm\ Hg/K})(273.16\,\mathrm{K})}{(1.202\ \mathrm{mm\ Hg/K})} = 348\,\mathrm{K}.$$

Actually, we could have assumed A was negative, and then the answer would be 198 K.

P21-5 Start with a differential form for Eq. 21-8, $dL/dT = \alpha L_0$, rearrange, and integrate:

$$\begin{aligned} \int_{L_0}^{L} dL &= \int_{T_0}^{T} \alpha L_0\, dT, \\ L - L_0 &= L_0 \int_{T_0}^{T} \alpha\, dT, \\ L &= L_0 \left(1 + \int_{T_0}^{T} \alpha\, dT\right). \end{aligned}$$

P21-6 $\Delta L = \alpha L \Delta T$, so

$$\frac{\Delta T}{\Delta t} = \frac{1}{\alpha L}\frac{\Delta L}{\Delta t} = \frac{(96\times 10^{-9}\text{m/s})}{(23\times 10^{-6}/\text{C}^\circ)(1.8\times 10^{-2}\text{m})} = 0.23^\circ\text{C/s}.$$

P21-7 (a) Consider the work that was done for Ex. 21-27. The length of rod a is

$$L_a = L_{a,0}(1 + \alpha_a \Delta T),$$

while the length of rod b is

$$L_b = L_{b,0}(1 + \alpha_b \Delta T).$$

The difference is

$$\begin{aligned} L_a - L_b &= L_{a,0}(1 + \alpha_a \Delta T) - L_{b,0}(1 + \alpha_b \Delta T), \\ &= L_{a,0} - L_{b,0} + (L_{a,0}\alpha_a - L_{b,0}\alpha_b)\Delta T, \end{aligned}$$

which will be a constant is $L_{a,0}\alpha_a = L_{b,0}\alpha_b$ or

$$L_{i,0} \propto 1/\alpha_i.$$

(b) We want $L_{a,0} - L_{b,0} = 0.30\,\text{m}$ so

$$k/\alpha_a - k/\alpha_b = 0.30\,\text{m},$$

where k is a constant of proportionality;

$$k = (0.30\,\text{m})/\left(1/(11\times 10^{-6}/\text{C}^\circ) - 1/(19\times 10^{-6}/\text{C}^\circ)\right) = 7.84\times 10^{-6}\text{m/C}^\circ.$$

The two lengths are

$$L_a = (7.84\times 10^{-6}\text{m/C}^\circ)/(11\times 10^{-6}/\text{C}^\circ) = 0.713\,\text{m}$$

for steel and

$$L_b = (7.84\times 10^{-6}\text{m/C}^\circ)/(19\times 10^{-6}/\text{C}^\circ) = 0.413\,\text{m}$$

for brass.

P21-8 The fractional increase in length of the bar is $\Delta L/L_0 = \alpha \Delta T$. The right triangle on the left has base $L_0/2$, height x, and hypotenuse $(L_0 + \Delta L)/2$. Then

$$x = \frac{1}{2}\sqrt{(L_0 + \Delta L)^2 - L_0^2} = \frac{L_0}{2}\sqrt{2\frac{\Delta L}{L_0}}.$$

With numbers,

$$x = \frac{(3.77\,\text{m})}{2}\sqrt{2(25\times 10^{-6}/\text{C}^\circ)(32\,\text{C}^\circ)} = 7.54\times 10^{-2}\text{m}.$$

P21-9 We want to evaluate $V = V_0(1 + \int \beta\, dT)$; the integral is the area under the graph; the graph looks like a triangle, so the result is

$$V = V_0[1 + (16\,\text{C}^\circ)(0.0002/\text{C}^\circ)/2] = (1.0016)V_0.$$

The density is then

$$\rho = \rho_0(V_0/V) = (1000\,\text{kg/m}^3)/(1.0016) = 0.9984\,\text{kg/m}^3.$$

P21-10 At 0.00°C the glass bulb is filled with mercury to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(\beta - 3\alpha)\Delta T.$$

Since $T_0 = 0.0°\text{C}$, then $\Delta T = T$, if it is measured in °C. The amount of mercury in the capillary is ΔV, and since the cross sectional area is fixed at A, then the length is $L = \Delta V/A$, or

$$L = \frac{V}{A}(\beta - 3\alpha)\Delta T.$$

P21-11 Let a, b, and c correspond to aluminum, steel, and invar, respectively. Then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

