E20-1 (a) $t = x/v = (0.20\,\text{m})/(0.941)(3.00 \times 10^8 \text{m/s}) = 7.1 \times 10^{-10}\text{s}$.
(b) $y = -gt^2/2 = -(9.81\,\text{m/s}^2)(7.1 \times 10^{-10}\text{s})^2/2 = 2.5 \times 10^{-18}\text{m}$.

E20-2 $L = L_0\sqrt{1 - u^2/c^2} = (2.86\,\text{m})\sqrt{1 - (0.999987)^2} = 1.46\text{ cm}$.

E20-3 $L = L_0\sqrt{1 - u^2/c^2} = (1.68\,\text{m})\sqrt{1 - (0.632)^2} = 1.30\,\text{m}$.

E20-4 Solve $\Delta t = \Delta t_0/\sqrt{1 - u^2/c^2}$ for u:

$$u = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = (3.00 \times 10^8 \text{m/s})\sqrt{1 - \left(\frac{(2.20\,\mu\text{s})}{(16.0\,\mu\text{s})}\right)^2} = 2.97 \times 10^8 \text{m/s}.$$

E20-5 We can apply $\Delta x = v\Delta t$ to find the time the particle existed before it decayed. Then

$$\Delta t = \frac{x}{v}\frac{(1.05 \times 10^{-3}\,\text{m})}{(0.992)(3.00 \times 10^8\,\text{m/s})} = 3.53 \times 10^{-12}\,\text{s}.$$

The *proper lifetime* of the particle is

$$\Delta t_0 = \Delta t\sqrt{1 - u^2/c^2} = (3.53 \times 10^{-12}\,\text{s})\sqrt{1 - (0.992)^2} = 4.46 \times 10^{-13}\,\text{s}.$$

E20-6 Apply Eq. 20-12:
$$v = \frac{(0.43c) + (0.587c)}{1 + (0.43c)(0.587c)/c^2} = 0.812c.$$

E20-7 (a) $L = L_0\sqrt{1 - u^2/c^2} = (130\,\text{m})\sqrt{1 - (0.740)^2} = 87.4\,\text{m}$
(b) $\Delta t = L/v = (87.4\,\text{m})/(0.740)(3.00 \times 10^8 \text{m/s}) = 3.94 \times 10^{-7}\text{s}$.

E20-8 $\Delta t = \Delta t_0/\sqrt{1 - u^2/c^2} = (26\text{ ns})\sqrt{1 - (0.99)^2} = 184\text{ ns}$. Then

$$L = v\Delta t = (0.99)(3.00 \times 10^8 \text{m/s})(184 \times 10^{-9}\text{s}) = 55\,\text{m}.$$

E20-9 (a) $v_\text{g} = 2v = (7.91 + 7.91)\text{ km/s} = 15.82\text{ km/s}$.
(b) A relativistic treatment yields $v_\text{r} = 2v/(1 + v^2/c^2)$. The fractional error is

$$\frac{v_\text{g}}{v_\text{r}} - 1 = \left(1 + \frac{v^2}{c^2}\right) - 1 = \frac{v^2}{c^2} = \frac{(7.91 \times 10^3 \text{m/s})^2}{(3.00 \times 10^8 \text{m/s})^2} = 6.95 \times 10^{-10}.$$

E20-10 Invert Eq. 20-15 to get $\beta = \sqrt{1 - 1/\gamma^2}$.
(a) $\beta = \sqrt{1 - 1/(1.01)^2} = 0.140$.
(b) $\beta = \sqrt{1 - 1/(10.0)^2} = 0.995$.
(c) $\beta = \sqrt{1 - 1/(100)^2} = 0.99995$.
(d) $\beta = \sqrt{1 - 1/(1000)^2} = 0.9999995$.

E20-11   The distance traveled by the particle is $(6.0 \text{ y})c$; the time required for the particle to travel this distance is 8.0 years. Then the speed of the particle is

$$v = \frac{\Delta x}{\Delta t} = \frac{(6.0 \text{ y})c}{(8.0 \text{ y})} = \frac{3}{4}c.$$

The speed parameter β is given by

$$\beta = \frac{v}{c} = \frac{\frac{3}{4}c}{c} = \frac{3}{4}.$$

E20-12 $\gamma = 1/\sqrt{1 - (0.950)^2} = 3.20$. Then

$$
\begin{aligned}
x' &= (3.20)[(1.00 \times 10^5 \text{m}) - (0.950)(3.00 \times 10^8 \text{m/s})(2.00 \times 10^{-4}\text{s})] = 1.38 \times 10^5 \text{m}, \\
t' &= (3.20)[(2.00 \times 10^{-4}\text{s}) - (1.00 \times 10^5 \text{m})(0.950)/(3.00 \times 10^8 \text{m/s})] = -3.73 \times 10^{-4}\text{s}.
\end{aligned}
$$

