

- E18-1** (a) $f = v/\lambda = (243 \text{ m/s})/(0.0327 \text{ m}) = 7.43 \times 10^3 \text{ Hz}$.
 (b) $T = 1/f = 1.35 \times 10^{-4} \text{ s}$.

- E18-2** (a) $f = (12)/(30 \text{ s}) = 0.40 \text{ Hz}$.
 (b) $v = (15 \text{ m})/(5.0 \text{ s}) = 3.0 \text{ m/s}$.
 (c) $\lambda = v/f = (3.0 \text{ m/s})/(0.40 \text{ Hz}) = 7.5 \text{ m}$.

E18-3 (a) The time for a particular point to move from maximum displacement to zero displacement is one-quarter of a period; the point must then go to maximum negative displacement, zero displacement, and finally maximum positive displacement to complete a cycle. So the period is $4(178 \text{ ms}) = 712 \text{ ms}$.

- (b) The frequency is $f = 1/T = 1/(712 \times 10^{-3} \text{ s}) = 1.40 \text{ Hz}$.
 (c) The wave-speed is $v = f\lambda = (1.40 \text{ Hz})(1.38 \text{ m}) = 1.93 \text{ m/s}$.

E18-4 Use Eq. 18-9, except let $f = 1/T$:

$$y = (0.0213 \text{ m}) \sin 2\pi \left(\frac{x}{(0.114 \text{ m})} - (385 \text{ Hz})t \right) = (0.0213 \text{ m}) \sin [(55.1 \text{ rad/m})x - (2420 \text{ rad/s})t].$$

E18-5 The dimensions for tension are $[F] = [M][L]/[T]^2$ where M stands for mass, L for length, T for time, and F stands for force. The dimensions for linear mass density are $[M]/[L]$. The dimensions for velocity are $[L]/[T]$.

Inserting this into the expression $v = F^a/\mu^b$,

$$\begin{aligned} \frac{[L]}{[T]} &= \left(\frac{[M][L]}{[T]^2} \right)^a / \left(\frac{[M]}{[L]} \right)^b, \\ \frac{[L]}{[T]} &= \frac{[M]^a [L]^a}{[T]^{2a}} \frac{[L]^b}{[M]^b}, \\ \frac{[L]}{[T]} &= \frac{[M]^{a-b} [L]^{a+b}}{[T]^{2a}} \end{aligned}$$

There are three equations here. One for time, $-1 = -2a$; one for length, $1 = a + b$; and one for mass, $0 = a - b$. We need to satisfy all three equations. The first is fairly quick; $a = 1/2$. Either of the other equations can be used to show that $b = 1/2$.

- E18-6** (a) $y_m = 2.30 \text{ mm}$.
 (b) $f = (588 \text{ rad/s})/(2\pi \text{ rad}) = 93.6 \text{ Hz}$.
 (c) $v = (588 \text{ rad/s})/(1822 \text{ rad/m}) = 0.323 \text{ m/s}$.
 (d) $\lambda = (2\pi \text{ rad})/(1822 \text{ rad/m}) = 3.45 \text{ mm}$.
 (e) $u_y = y_m \omega = (2.30 \text{ mm})(588 \text{ rad/s}) = 1.35 \text{ m/s}$.

- E18-7** (a) $y_m = 0.060 \text{ m}$.
 (b) $\lambda = (2\pi \text{ rad})/(2.0\pi \text{ rad/m}) = 1.0 \text{ m}$.
 (c) $f = (4.0\pi \text{ rad/s})/(2\pi \text{ rad}) = 2.0 \text{ Hz}$.
 (d) $v = (4.0\pi \text{ rad/s})/(2.0\pi \text{ rad/m}) = 2.0 \text{ m/s}$.
 (e) Since the second term is positive the wave is moving in the $-x$ direction.
 (f) $u_y = y_m \omega = (0.060 \text{ m})(4.0\pi \text{ rad/s}) = 0.75 \text{ m/s}$.

E18-8 $v = \sqrt{F/\mu} = \sqrt{(487 \text{ N})/[(0.0625 \text{ kg})/(2.15 \text{ m})]} = 129 \text{ m/s}$.

E18-9 We'll first find the linear mass density by rearranging Eq. 18-19,

$$\mu = \frac{F}{v^2}$$

Since this is the same string, we expect that changing the tension will not significantly change the linear mass density. Then for the two different instances,

$$\frac{F_1}{v_1^2} = \frac{F_2}{v_2^2}$$

We want to know the new tension, so

$$F_2 = F_1 \frac{v_2^2}{v_1^2} = (123 \text{ N}) \frac{(180 \text{ m/s})^2}{(172 \text{ m/s})^2} = 135 \text{ N}$$

E18-10 First $v = (317 \text{ rad/s})/(23.8 \text{ rad/m}) = 13.32 \text{ m/s}$. Then

$$\mu = F/v^2 = (16.3 \text{ N})/(13.32 \text{ m/s})^2 = 0.0919 \text{ kg/m}.$$

E18-11 (a) $y_m = 0.05 \text{ m}$.