We can replace a with $a_0(1 + \alpha_a \Delta T)$, and write similar expressions for b and c. Since $a_0 = b_0 = c_0$, this can be simplified to

$$\cos C = \frac{(1 + \alpha_a\Delta T)^2 + (1 + \alpha_b\Delta T)^2 - (1 + \alpha_c\Delta T)^2}{2(1 + \alpha_a\Delta T)(1 + \alpha_b\Delta T)}.$$

Expand this as a Taylor series in terms of ΔT, and we find

$$\cos C \approx \frac{1}{2} + \frac{1}{2}\left(\alpha_a + \alpha_b - 2\alpha_c\right)\Delta T.$$

Now solve:

$$\Delta T = \frac{2\cos(59.95^\circ) - 1}{(23\times10^{-6}/\text{C}^\circ) + (11\times10^{-6}/\text{C}^\circ) - 2(0.7\times10^{-6}/\text{C}^\circ)} = 46.4^\circ\text{C}.$$

The final temperature is then 66.4°C.

P21-12 The bottom of the iron bar moves downward according to $\Delta L = \alpha L \Delta T$. The center of mass of the iron bar is located in the center; it moves downward half the distance. The mercury expands in the glass upwards; subtracting off the distance the iron moves we get

$$\Delta h = \beta h \Delta T - \Delta L = (\beta h - \alpha L)\Delta T.$$

The center of mass in the mercury is located in the center. If the center of mass of the system is to remain constant we require

$$m_\text{i}\Delta L/2 = m_\text{m}(\Delta h - \Delta L)/2;$$

or, since $\rho = mV = mAy$,

$$\rho_\text{i}\alpha L = \rho_\text{m}(\beta h - 2\alpha L).$$

Solving for h,

$$h = \frac{(12\times10^{-6}/\text{C}^\circ)(1.00\,\text{m})[(7.87\times10^3\text{kg/m}^3) + 2(13.6\times10^3\text{kg/m}^3)]}{(13.6\times10^3\text{kg/m}^3)(18\times10^{-5}/\text{C}^\circ)} = 0.17\,\text{m}.$$

P21-13 The volume of the block which is beneath the surface of the mercury displaces a mass of mercury equal to the mass of the block. The mass of the block is independent of the temperature but the volume of the displaced mercury changes according to

$$V_{\rm m} = V_{\rm m,0}(1 + \beta_{\rm m}\Delta T).$$

This volume is equal to the depth which the block sinks times the cross sectional area of the block (which *does* change with temperature). Then

$$h_{\rm s}{h_{\rm b}}^2 = h_{\rm s,0}{h_{\rm b,0}}^2(1 + \beta_{\rm m}\Delta T),$$

where $h_{\rm s}$ is the depth to which the block sinks and $h_{\rm b,0} = 20$ cm is the length of the side of the block. But

$$h_{\rm b} = h_{\rm b,0}(1 + \alpha_{\rm b}\Delta T),$$

so

$$h_{\rm s} = h_{\rm s,0}\frac{1 + \beta_{\rm m}\Delta T}{(1 + \alpha_{\rm b}\Delta T)^2}.$$

Since the changes are small we can expand the right hand side using the binomial expansion; keeping terms only in ΔT we get

$$h_{\rm s} \approx h_{\rm s,0}(1 + (\beta_{\rm m} - 2\alpha_{\rm b})\Delta T),$$

which means the block will sink a distance $h_{\rm s} - h_{\rm s,0}$ given by

$$h_{\rm s,0}(\beta_{\rm m} - 2\alpha_{\rm b})\Delta T = h_{\rm s,0}\left[(1.8\times10^{-4}/{\rm C}^\circ) - 2(23\times10^{-6}/{\rm C}^\circ)\right](50\,{\rm C}^\circ) = (6.7\times10^{-3})h_{\rm s,0}.$$

In order to finish we need to know how much of the block was submerged in the first place. Since the fraction submerged is equal to the ratio of the densities, we have

$$h_{\rm s,0}/h_{\rm b,0} = \rho_{\rm b}/\rho_{\rm m} = (2.7\times10^3{\rm kg/m^3})/(1.36\times10^4{\rm kg/m^3}),$$

so $h_{\rm s,0} = 3.97$ cm, and the change in depth is 0.27 mm.