E20-13   (a) $\gamma = 1/\sqrt{1 - (0.380)^2} = 1.081$. Then

$$
\begin{aligned}
x' &= (1.081)[(3.20 \times 10^8 \text{m}) - (0.380)(3.00 \times 10^8 \text{m/s})(2.50 \text{ s})] = 3.78 \times 10^7 \text{m}, \\
t' &= (1.081)[(2.50 \text{ s}) - (3.20 \times 10^8 \text{m})(0.380)/(3.00 \times 10^8 \text{m/s})] = 2.26 \text{ s}.
\end{aligned}
$$

(b) $\gamma = 1/\sqrt{1 - (0.380)^2} = 1.081$. Then

$$
\begin{aligned}
x' &= (1.081)[(3.20 \times 10^8 \text{m}) - (-0.380)(3.00 \times 10^8 \text{m/s})(2.50 \text{ s})] = 6.54 \times 10^8 \text{m}, \\
t' &= (1.081)[(2.50 \text{ s}) - (3.20 \times 10^8 \text{m})(-0.380)/(3.00 \times 10^8 \text{m/s})] = 3.14 \text{ s}.
\end{aligned}
$$

E20-14

E20-15   (a) $v'_x = (-u)/(1 - 0)$ and $v'_y = c\sqrt{1 - u^2/c^2}$.
(b) $(v'_x)^2 + (v'_y)^2 = u^2 + c^2 - u^2 = c^2$.

E20-16 $v' = (0.787c + 0.612c)/[1 + (0.787)(0.612)] = 0.944c$.

E20-17   (a) The first part is easy; we appear to be moving away from A at the same speed as A appears to be moving away from us: $0.347c$.
(b) Using the velocity transformation formula, Eq. 20-18,

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0.347c) - (-0.347c)}{1 - (-0.347c)(0.347c)/c^2} = 0.619c.$$

The negative sign reflects the fact that these two velocities are in *opposite* directions.

E20-18 $v' = (0.788c - 0.413c)/[1 + (0.788)(-0.413)] = 0.556c$.

E20-19   (a) $\gamma = 1/\sqrt{1 - (0.8)^2} = 5/3$.

$$
\begin{aligned}
v'_x &= \frac{v_x}{\gamma(1 - uv_y/c^2)} = \frac{3(0.8c)}{5[1 - (0)]} = \frac{12}{25}c, \\
v'_y &= \frac{v_y - u}{1 - uv_y/c^2} = \frac{(0) - (0.8c)}{1 - (0)} = -\frac{4}{5}c.
\end{aligned}
$$

Then $v' = c\sqrt{(-4/5)^2 + (12/25)^2} = 0.933c$ directed $\theta = \arctan(-12/20) = 31°$ East of South.

(b) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$
\begin{aligned}
v'_x &= \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0)-(-0.8c)}{1-(0)} = +\frac{4}{5}c, \\
v'_y &= \frac{v_y}{\gamma(1-uv_x/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c.
\end{aligned}
$$

Then $v' = c\sqrt{(4/5)^2+(12/25)^2} = 0.933c$ directed $\theta = \arctan(20/12) = 59°$ West of North.

E20-20 This exercise should occur in Section 20-9.
    (a) $v = 2\pi(6.37\times10^6\mathrm{m})c/(1\,\mathrm{s})(3.00\times10^8\mathrm{m/s}) = 0.133c$.
    (b) $K = (\gamma-1)mc^2 = (1/\sqrt{1-(0.133)^2}-1)(511\text{ keV}) = 4.58$ keV.
    (c) $K_c = mv^2/2 = mc^2(v^2/c^2)/2 = (511\text{ keV})(0.133)^2/2 = 4.52$ keV. The percent error is

$$
(4.52-4.58)/(4.58) = -1.31\%.
$$

E20-21 $\Delta L = L' - L_0$ so

$$
\Delta L = 2(6.370\times10^6\mathrm{m})(1-\sqrt{1-(29.8\times10^3\mathrm{m/s})^2/(3.00\times10^8\mathrm{m/s})^2}) = 6.29\times10^{-2}\mathrm{m}.
$$

E20-22 (a) $\Delta L/L_0 = 1 - L'/L_0$ so

$$
\Delta L = (1-\sqrt{1-(522\,\mathrm{m/s})^2/(3.00\times10^8\mathrm{m/s})^2}) = 1.51\times10^{-12}.
$$

(b) We want to solve $\Delta t - \Delta t' = 1\,\mu\mathrm{s}$, or

$$
1\,\mu\mathrm{s} = \Delta t(1-1/\sqrt{1-(522\,\mathrm{m/s})^2/(3.00\times10^8\mathrm{m/s})^2}),
$$

which has solution $\Delta t = 6.61\times10^5\mathrm{s}$. That's 7.64 days.