(b) $\lambda = (0.55 \text{ m}) - (0.15 \text{ m}) = 0.40 \text{ m}$.

(c) $v = \sqrt{F/\mu} = \sqrt{(3.6 \text{ N})/(0.025 \text{ kg/m})} = 12 \text{ m/s}$.

(d) $T = 1/f = \lambda/v = (0.40 \text{ m})/(12 \text{ m/s}) = 3.33 \times 10^{-2} \text{ s}$.

(e) $u_y = y_m \omega = 2\pi y_m/T = 2\pi(0.05 \text{ m})/(3.33 \times 10^{-2} \text{ s}) = 9.4 \text{ m/s}$.

(f) $k = (2\pi \text{ rad})/(0.40 \text{ m}) = 5.0\pi \text{ rad/m}$; $\omega = kv = (5.0\pi \text{ rad/m})(12 \text{ m/s}) = 60\pi \text{ rad/s}$. The phase angle can be found from

$$(0.04 \text{ m}) = (0.05 \text{ m}) \sin(\phi),$$

or $\phi = 0.93 \text{ rad}$. Then

$$y = (0.05 \text{ m}) \sin[(5.0\pi \text{ rad/m})x + (60\pi \text{ rad/s})t + (0.93 \text{ rad})].$$

E18-12 (a) The tensions in the two strings are equal, so $F = (0.511 \text{ kg})(9.81 \text{ m/s}^2)/2 = 2.506 \text{ N}$. The wave speed in string 1 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506 \text{ N})/(3.31 \times 10^{-3} \text{ kg/m})} = 27.5 \text{ m/s},$$

while the wave speed in string 2 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506 \text{ N})/(4.87 \times 10^{-3} \text{ kg/m})} = 22.7 \text{ m/s}.$$

(b) We have $\sqrt{F_1/\mu_1} = \sqrt{F_2/\mu_2}$, or $F_1/\mu_1 = F_2/\mu_2$. But $F_i = M_i g$, so $M_1/\mu_1 = M_2/\mu_2$. Using $M = M_1 + M_2$,

$$\begin{aligned} \frac{M_1}{\mu_1} &= \frac{M - M_1}{\mu_2}, \\ \frac{M_1}{\mu_1} + \frac{M_1}{\mu_2} &= \frac{M}{\mu_2}, \\ M_1 &= \frac{M}{\mu_2} \left/ \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right) \right., \\ &= \frac{(0.511 \text{ kg})}{(4.87 \times 10^{-3} \text{ kg/m})} \left/ \left(\frac{1}{(3.31 \times 10^{-3} \text{ kg/m})} + \frac{1}{(4.87 \times 10^{-3} \text{ kg/m})} \right) \right., \\ &= 0.207 \text{ kg} \end{aligned}$$

and $M_2 = (0.511 \text{ kg}) - (0.207 \text{ kg}) = 0.304 \text{ kg}$.

E18-13 We need to know the wave speed before we do anything else. This is found from Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{(248 \text{ N})}{(0.0978 \text{ kg})/(10.3 \text{ m})}} = 162 \text{ m/s}.$$

The two pulses travel in opposite directions on the wire; one travels a distance x_1 in a time t , the other travels a distance x_2 in a time $t + 29.6 \text{ ms}$, and since the pulses meet, we have $x_1 + x_2 = 10.3 \text{ m}$.

Our equations are then $x_1 = vt = (162 \text{ m/s})t$, and $x_2 = v(t + 29.6 \text{ ms}) = (162 \text{ m/s})(t + 29.6 \text{ ms}) = (162 \text{ m/s})t + 4.80 \text{ m}$. We can add these two expressions together to solve for the time t at which the pulses meet,

$$10.3 \text{ m} = x_1 + x_2 = (162 \text{ m/s})t + (162 \text{ m/s})t + 4.80 \text{ m} = (324 \text{ m/s})t + 4.80 \text{ m}.$$

which has solution $t = 0.0170 \text{ s}$. The two pulses meet at $x_1 = (162 \text{ m/s})(0.0170 \text{ s}) = 2.75 \text{ m}$, or $x_2 = 7.55 \text{ m}$.

E18-14 (a) $\partial y / \partial r = (A/r)k \cos(kr - \omega t) - (A/r^2) \sin(kr - \omega t)$. Multiply this by r^2 , and then find

$$\frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = Ak \cos(kr - \omega t) - Ak^2 r \sin(kr - \omega t) - Ak \cos(kr - \omega t).$$

Simplify, and then divide by r^2 to get

$$\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = -(Ak^2/r) \sin(kr - \omega t).$$

Now find $\partial^2 y / \partial t^2 = -A\omega^2 \sin(kr - \omega t)$. But since $1/v^2 = k^2/\omega^2$, the two sides are equal.

(b) $[\text{length}]^2$.