P21-14 The area of glass expands according to $\Delta A_{\rm g} = 2\alpha_{\rm g}A_{\rm g}\Delta T$. The are of Dumet wire expands according to

$$\Delta A_{\rm c} + \Delta A{\rm i} = 2(\alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i})\Delta T.$$

We need these to be equal, so

$$\begin{aligned}
\alpha_{\rm g}A_{\rm g} &= \alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i},\\
\alpha_{\rm g}{r_{\rm g}}^2 &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\alpha_{\rm g}({r_{\rm c}}^2 + {r_{\rm i}}^2) &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\frac{{r_{\rm i}}^2}{{r_{\rm c}}^2} &= \frac{\alpha_{\rm c} - \alpha_{\rm g}}{\alpha_{\rm c} - \alpha_{\rm i}}.
\end{aligned}$$

P21-15

P21-16 $V_2 = V_1(p_1/p_2)(T_1/T_2)$, so

$$V_2 = (3.47\,{\rm m^3})[(76\text{ cm Hg})/(36\text{ cm Hg})][(225\,{\rm K})/(295\,{\rm K})] = 5.59\,{\rm m^3}.$$

P21-17 Call the containers one and two so that $V_1 = 1.22$ L and $V_2 = 3.18$ L. Then the initial number of moles in the two containers are

$$n_{1,\mathrm{i}} = \frac{p_\mathrm{i} V_1}{RT_\mathrm{i}} \text{ and } n_{2,\mathrm{i}} = \frac{p_\mathrm{i} V_2}{RT_\mathrm{i}}.$$

The total is

$$n = p_\mathrm{i}(V_1 + V_2)/(RT_\mathrm{i}).$$

Later the temperatures are changed and then the number of moles of gas in each container is

$$n_{1,\mathrm{f}} = \frac{p_\mathrm{f} V_1}{RT_{1,\mathrm{f}}} \text{ and } n_{2,\mathrm{f}} = \frac{p_\mathrm{f} V_2}{RT_{2,\mathrm{f}}}.$$

The total is still n, so

$$\frac{p_\mathrm{f}}{R}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right) = \frac{p_\mathrm{i}(V_1 + V_2)}{RT_\mathrm{i}}.$$

We can solve this for the final pressure, so long as we remember to convert all temperatures to Kelvins,

$$p_\mathrm{f} = \frac{p_\mathrm{i}(V_1 + V_2)}{T_\mathrm{i}}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right)^{-1},$$

or

$$p_\mathrm{f} = \frac{(1.44\text{ atm})(1.22\text{L} + 3.18\text{ L})}{(289\,\text{K})}\left(\frac{(1.22\text{ L})}{(289\,\text{K})} + \frac{(3.18\text{ L})}{(381\,\text{K})}\right)^{-1} = 1.74\text{ atm}.$$

P21-18 Originally $n_A = p_A V_A/RT_A$ and $n_B = p_B V_B/RT_B$; $V_B = 4V_A$. Label the final state of A as C and the final state of B as D. After mixing, $n_C = p_C V_A/RT_A$ and $n_D = p_D V_B/RT_B$, but $P_C = P_D$ and $n_A + n_B = n_C + n_D$. Then

$$p_A/T_A + 4p_B/T_B = p_C(1/T_A + 4/T_B),$$

or

$$p_C = \frac{(5\times10^5\text{Pa})/(300\,\text{K}) + 4(1\times10^5\text{Pa})/(400\,\text{K})}{1/(300\,\text{K}) + 4/(400\,K)} = 2.00\times10^5\text{Pa}.$$

P21-19 If the temperature is uniform then all that is necessary is to substitute $p_0 = nRT/V$ and $p = nRT/V$; cancel RT from both sides, and then equate n/V with n_V.

P21-20 Use the results of Problem 15-19. The initial pressure inside the bubble is $p_\mathrm{i} = p_0 + 4\gamma/r_\mathrm{i}$. The final pressure inside the bell jar is zero, so$p_\mathrm{f} = 4\gamma/r_\mathrm{f}$. The initial and final pressure inside the bubble are related by $p_\mathrm{i} {r_\mathrm{i}}^3 = p_\mathrm{f} {r_\mathrm{f}}^3 = 4\gamma {r_\mathrm{f}}^2$. Now for numbers:

$$p_\mathrm{i} = (1.01\times10^5\text{Pa}) + 4(2.5\times10^{-2}\text{N/m})/(2.0\times10^{-3}\text{m}) = 1.0105\times10^5\text{Pa}.$$

and

$$r_\mathrm{f} = \sqrt{\frac{(1.0105\times10^5\text{Pa})(2.0\times10^{-3}\text{m})^3}{4(2.5\times10^{-2}\text{N/m})}} = 8.99\times10^{-2}\text{m}.$$

P21-21

P21-22