E20-23 The length of the ship as measured in the "certain" reference frame is

$$
L = L_0\sqrt{1-v^2/c^2} = (358\,\mathrm{m})\sqrt{1-(0.728)^2} = 245\,\mathrm{m}.
$$

In a time Δt the ship will move a distance $x_1 = v_1\Delta t$ while the micrometeorite will move a distance $x_2 = v_2\Delta t$; since they are moving toward each other then the micrometeorite will pass then ship when $x_1 + x_2 = L$. Then

$$
\Delta t = L/(v_1+v_2) = (245\,\mathrm{m})/[(0.728+0.817)(3.00\times10^8\mathrm{m/s})] = 5.29\times10^{-7}\,\mathrm{s}.
$$

This answer is the time measured in the "certain" reference frame. We can use Eq. 20-21 to find the time as measured on the ship,

$$
\Delta t = \frac{\Delta t' + u\Delta x'/c^2}{\sqrt{1-u^2/c^2}} = \frac{(5.29\times10^{-7}\,\mathrm{s})+(0.728c)(116\,\mathrm{m})/c^2}{\sqrt{1-(0.728)^2}} = 1.23\times10^{-6}\mathrm{s}.
$$

E20-24 (a) $\gamma = 1/\sqrt{1-(0.622)^2} = 1.28$.
    (b) $\Delta t = (183\,\mathrm{m})/(0.622)(3.00\times10^8\mathrm{m/s}) = 9.81\times10^{-7}\mathrm{s}$. On the clock, however,

$$
\Delta t' = \Delta t/\gamma = (9.81\times10^{-7}\mathrm{s})/(1.28) = 7.66\times10^{-7}\mathrm{s}.
$$

E20-25   (a) $\Delta t = (26.0 \text{ ly})/(0.988)(1.00 \text{ ly/y}) = 26.3$ y.
(b) The signal takes 26 years to return, so $26 + 26.3 = 52.3$ years.
(c) $\Delta t' = (26.3 \text{ y})\sqrt{1 - (0.988)^2} = 4.06$ y.

E20-26   (a) $\gamma = (1000 \text{ y})(1 \text{ y}) = 1000$;

$$v = c\sqrt{1 - 1/\gamma^2} \approx c(1 - 1/2\gamma^2) = 0.9999995c$$

(b) No.

E20-27 $(5.61 \times 10^{29} \text{ MeV}/c^2)c/(3.00 \times 10^8 \text{m/s}) = 1.87 \times 10^{21}$ MeV/c.

E20-28 $p^2 = m^2 c^2 = m^2 v^2/(1 - v^2/c^2)$, so $2v^2/c^2 = 1$, or $v = \sqrt{2}c$.

⏹ **E20-29**   The magnitude of the momentum of a relativistic particle in terms of the magnitude of the velocity is given by Eq. 20-23,
$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$
The speed parameter, β, is what we are looking for, so we need to rearrange the above expression for the quantity v/c.

$$
\begin{aligned}
p/c &= \frac{mv/c}{\sqrt{1 - v^2/c^2}}, \\
\frac{p}{c} &= \frac{m\beta}{\sqrt{1 - \beta^2}}, \\
\frac{mc}{p} &= \frac{\sqrt{1 - \beta^2}}{\beta}, \\
\frac{mc}{p} &= \sqrt{1/\beta^2 - 1}.
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
\frac{mc^2}{pc} &= \sqrt{1/\beta^2 - 1}, \\
\left(\frac{mc^2}{pc}\right)^2 &= \frac{1}{\beta^2} - 1, \\
\sqrt{\left(\frac{mc^2}{pc}\right)^2 + 1} &= \frac{1}{\beta}, \\
\frac{pc}{\sqrt{m^2 c^4 + p^2 c^2}} &= \beta
\end{aligned}
$$

(a) For the electron,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(0.511 \text{ MeV}/c^2)^2 c^4 + (12.5 \text{ MeV}/c)^2 c^2}} = 0.999.$$

(b) For the proton,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(938 \text{ MeV}/c^2)^2 c^4 + (12.5 \text{ MeV}/c)^2 c^2}} = 0.0133.$$

E20-30 $K = mc^2(\gamma - 1)$, so $\gamma = 1 + K/mc^2$. $\beta = \sqrt{1 - 1/\gamma^2}$.
   (a) $\gamma = 1 + (1.0 \text{ keV})/(511 \text{ keV}) = 1.00196$. $\beta = 0.0625c$.
   (b) $\gamma = 1 + (1.0 \text{ MeV})/(0.511 \text{ MeV}) = 2.96$. $\beta = 0.941c$.
   (c) $\gamma = 1 + (1.0 \text{ GeV})/(0.511 \text{ MeV}) = 1960$. $\beta = 0.99999987c$.