E18-15 The linear mass density is $\mu = (0.263 \text{ kg})/(2.72 \text{ m}) = 9.669 \times 10^{-2} \text{ kg/m}$. The wave speed is $v = \sqrt{(36.1 \text{ N})/(9.669 \times 10^{-2} \text{ kg/m})} = 19.32 \text{ m/s}$.

$P_{\text{av}} = \frac{1}{2} \mu \omega^2 y_m^2 v$, so

$$\omega = \sqrt{\frac{2(85.5 \text{ W})}{(9.669 \times 10^{-2} \text{ kg/m})(7.70 \times 10^{-3} \text{ m})^2(19.32 \text{ m/s})}} = 1243 \text{ rad/s}.$$

Then $f = (1243 \text{ rad/s})/2\pi = 199 \text{ Hz}$.

E18-16 (a) If the medium absorbs no energy then the power flow through any closed surface which contains the source must be constant. Since for a cylindrical surface the area grows as r , then intensity must fall off as $1/r$.

(b) Intensity is proportional to the amplitude squared, so the amplitude must fall off as $1/\sqrt{r}$.

E18-17 The intensity is the average power per unit area (Eq. 18-33); as you get farther from the source the intensity falls off because the perpendicular area increases. At some distance r from the source the total possible area is the area of a spherical shell of radius r , so intensity as a function of distance from the source would be

$$I = \frac{P_{\text{av}}}{4\pi r^2}$$

We are given two intensities: $I_1 = 1.13 \text{ W/m}^2$ at a distance r_1 ; $I_2 = 2.41 \text{ W/m}^2$ at a distance $r_2 = r_1 - 5.30 \text{ m}$. Since the average power of the source is the same in both cases we can equate these two values as

$$\begin{aligned} 4\pi r_1^2 I_1 &= 4\pi r_2^2 I_2, \\ 4\pi r_1^2 I_1 &= 4\pi (r_1 - d)^2 I_2, \end{aligned}$$

where $d = 5.30 \text{ m}$, and then solve for r_1 . Doing this we find a quadratic expression which is

$$\begin{aligned} r_1^2 I_1 &= (r_1^2 - 2dr_1 + d^2) I_2, \\ 0 &= \left(1 - \frac{I_1}{I_2}\right) r_1^2 - 2dr_1 + d^2, \\ 0 &= \left(1 - \frac{(1.13 \text{ W/m}^2)}{(2.41 \text{ W/m}^2)}\right) r_1^2 - 2(5.30 \text{ m})r_1 + (5.30 \text{ m})^2, \\ 0 &= (0.531)r_1^2 - (10.6 \text{ m})r_1 + (28.1 \text{ m}^2). \end{aligned}$$

The solutions to this are $r_1 = 16.8 \text{ m}$ and $r_1 = 3.15 \text{ m}$; but since the person walked 5.3 m toward the lamp we will assume they started at least that far away. Then the power output from the light is

$$P = 4\pi r_1^2 I_1 = 4\pi (16.8 \text{ m})^2 (1.13 \text{ W/m}^2) = 4.01 \times 10^3 \text{ W}.$$

E18-18 Energy density is energy per volume, or $u = U/V$. A wave front of cross sectional area A sweeps out a volume of $V = Al$ when it travels a distance l . The wave front travels that distance l in a time $t = l/v$. The energy flow per time is the power, or $P = U/t$. Combine this with the definition of intensity, $I = P/A$, and

$$I = \frac{P}{A} = \frac{U}{At} = \frac{uV}{At} = \frac{uAl}{At} = uv.$$

E18-19 Refer to Eq. 18-40, where the amplitude of the combined wave is

$$2y_m \cos(\Delta\phi/2),$$

where y_m is the amplitude of the combining waves. Then

$$\cos(\Delta\phi/2) = (1.65y_m)/(2y_m) = 0.825,$$

which has solution $\Delta\phi = 68.8^\circ$.

E18-20 Consider only the point $x = 0$. The displacement y at that point is given by

$$y = y_{m1} \sin(\omega t) + y_{m2} \sin(\omega t + \pi/2) = y_{m1} \sin(\omega t) + y_{m2} \cos(\omega t).$$

This can be written as

$$y = y_m (A_1 \sin \omega t + A_2 \cos \omega t),$$

where $A_i = y_{mi}/y_m$. But if y_m is judiciously chosen, $A_1 = \cos \beta$ and $A_2 = \sin \beta$, so that

$$y = y_m \sin(\omega t + \beta).$$

Since we then require $A_1^2 + A_2^2 = 1$, we must have

$$y_m = \sqrt{(3.20 \text{ cm})^2 + (4.19 \text{ cm})^2} = 5.27 \text{ cm}.$$

E18-21 The easiest approach is to use a phasor representation of the waves.

Write the phasor components as

$$\begin{aligned}x_1 &= y_{m1} \cos \phi_1, \\y_1 &= y_{m1} \sin \phi_1, \\x_2 &= y_{m2} \cos \phi_2, \\y_2 &= y_{m2} \sin \phi_2,\end{aligned}$$

and then use the cosine law to find the magnitude of the resultant.

The phase angle can be found from the arcsine of the opposite over the hypotenuse.

E18-22 (a) The diagrams for all times except $t = 15$ ms should show two distinct pulses, first moving closer together, then moving farther apart. The pulses do not flip over when passing each other. The $t = 15$ ms diagram, however, should simply be a flat line.

E18-23 Use a program such as Maple or Mathematica to plot this.

E18-24 Use a program such as Maple or Mathematica to plot this.

E18-25 (a) The wave speed can be found from Eq. 18-19; we need to know the linear mass density, which is $\mu = m/L = (0.122 \text{ kg})/(8.36 \text{ m}) = 0.0146 \text{ kg/m}$. The wave speed is then given by

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(96.7 \text{ N})}{(0.0146 \text{ kg/m})}} = 81.4 \text{ m/s}.$$

(b) The longest possible standing wave will be twice the length of the string; so $\lambda = 2L = 16.7 \text{ m}$.

(c) The frequency of the wave is found from Eq. 18-13, $v = f\lambda$.

$$f = \frac{v}{\lambda} = \frac{(81.4 \text{ m/s})}{(16.7 \text{ m})} = 4.87 \text{ Hz}$$

E18-26 (a) $v = \sqrt{(152 \text{ N})/(7.16 \times 10^{-3} \text{ kg/m})} = 146 \text{ m/s}$.

(b) $\lambda = (2/3)(0.894 \text{ m}) = 0.596 \text{ m}$.

(c) $f = v/\lambda = (146 \text{ m/s})/(0.596 \text{ m}) = 245 \text{ Hz}$.

E18-27 (a) $y = -3.9 \text{ cm}$.

(b) $y = (0.15 \text{ m}) \sin[(0.79 \text{ rad/m})x + (13 \text{ rad/s})t]$.

(c) $y = 2(0.15 \text{ m}) \sin[(0.79 \text{ rad/m})(2.3 \text{ m})] \cos[(13 \text{ rad/s})(0.16 \text{ s})] = -0.14 \text{ m}$.

E18-28 (a) The amplitude is half of 0.520 cm , or 2.60 mm . The speed is

$$v = (137 \text{ rad/s})/(1.14 \text{ rad/cm}) = 1.20 \text{ m/s}.$$

(b) The nodes are $(\pi \text{ rad})/(1.14 \text{ rad/cm}) = 2.76 \text{ cm}$ apart.

(c) The velocity of a particle on the string at position x and time t is the derivative of the wave equation with respect to time, or

$$u_y = -(0.520 \text{ cm})(137 \text{ rad/s}) \sin[(1.14 \text{ rad/cm})(1.47 \text{ cm})] \sin[(137 \text{ rad/s})(1.36 \text{ s})] = -0.582 \text{ m/s}.$$

E18-29 (a) We are given the wave frequency and the wave-speed, the wavelength is found from Eq. 18-13,

$$\lambda = \frac{v}{f} = \frac{(388 \text{ m/s})}{(622 \text{ Hz})} = 0.624 \text{ m}$$

The standing wave has four loops, so from Eq. 18-45

$$L = n\frac{\lambda}{2} = (4)\frac{(0.624 \text{ m})}{2} = 1.25 \text{ m}$$

is the length of the string.

(b) We can just write it down,

$$y = (1.90 \text{ mm}) \sin[(2\pi/0.624 \text{ m})x] \cos[(2\pi 622 \text{ s}^{-1})t].$$

E18-30 (a) $f_n = nv/2L = (1)(250 \text{ m/s})/2(0.150 \text{ m}) = 833 \text{ Hz}$.

(b) $\lambda = v/f = (348 \text{ m/s})/(833 \text{ Hz}) = 0.418 \text{ m}$.

E18-31 $v = \sqrt{F/\mu} = \sqrt{FL/m}$. Then $f_n = nv/2L = n\sqrt{F/4mL}$, so

$$f_1 = (1)\sqrt{(236 \text{ N})/4(0.107 \text{ kg})(9.88 \text{ m})} = 7.47 \text{ Hz},$$

and $f_2 = 2f_1 = 14.9 \text{ Hz}$ while $f_3 = 3f_1 = 22.4 \text{ Hz}$.

E18-32 (a) $v = \sqrt{F/\mu} = \sqrt{FL/m} = \sqrt{(122 \text{ N})(1.48 \text{ m})/(8.62 \times 10^{-3} \text{ kg})} = 145 \text{ m/s}$.

(b) $\lambda_1 = 2(1.48 \text{ m}) = 2.96 \text{ m}$; $\lambda_2 = 1.48 \text{ m}$.

(c) $f_1 = (145 \text{ m/s})/(2.96 \text{ m}) = 49.0 \text{ Hz}$; $f_2 = (145 \text{ m/s})/(1.48 \text{ m}) = 98.0 \text{ Hz}$.