E20-31 The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

We rearrange this to solve for $\beta = v/c$,

$$\beta = \sqrt{1 - \left(\frac{mc^2}{K + mc^2}\right)^2}.$$

It is actually *much* easier to find γ, since

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

so $K = \gamma mc^2 - mc^2$ implies

$$\gamma = \frac{K + mc^2}{mc^2}$$

   (a) For the electron,

$$\beta = \sqrt{1 - \left(\frac{(0.511 \text{ MeV}/c^2)c^2}{(10 \text{ MeV}) + (0.511 \text{ MeV}/c^2)c^2}\right)^2} = 0.9988,$$

and

$$\gamma = \frac{(10 \text{ MeV}) + (0.511 \text{ MeV}/c^2)c^2}{(0.511 \text{ MeV}/c^2)c^2} = 20.6.$$

   (b) For the proton,

$$\beta = \sqrt{1 - \left(\frac{(938 \text{ MeV}/c^2)c^2}{(10 \text{ MeV}) + (938 \text{ MeV}/c^2)c^2}\right)^2} = 0.0145,$$

and

$$\gamma = \frac{(10 \text{ MeV}) + (938 \text{ MeV}/c^2)c^2}{(938 \text{ MeV}/c^2)c^2} = 1.01.$$

   (b) For the alpha particle,

$$\beta = \sqrt{1 - \left(\frac{4(938 \text{ MeV}/c^2)c^2}{(10 \text{ MeV}) + 4(938 \text{ MeV}/c^2)c^2}\right)^2} = 0.73,$$

and

$$\gamma = \frac{(10 \text{ MeV}) + 4(938 \text{ MeV}/c^2)c^2}{4(938 \text{ MeV}/c^2)c^2} = 1.0027.$$

E20-32 $\gamma = 1/\sqrt{1 - (0.99)^2} = 7.089$.
   (a) $E = \gamma mc^2 = (7.089)(938.3 \text{ MeV}) = 6650 \text{ MeV}$. $K = E - mc^2 = 5710 \text{ MeV}$. $p = mv\gamma = (938.3 \text{ MeV}/c^2)(0.99c)(7.089) = 6580 \text{ MeV}/c$.
   (b) $E = \gamma mc^2 = (7.089)(0.511 \text{ MeV}) = 3.62 \text{ MeV}$. $K = E - mc^2 = 3.11 \text{ MeV}$. $p = mv\gamma = (0.511 \text{ MeV}/c^2)(0.99c)(7.089) = 3.59 \text{ MeV}/c$.

E20-33 $\Delta m/\Delta t = (1.2\times10^{41}\text{W})/(3.0\times10^8\text{m/s})^2 = 1.33\times10^{24}\text{kg/s}$, which is

$$\frac{\Delta m}{\Delta t} = \frac{(1.33\times10^{24}\text{kg/s})(3.16\times10^7\text{s/y})}{(1.99\times10^{30}\text{kg/sun})} = 21.1$$

E20-34   (a) If $K = E - mc^2 = 2mc^2$, then $E = 3mc^2$, so $\gamma = 3$, and

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(3)^2} = 0.943c.$$

(b) If $E = 2mc^2$, then $\gamma = 2$, and

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(2)^2} = 0.866c.$$

E20-35   (a) The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = mc^2\left(\left(1 - \beta^2\right)^{-1/2} - 1\right).$$

We want to expand the $1 - \beta^2$ part for small β,

$$\left(1 - \beta^2\right)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots$$

Inserting this into the kinetic energy expression,

$$K = \frac{1}{2}mc^2\beta^2 + \frac{3}{8}mc^2\beta^4 + \cdots$$

But $\beta = v/c$, so

$$K = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

(b) We want to know when the error because of neglecting the second (and higher) terms is 1%; or

$$0.01 = (\frac{3}{8}m\frac{v^4}{c^2})/(\frac{1}{2}mv^2) = \frac{3}{4}\left(\frac{v}{c}\right)^2.$$

This will happen when $v/c = \sqrt{(0.01)4/3} = 0.115$.