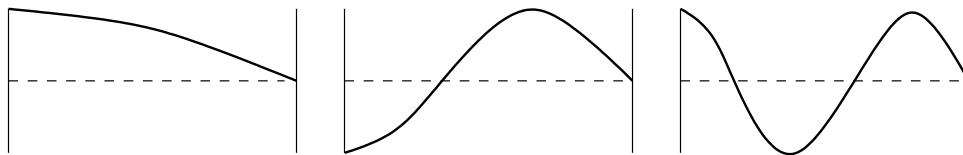
E18-33 Although the tied end of the string forces it to be a node, the fact that the other end is loose means that it should be an anti-node. The discussion of Section 18-10 indicated that the spacing between nodes is always $\lambda/2$. Since anti-nodes occur between nodes, we can expect that the distance between a node and the nearest anti-node is $\lambda/4$.

The longest possible wavelength will have one node at the tied end, an anti-node at the loose end, and no other nodes or anti-nodes. In this case $\lambda/4 = 120 \text{ cm}$, or $\lambda = 480 \text{ cm}$.

The next longest wavelength will have a node somewhere in the middle region of the string. But this means that there must be an anti-node between this new node and the node at the tied end of the string. Moving from left to right, we then have an anti-node at the loose end, a node, and anti-node, and finally a node at the tied end. There are four points, each separated by $\lambda/4$, so the wavelength would be given by $3\lambda/4 = 120 \text{ cm}$, or $\lambda = 160 \text{ cm}$.

To progress to the next wavelength we will add another node, and another anti-node. This will add another two lengths of $\lambda/4$ that need to be fit onto the string; hence $5\lambda/4 = 120 \text{ cm}$, or $\lambda = 100 \text{ cm}$.

In the figure below we have sketched the first three standing waves.



E18-34 (a) Note that $f_n = nf_1$. Then $f_{n+1} - f_n = f_1$. Since there is no resonant frequency between these two then they must differ by 1, and consequently $f_1 = (420 \text{ Hz}) - (315 \text{ Hz}) = 105 \text{ Hz}$.

(b) $v = f\lambda = (105 \text{ Hz})[2(0.756 \text{ m})] = 159 \text{ m/s}$.

P18-1 (a) $\lambda = v/f$ and $k = 360^\circ/\lambda$. Then

$$x = (55^\circ)\lambda/(360^\circ) = 55(353 \text{ m/s})/360(493 \text{ Hz}) = 0.109 \text{ m}.$$

(b) $\omega = 360^\circ f$, so

$$\phi = \omega t = (360^\circ)(493 \text{ Hz})(1.12 \times 10^{-3} \text{ s}) = 199^\circ.$$

P18-2 $\omega = (2\pi \text{ rad})(548 \text{ Hz}) = 3440 \text{ rad/s}$; $\lambda = v/f$ and then

$$k = (2\pi \text{ rad})/[(326 \text{ m/s})/(548 \text{ Hz})] = 10.6 \text{ rad/m}.$$

Finally, $y = (1.12 \times 10^{-2} \text{ m}) \sin[(10.6 \text{ rad/m})x + (3440 \text{ rad/s})t]$.

P18-3 (a) This problem really isn't as bad as it might look. The tensile stress S is tension per unit cross sectional area, so

$$S = \frac{F}{A} \text{ or } F = SA.$$

We already know that linear mass density is $\mu = m/L$, where L is the length of the wire. Substituting into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{SA}{m/L}} \sqrt{\frac{S}{m/(AL)}}.$$

But AL is the volume of the wire, so the denominator is just the mass density ρ .

(b) The maximum speed of the transverse wave will be

$$v = \sqrt{\frac{S}{\rho}} = \sqrt{\frac{(720 \times 10^6 \text{ Pa})}{(7800 \text{ kg/m}^3)}} = 300 \text{ m/s}.$$

P18-4 (a) $f = \omega/2\pi = (4.08 \text{ rad/s})/(2\pi \text{ rad}) = 0.649 \text{ Hz}$.

(b) $\lambda = v/f = (0.826 \text{ m/s})/(0.649 \text{ Hz}) = 1.27 \text{ m}$.

(c) $k = (2\pi \text{ rad})/(1.27 \text{ m}) = 4.95 \text{ rad/m}$, so

$$y = (5.12 \text{ cm}) \sin[(4.95 \text{ rad/m})x - (4.08 \text{ rad/s})t + \phi],$$

where ϕ is determined by $(4.95 \text{ rad/m})(9.60 \times 10^{-2} \text{ m}) + \phi = (1.16 \text{ rad})$, or $\phi = 0.685 \text{ rad}$.

(d) $F = \mu v^2 = (0.386 \text{ kg/m})(0.826 \text{ m/s})^2 = 0.263 \text{ m/s}$.

P18-5 We want to show that $dy/dx = u_y/v$. The easy way, although not mathematically rigorous:

$$\frac{dy}{dx} = \frac{dy}{dx} \frac{dt}{dt} = \frac{dy}{dt} \frac{dt}{dx} = u_y \frac{1}{v} = \frac{u_y}{x}.$$

P18-6 The maximum value for u_y occurs when the cosine function in Eq. 18-14 returns unity. Consequently, $u_m/y_m = \omega$.

P18-7 (a) The linear mass density changes as the rubber band is stretched! In this case,

$$\mu = \frac{m}{L + \Delta L}.$$

The tension in the rubber band is given by $F = k\Delta L$. Substituting this into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta L(L + \Delta L)}{m}}.$$

(b) We want to know the time it will take to travel the length of the rubber band, so

$$v = \frac{L + \Delta L}{t} \text{ or } t = \frac{L + \Delta L}{v}.$$

Into this we will substitute our expression for wave speed

$$t = (L + \Delta L) \sqrt{\frac{m}{k\Delta L(L + \Delta L)}} = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}}$$

We have two possibilities to consider: either $\Delta L \ll L$ or $\Delta L \gg L$. In either case we are only interested in the part of the expression with $L + \Delta L$; whichever term is much larger than the other will be the only significant part.

Then if $\Delta L \ll L$ we get $L + \Delta L \approx L$ and

$$t = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}} \approx \sqrt{\frac{mL}{k\Delta L}},$$

so that t is proportional to $1/\sqrt{\Delta L}$.

But if $\Delta L \gg L$ we get $L + \Delta L \approx \Delta L$ and

$$t = \sqrt{\frac{m(L + \Delta L)}{k\Delta L}} \approx \sqrt{\frac{m\Delta L}{k\Delta L}} = \sqrt{\frac{m}{k}},$$

so that t is constant.

P18-8 (a) The tension in the rope at some point is a function of the weight of the cable beneath it. If the bottom of the rope is $y = 0$, then the weight beneath some point y is $W = y(m/L)g$. The speed of the wave at that point is $v = \sqrt{T/(m/L)} = \sqrt{y(M/L)g/(m/L)} = \sqrt{gy}$.

(b) $dy/dt = \sqrt{gy}$, so

$$\begin{aligned} dt &= \frac{dy}{\sqrt{gy}}, \\ t &= \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{L/g}. \end{aligned}$$

(c) No.

P18-9 (a) $M = \int \mu dx$, so

$$M = \int_0^L kx dx = \frac{1}{2}kL^2.$$

Then $k = 2M/L^2$.

(b) $v = \sqrt{F/\mu} = \sqrt{F/kx}$, then

$$\begin{aligned} dt &= \sqrt{kx/F} dx, \\ t &= \int_0^L \sqrt{2M/F L^2} \sqrt{x} dx = \frac{2}{3} \sqrt{2M/F L^2} L^{3/2} = \sqrt{8ML/9F}. \end{aligned}$$

P18-10 Take a cue from pressure and surface tension. In the rotating *non-inertial* reference frame for which the hoop appears to be at rest there is an effective force per unit length acting to push on each part of the loop directly away from the center. This force per unit length has magnitude

$$\frac{\Delta F}{\Delta L} = (\Delta m / \Delta L) \frac{v^2}{r} = \mu \frac{v^2}{r}.$$

There must be a tension T in the string to hold the loop together. Imagine the loop to be replaced with two semicircular loops. Each semicircular loop has a diameter part; the force tending to pull off the diameter section is $(\Delta F / \Delta L)2r = 2\mu v^2$. There are two connections to the diameter section, so the tension in the string must be half the force on the diameter section, or $T = \mu v^2$.

The wave speed is $v_w = \sqrt{T/\mu} = v$.

Note that the wave on the string can travel in either direction relative to an inertial observer. One wave will appear to be fixed in space; the other will move around the string with twice the speed of the string.

P18-11 If we assume that Handel wanted his violins to play in tune with the other instruments then all we need to do is find an instrument from Handel's time that will accurately keep pitch over a period of several hundred years. Most instruments won't keep pitch for even a few days because of temperature and humidity changes; some (like the piccolo?) can't even play in tune for more than a few notes! But if someone found a tuning fork...

Since the length of the string doesn't change, and we are using a string with the same mass density, the only choice is to change the tension. But $f \propto v \propto \sqrt{T}$, so the percentage change in the tension of the string is

$$\frac{T_f - T_i}{T_i} = \frac{f_f^2 - f_i^2}{f_i^2} = \frac{(440 \text{ Hz})^2 - (422.5 \text{ Hz})^2}{(422.5 \text{ Hz})^2} = 8.46\%.$$

P18-12

P18-13 (a) The point sources emit spherical waves; the solution to the appropriate wave equation is found in Ex. 18-14:

$$y_i = \frac{A}{r_i} \sin(kr_i - \omega t).$$

If r_i is sufficiently large compared to A , and $r_1 \approx r_2$, then let $r_1 = r - \delta r$ and $r_2 = r + \delta r$;

$$\frac{A}{r_1} + \frac{A}{r_2} \approx \frac{2A}{r},$$

with an error of order $(\delta r/r)^2$. So ignore it.