E20-36 $K_c = (1000\,\text{kg})(20\,\text{m/s})^2/2 = 2.0\times10^5\,\text{J}$. The relativistic calculation is slightly harder:

$$
\begin{aligned}
K_r &= (1000\,\text{kg})(3\times10^8\text{m/s})^2(1/\sqrt{1 - (20\,\text{m/s})^2/(3\times10^8\text{m/s})^2} - 1), \\
&\approx (1000\,\text{kg})\left[\frac{1}{2}(20\,\text{m/s})^2 + \frac{3}{8}(20\,\text{m/s})^4/(3\times10^8\text{m/s})^2 + ...\right], \\
&= 2.0\times10^5\,\text{J} + 6.7\times10^{-10}\,\text{J}.
\end{aligned}
$$

E20-37   Start with Eq. 20-34 in the form

$$E^2 = (pc)^2 + (mc^2)^2$$

The rest energy is mc^2, and if the total energy is three times this then $E = 3mc^2$, so

$$
\begin{aligned}
(3mc^2)^2 &= (pc)^2 + (mc^2)^2, \\
8(mc^2)^2 &= (pc)^2, \\
\sqrt{8}\,mc &= p.
\end{aligned}
$$

E20-38 The initial kinetic energy is

$$K_i = \frac{1}{2}m\left(\frac{2v}{1+v^2/c^2}\right) = \frac{2mv^2}{(1+v^2/c^2)^2}.$$

The final kinetic energy is

$$K_f = 2\frac{1}{2}m\left(v\sqrt{2-v^2/c^2}\right)^2 = mv^2(2-v^2/c^2).$$

E20-39 This exercise is much more involved than the previous one!
The initial kinetic energy is

$$
\begin{aligned}
K_i &= \frac{mc^2}{\sqrt{1-\left(\frac{2v}{1+v^2/c^2}\right)^2/c^2}} - mc^2, \\
&= \frac{mc^2(1+v^2/c^2)}{\sqrt{(1+v^2/c^2)^2 - 4v^2/c^2}} - mc^2, \\
&= \frac{m(c^2+v^2)}{1-v^2/c^2} - \frac{m(c^2-v^2)}{1-v^2/c^2}, \\
&= \frac{2mv^2}{1-v^2/c^2}.
\end{aligned}
$$

The final kinetic energy is

$$
\begin{aligned}
K_f &= 2\frac{mc^2}{\sqrt{1-\left(v\sqrt{2-v^2/c^2}\right)^2/c^2}} - 2mc^2, \\
&= 2\frac{mc^2}{\sqrt{1-(v^2/c^2)(2-v^2/c^2)}} - 2mc^2, \\
&= 2\frac{mc^2}{1-v^2/c^2} - 2mc^2, \\
&= 2\frac{mc^2}{1-v^2/c^2} - 2\frac{m(c^2-v^2)}{1-v^2/c^2}, \\
&= \frac{2mv^2}{1-v^2/c^2}.
\end{aligned}
$$

E20-40 For a particle with mass, $\gamma = K/mc^2 + 1$. For the electron, $\gamma = (0.40)/(0.511) + 1 = 1.78$.
For the proton, $\gamma = (10)/(938) + 1 = 1.066$.
For the photon, $pc = E$. For a particle with mass, $pc = \sqrt{(K+mc^2)^2 - m^2c^4}$. For the electron,

$$pc = \sqrt{[(0.40 \text{ MeV}) + (0.511 \text{ MeV})]^2 - (0.511 \text{ MeV})^2} = 0.754 \text{ MeV}.$$

For the proton,

$$pc = \sqrt{[(10 \text{ MeV}) + (938 \text{ MeV})]^2 - (938 \text{ MeV})^2} = 137 \text{ MeV}.$$

(a) Only photons move at the speed of light, so it is moving the fastest.
(b) The proton, since it has smallest value for γ.
(c) The proton has the greatest momentum.
(d) The photon has the least.

E20-41   Work is change in energy, so

$$W = mc^2/\sqrt{1 - (v_f/c)^2} - mc^2/\sqrt{1 - (v_i/c)^2}.$$

(a) Plug in the numbers,

$$W = (0.511 \text{ MeV})(1/\sqrt{1 - (0.19)^2} - 1/\sqrt{1 - (0.18)^2}) = 0.996 \text{ keV}.$$

(b) Plug in the numbers,

$$W = (0.511 \text{ MeV})(1/\sqrt{1 - (0.99)^2} - 1/\sqrt{1 - (0.98)^2}) = 1.05 \text{ MeV}.$$

E20-42 $E = 2\gamma m_0 c^2 = mc^2$, so

$$m = 2\gamma m_0 = 2(1.30 \text{ mg})/\sqrt{1 - (0.580)^2} = 3.19 \text{ mg}.$$

E20-43   (a) Energy conservation requires $E_k = 2E_\pi$, or $m_k c^2 = 2\gamma m_\pi c^2$. Then

$$\gamma = (498 \text{ MeV})/2(140 \text{ MeV}) = 1.78$$

This corresponds to a speed of $v = c\sqrt{1 - 1/(1.78)^2} = 0.827c$.
   (b) $\gamma = (498 \text{ MeV} + 325 \text{ MeV})/(498 \text{ MeV}) = 1.65$, so $v = c\sqrt{1 - 1/(1.65)^2} = 0.795c$.
   (c) The lab frame velocities are then