Then

$$\begin{aligned} y_1 + y_2 &\approx \frac{A}{r} [\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)], \\ &= \frac{2A}{r} \sin(kr - \omega t) \cos \frac{k}{2}(r_1 - r_2), \\ y_m &= \frac{2A}{r} \cos \frac{k}{2}(r_1 - r_2). \end{aligned}$$

(b) A maximum (minimum) occurs when the operand of the cosine, $k(r_1 - r_2)/2$ is an integer multiple of π (a half odd-integer multiple of π)

P18-14 The direct wave travels a distance d from S to D . The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D . The wave which reflects off the layer after it has risen a distance h travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n\lambda,$$

and later we have destructive interference so

$$\sqrt{d^2 + 4(H+h)^2} - d = (n + 1/2)\lambda.$$

We don't know n , but we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = \lambda/2$$

P18-15 The wavelength is

$$\lambda = v/f = (3.00 \times 10^8 \text{ m/s}) / (13.0 \times 10^6 \text{ Hz}) = 23.1 \text{ m}.$$

The direct wave travels a distance d from S to D . The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D . The wave which reflects off the layer one minute later travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n_1\lambda,$$

and then one minute later we have

$$\sqrt{d^2 + 4(H+h)^2} - d = n_2\lambda.$$

We don't know either n_1 or n_2 , but we do know the difference is 6, so we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = 6\lambda$$

We could use that expression as written, do some really obnoxious algebra, and then get the answer. But we don't want to; we want to take advantage of the fact that h is small compared to d and H . Then the first term can be written as

$$\begin{aligned} \sqrt{d^2 + 4(H+h)^2} &= \sqrt{d^2 + 4H^2 + 8Hh + 4h^2}, \\ &\approx \sqrt{d^2 + 4H^2 + 8Hh}, \\ &\approx \sqrt{d^2 + 4H^2} \sqrt{1 + \frac{8H}{d^2 + 4H^2}h}, \\ &\approx \sqrt{d^2 + 4H^2} \left(1 + \frac{1}{2} \frac{8H}{d^2 + 4H^2}h \right). \end{aligned}$$

Between the second and the third lines we factored out $d^2 + 4H^2$; that last line is from the binomial expansion theorem. We put this into the previous expression, and

$$\begin{aligned} \sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} &= 6\lambda, \\ \sqrt{d^2 + 4H^2} \left(1 + \frac{4H}{d^2 + 4H^2}h \right) - \sqrt{d^2 + 4H^2} &= 6\lambda, \\ \frac{4H}{\sqrt{d^2 + 4H^2}}h &= 6\lambda. \end{aligned}$$

Now what were we doing? We were trying to find the speed at which the layer is moving. We know H , d , and λ ; we can then find h ,

$$h = \frac{6(23.1 \text{ m})}{4(510 \times 10^3 \text{ m})} \sqrt{(230 \times 10^3 \text{ m})^2 + 4(510 \times 10^3 \text{ m})^2} = 71.0 \text{ m}.$$

The layer is then moving at $v = (71.0 \text{ m})/(60 \text{ s}) = 1.18 \text{ m/s}$.

P18-16 The equation of the standing wave is

$$y = 2y_m \sin kx \cos \omega t.$$

The transverse speed of a point on the string is the derivative of this, or

$$u_y = -2y_m \omega \sin kx \sin \omega t,$$

this has a maximum value when $\omega t - \pi/2$ is an integer multiple of π . The maximum value is

$$u_m = 2y_m \omega \sin kx.$$

Each mass element on the string dm then has a maximum kinetic energy

$$dK_m = (dm/2)u_m^2 = y_m^2 \omega^2 \sin^2 kx dm.$$

Using $dm = \mu dx$, and integrating over one loop from $kx = 0$ to $kx = \pi$, we get

$$K_m = y_m^2 \omega^2 \mu / 2k = 2\pi^2 y_m^2 f \mu v.$$

P18-17 (a) For 100% reflection the amplitudes of the incident and reflected wave are equal, or $A_i = A_r$, which puts a zero in the denominator of the equation for SWR. If there is no reflection, $A_r = 0$ leaving the expression for SWR to reduce to $A_i/A_i = 1$.

(b) $P_r/P_i = A_r^2/A_i^2$. Do the algebra:

$$\begin{aligned} \frac{A_i + A_r}{A_i - A_r} &= \text{SWR}, \\ A_i + A_r &= \text{SWR}(A_i - A_r), \\ A_r(\text{SWR} + 1) &= A_i(\text{SWR} - 1), \\ A_r/A_i &= (\text{SWR} - 1)/(\text{SWR} + 1). \end{aligned}$$

Square this, and multiply by 100.