$$v_1' = \frac{(0.795) + (-0.827)}{1 + (0.795)(-0.827)}c = -0.0934c,$$

and

$$v_2' = \frac{(0.795) + (0.827)}{1 + (0.795)(0.827)}c = 0.979c,$$

The corresponding kinetic energies are

$$K_1 = (140 \text{ MeV})(1/\sqrt{1 - (-0.0934)^2} - 1) = 0.614 \text{ MeV}$$

and

$$K_1 = (140 \text{ MeV})(1/\sqrt{1 - (0.979)^2} - 1) = 548 \text{ MeV}$$

E20-44

P20-1   (a) $\gamma = 2$, so $v = \sqrt{1 - 1/(2)^2} = 0.866c$.
   (b) $\gamma = 2$.

P20-2   (a) Classically, $v' = (0.620c) + (0.470c) = 1.09c$. Relativistically,

$$v' = \frac{(0.620c) + (0.470c)}{1 + (0.620)(0.470)} = 0.844c.$$

(b) Classically, $v' = (0.620c) + (-0.470c) = 0.150c$. Relativistically,

$$v' = \frac{(0.620c) + (-0.470c)}{1 + (0.620)(-0.470)} = 0.211c.$$

P20-3 (a) $\gamma = 1/\sqrt{1 - (0.247)^2} = 1.032$. Use the equations from Table 20-2.

$$\Delta t = (1.032)[(0) - (0.247)(30.4 \times 10^3 \text{m})/(3.00 \times 10^8 \text{m/s})] = -2.58 \times 10^{-5} \text{s}.$$

(b) The red flash appears to go first.

P20-4 Once again, the "pico" should have been a μ.
$\gamma = 1/\sqrt{1 - (0.60)^2} = 1.25$. Use the equations from Table 20-2.

$$\Delta t = (1.25)[(4.0 \times 10^{-6} \text{s}) - (0.60)(3.0 \times 10^3 \text{m})/(3.00 \times 10^8 \text{m/s})] = -2.5 \times 10^{-6} \text{s}.$$

| **P20-5** | We can choose our coordinate system so that u is directed along the x axis without any loss of generality. Then, according to Table 20-2,

$$\begin{aligned}
\Delta x' &= \gamma(\Delta x - u\Delta t), \\
\Delta y' &= \Delta y, \\
\Delta z' &= \Delta z, \\
c\Delta t' &= \gamma(c\Delta t - u\Delta x/c).
\end{aligned}$$

Square these expressions,

$$\begin{aligned}
(\Delta x')^2 &= \gamma^2(\Delta x - u\Delta t)^2 = \gamma^2\left((\Delta x)^2 - 2u(\Delta x)(\Delta t) + (\Delta t)^2\right), \\
(\Delta y')^2 &= (\Delta y)^2, \\
(\Delta z')^2 &= (\Delta z)^2, \\
c^2(\Delta t')^2 &= \gamma^2(c\Delta t - u\Delta x/c)^2 = \gamma^2\left(c^2(\Delta t)^2 - 2u(\Delta t)(\Delta x) + u^2(\Delta x)^2/c^2\right).
\end{aligned}$$

We'll add the first three equations and then subtract the fourth. The left hand side is the equal to

$$(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2,$$

while the right hand side will equal

$$\gamma^2\left((\Delta x)^2 + u^2(\Delta t)^2 - c^2(\Delta t)^2 - u^2/c^2(\Delta x)^2\right) + (\Delta y)^2 + (\Delta z)^2,$$

which can be rearranged as

$$\gamma^2\left(1 - u^2/c^2\right)(\Delta x)^2 + \gamma^2\left(u^2 - c^2\right)(\Delta t)^2 + (\Delta y)^2 + (\Delta z)^2,$$
$$\gamma^2\left(1 - u^2/c^2\right)(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2\gamma^2\left(1 - u^2/c^2\right)(\Delta t)^2.$$

But

$$\gamma^2 = \frac{1}{1 - u^2/c^2},$$

so the previous expression will simplify to

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2.$$

P20-6 (a) $v_x = [(0.780c) + (0.240c)]/[1 + (0.240)(0.780)] = 0.859c$.
(b) $v_x = [(0) + (0.240c)]/[1 + (0)] = 0.240c$, while

$$v_y = (0.780c)\sqrt{1 - (0.240)^2}/[1 + (0)] = 0.757c.$$

Then $v = \sqrt{(0.240c)^2 + (0.757c)^2} = 0.794c$.
(b) $v_x' = [(0) - (0.240c)]/[1 + (0)] = -0.240c$, while

$$v_y' = (0.780c)\sqrt{1 - (0.240)^2}/[1 + (0)] = 0.757c.$$

Then $v' = \sqrt{(-0.240c)^2 + (0.757c)^2} = 0.794c$.

P20-7   If we look back at the boost equation we might notice that it looks *very* similar to the rule for the tangent of the sum of two angles. It is exactly the same as the rule for the *hyperbolic* tangent,

$$\tanh(\alpha_1 + \alpha_2) = \frac{\tanh\alpha_1 + \tanh\alpha_2}{1 + \tanh\alpha_1 \tanh\alpha_2}.$$

This means that each boost of $\beta = 0.5$ is the same as a "hyperbolic" rotation of α_r where $\tanh\alpha_r = 0.5$. We need only add these rotations together until we get to α_f, where $\tanh\alpha_f = 0.999$.

$\alpha_f = 3.800$, and $\alpha_R = 0.5493$. We can fit $(3.800)/(0.5493) = 6.92$ boosts, but we need an integral number, so there are seven boosts required. The final speed after these seven boosts will be $0.9991c$.

P20-8   (a) If $\Delta x' = 0$, then $\Delta x = u\Delta t$, or

$$u = (730\,\mathrm{m})/(4.96 \times 10^{-6}\mathrm{s}) = 1.472 \times 10^8 \mathrm{m/s} = 0.491c.$$

(b) $\gamma = 1/\sqrt{1 - (0.491)^2} = 1.148$,

$$\Delta t' = (1.148)[(4.96 \times 10^{-6}\mathrm{s}) - (0.491)(730\,\mathrm{m})/(3 \times 10^8)] = 4.32 \times 10^{-6}\mathrm{s}.$$

P20-9   Since the maximum value for u is c, then the minimum Δt is

$$\Delta t \geq (730\,\mathrm{m})/(3.00 \times 10^8 \mathrm{m/s}) = 2.43 \times 10^{-6}\mathrm{s}.$$

P20-10   (a) Yes.

(b) The speed will be very close to the speed of light, consequently $\gamma \approx (23,000)/(30) = 766.7$. Then

$$v = \sqrt{1 - 1/\gamma^2} \approx 1 - 1/2\gamma^2 = 1 - 1/2(766.7)^2 = 0.99999915c.$$

P20-11   (a) $\Delta t' = (5.00\,\mu\mathrm{s})\sqrt{1 - (0.6)^2} = 4.00\,\mu\mathrm{s}$.

(b) Note: it takes time for the reading on the S' clock to be seen by the S clock. In this case, $\Delta t_1 + \Delta t_2 = 5.00\,\mu\mathrm{s}$, where $\Delta t_1 = x/u$ and $\Delta t_2 = x/c$. Solving for Δt_1,

$$\Delta t_1 = \frac{(5.00\,\mu\mathrm{s})/(0.6c)}{1/(0.6c) + 1/c} = 3.125\,\mathrm{s},$$

and

$$\Delta t_1' = (3.125\,\mu\mathrm{s})\sqrt{1 - (0.6)^2} = 2.50\,\mu\mathrm{s}.$$

P20-12   The only change in the components of Δr occur parallel to the boost. Then we can choose the boost to be parallel to Δr and then

$$\Delta r' = \gamma[\Delta r - u(0)] = \gamma\Delta r \geq \Delta r,$$

since $\gamma \geq 1$.

P20-13   (a) Start with Eq. 20-34,

$$E^2 = (pc)^2 + (mc^2)^2,$$

and substitute into this $E = K + mc^2$,

$$K^2 + 2Kmc^2 + (mc^2)^2 = (pc)^2 + (mc^2)^2.$$

We can rearrange this, and then

$$K^2 + 2Kmc^2 = (pc)^2,$$
$$m = \frac{(pc)^2 - K^2}{2Kc^2}$$

(b) As $v/c \to 0$ we have $K \to \frac{1}{2}mv^2$ and $p \to mv$, the classical limits. Then the above expression becomes

$$m = \frac{m^2v^2c^2 - \frac{1}{4}m^2v^4}{mv^2c^2},$$
$$= m\frac{v^2c^2 - \frac{1}{4}v^4}{v^2c^2},$$
$$= m\left(1 - \frac{1}{4}\frac{v^2}{c^2}\right)$$

But $v/c \to 0$, so this expression reduces to $m = m$ in the classical limit, which is a *good thing*.

(c) We get

$$m = \frac{(121 \text{ MeV})^2 - (55.0 \text{ MeV})^2}{2(55.0 \text{ MeV})c^2} = 1.06 \text{ MeV}/c^2,$$

which is $(1.06 \text{ MeV}/c^2)/(0.511 \text{ MeV}/c^2) = 207m_e$. A muon.