P18-18 Measure with a ruler; I get $2A_{\max} = 1.1 \text{ cm}$ and $2A_{\min} = 0.5 \text{ cm}$.

(a) $\text{SWR} = (1.1/0.5) = 2.2$

(b) $(2.2 - 1)^2/(2.2 + 1)^2 = 0.14 \%$.

P18-19 (a) Call the three waves

$$\begin{aligned} y_i &= A \sin k_1(x - v_1 t), \\ y_t &= B \sin k_2(x - v_2 t), \\ y_r &= C \sin k_1(x + v_1 t), \end{aligned}$$

where the subscripts i, t, and r refer to the incident, transmitted, and reflected waves respectively.

Apply the principle of superposition. Just to the left of the knot the wave has amplitude $y_i + y_r$ while just to the right of the knot the wave has amplitude y_t . These two amplitudes must line up at the knot for all times t , or the knot will come undone. Remember the knot is at $x = 0$, so

$$\begin{aligned} y_i + y_r &= y_t, \\ A \sin k_1(-v_1t) + C \sin k_1(+v_1t) &= B \sin k_2(-v_2t), \\ -A \sin k_1v_1t + C \sin k_1v_1t &= -B \sin k_2v_2t \end{aligned}$$

We know that $k_1v_1 = k_2v_2 = \omega$, so the three sin functions are all equivalent, and can be canceled. This leaves $A = B + C$.

(b) We need to match more than the displacement, we need to match the slope just on either side of the knot. In that case we need to take the derivative of

$$y_i + y_r = y_t$$

with respect to x , and then set $x = 0$. First we take the derivative,

$$\begin{aligned} \frac{d}{dx}(y_i + y_r) &= \frac{d}{dx}(y_t), \\ k_1A \cos k_1(x - v_1t) + k_1C \cos k_1(x + v_1t) &= k_2B \cos k_2(x - v_2t), \end{aligned}$$

and then we set $x = 0$ and simplify,

$$\begin{aligned} k_1A \cos k_1(-v_1t) + k_1C \cos k_1(+v_1t) &= k_2B \cos k_2(-v_2t), \\ k_1A \cos k_1v_1t + k_1C \cos k_1v_1t &= k_2B \cos k_2v_2t. \end{aligned}$$

This last expression simplifies like the one in part (a) to give

$$k_1(A + C) = k_2B$$

We can combine this with $A = B + C$ to solve for C ,

$$\begin{aligned} k_1(A + C) &= k_2(A - C), \\ C(k_1 + k_2) &= A(k_2 - k_1), \\ C &= A \frac{k_2 - k_1}{k_1 + k_2}. \end{aligned}$$

If $k_2 < k_1$ C will be negative; this means the reflected wave will be inverted.

P18-20

P18-21 Find the wavelength from

$$\lambda = 2(0.924 \text{ m})/4 = 0.462 \text{ m}.$$

Find the wavespeed from

$$v = f\lambda = (60.0 \text{ Hz})(0.462 \text{ m}) = 27.7 \text{ m/s}.$$

Find the tension from

$$F = \mu v^2 = (0.0442 \text{ kg})(27.7 \text{ m/s})^2/(0.924 \text{ m}) = 36.7 \text{ N}.$$

P18-22 (a) The frequency of vibration f is the same for both the aluminum and steel wires; they do not, however, need to vibrate in the same mode. The speed of waves in the aluminum is v_1 , that in the steel is v_2 . The aluminum vibrates in a mode given by $n_1 = 2L_1f/v_1$, the steel vibrates in a mode given by $n_2 = 2L_2f/v_2$. Both n_1 and n_2 need be integers, so the ratio must be a rational fraction. Note that the ratio is independent of f , so that L_1 and L_2 must be chosen correctly for this problem to work at all!

This ratio is

$$\frac{n_2}{n_1} = \frac{L_2}{L_1} \sqrt{\frac{\mu_2}{\mu_1}} = \frac{(0.866 \text{ m})}{(0.600 \text{ m})} \sqrt{\frac{(7800 \text{ kg/m}^3)}{(2600 \text{ kg/m}^3)}} = 2.50 \approx \frac{5}{2}$$

Note that since the wires have the same tension and the same cross sectional area it is acceptable to use the volume density instead of the linear density in the problem.

The smallest integer solution is then $n_1 = 2$ and $n_2 = 5$. The frequency of vibration is then

$$f = \frac{n_1 v}{2L_1} = \frac{n_1}{2L_1} \sqrt{\frac{T}{\rho_1 A}} = \frac{(2)}{2(0.600 \text{ m})} \sqrt{\frac{(10.0 \text{ kg})(9.81 \text{ m/s}^2)}{(2600 \text{ kg/m}^3)(1.00 \times 10^{-6} \text{ m}^2)}} = 323 \text{ Hz.}$$

(b) There are three nodes in the aluminum and six in the steel. But one of those nodes is shared, and two are on the ends of the wire. The answer is then six.