P20-14 Since $E \gg mc^2$ the particle is ultra-relativistic and $v \approx c$. $\gamma = (135)/(0.1396) = 967$. Then the particle has a lab-life of $\Delta t' = (967)(35.0 \times 10^{-9}\text{s}) = 3.385 \times 10^{-5}\text{s}$. The distance traveled is

$$x = (3.00 \times 10^8 \text{m/s})(3.385 \times 10^{-5}\text{s}) = 1.016 \times 10^4 \text{m},$$

so the pion decays 110 km above the Earth.

P20-15 (a) A completely inelastic collision means the two particles, each of mass m_1, stick together after the collision, in effect becoming a new particle of mass m_2. We'll use the subscript 1 for moving particle of mass m_1, the subscript 0 for the particle which is originally at rest, and the subscript 2 for the new particle after the collision. We need to conserve momentum,

$$p_1 + p_0 = p_2,$$
$$\gamma_1 m_1 u_1 + (0) = \gamma_2 m_2 u_2,$$

and we need to conserve total energy,

$$E_1 + E_0 = E_2,$$
$$\gamma_1 m_1 c^2 + m_1 c^2 = \gamma_2 m_2 c^2,$$

Divide the momentum equation by the energy equation and then

$$\frac{\gamma_1 u_1}{\gamma_1 + 1} = u_2.$$

But $u_1 = c\sqrt{1 - 1/\gamma_1^2}$, so

$$u_2 = c\frac{\gamma_1\sqrt{1 - 1/\gamma_1^2}}{\gamma_1 + 1},$$
$$= c\frac{\sqrt{\gamma_1^2 - 1}}{\gamma_1 + 1},$$

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$$= c\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\gamma_1+1},$$

$$= c\sqrt{\frac{\gamma_1-1}{\gamma_1+1}}.$$

(b) Using the momentum equation,

$$m_2 = m_1\frac{\gamma_1 u_1}{\gamma_2 u_2},$$

$$= m_1\frac{c\gamma_1\sqrt{1-1/\gamma_1^2}}{u_2/\sqrt{1-(u_2/c)^2}},$$

$$= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(c/u_2)^2-1}},$$

$$= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(\gamma_1+1)/(\gamma_1-1)-1}},$$

$$= m_1\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\sqrt{(\gamma_1-1)/2}},$$

$$= m_1\sqrt{2(\gamma_1+1)}.$$

P20-16 (a) $K = W = \int F\,dx = \int(dp/dt)dx = \int(dx/dt)dp = \int v\,dp$.
(b) $dp = m\gamma\,dv + mv(d\gamma/dv)dv$. Now use Maple or Mathematica to save time, and get

$$dp = \frac{m\,dv}{(1-v^2/c^2)^{1/2}} + \frac{mv^2\,dv}{c^2(1-v^2/c^2)^{3/2}}.$$

Now integrate:

$$K = \int v\left(\frac{m}{(1-v^2/c^2)^{1/2}} + \frac{mv^2}{c^2(1-v^2/c^2)^{3/2}}\right)dv,$$

$$= \frac{mv^2}{\sqrt{1-v^2/c^2}}.$$

P20-17 (a) Since $E = K + mc^2$, then

$$E_{\text{new}} = 2E = 2mc^2 + 2K = 2mc^2(1+K/mc^2).$$

(b) $E_{\text{new}} = 2(0.938\text{ GeV}) + 2(100\text{ GeV}) = 202\text{ GeV}$.
(c) $K = (100\text{ GeV})/2 - (0.938\text{ GeV}) = 49.1\text{ GeV}$.

P20-18 (a) Assume only one particle is formed. That particle can later decay, but it sets the standard on energy and momentum conservation. The momentum of this one particle must equal that of the incident proton, or

$$p^2c^2 = [(mc^2+K)^2 - m^2c^4].$$

The initial energy was $K + 2mc^2$, so the mass of the "one" particle is given by

$$M^2c^4 = [(K+2mc^2)^2 - p^2c^2] = 2Kmc^2 + 4m^2c^4.$$

This is a measure of the available energy; the remaining energy is required to conserve momentum. Then

$$E_{\text{new}} = \sqrt{M^2c^4} = 2mc^2\sqrt{1+K/2mc^2}.$$

P20-19   The initial momentum is $m\gamma_i v_i$. The final momentum is $(M-m)\gamma_f v_f$. Manipulating the momentum conservation equation,

$$
\begin{aligned}
m\gamma_i v_i &= (M-m)\gamma_f v_f, \\
\frac{1}{m\gamma_i \beta_i} &= \frac{\sqrt{1-\beta_f^2}}{(M-m)\beta_f}, \\
\frac{M-m}{m\gamma_i \beta_i} &= \left(\frac{1}{\beta_f^2}-1\right), \\
\frac{M-m}{m\gamma_i \beta_i}+1 &= \frac{1}{\beta_f^2},
\end{aligned}
$$