E17-1 For a perfect spring $|F| = k|x|$. $x = 0.157\,\text{m}$ when $3.94\,\text{kg}$ is suspended from it. There would be two forces on the object— the force of gravity, $W = mg$, and the force of the spring, F. These two force must balance, so $mg = kx$ or

$$k = \frac{mg}{x} = \frac{(3.94\,\text{kg})(9.81\,\text{m/s}^2)}{(0.157\,\text{m})} = 0.246\,\text{N/m}.$$

Now that we know k, the spring constant, we can find the period of oscillations from Eq. 17-8,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(0.520\,\text{kg})}{(0.246\,\text{N/m})}} = 0.289\,\text{s}.$$

E17-2 (a) $T = 0.484\,\text{s}$.
(b) $f = 1/T = 1/(0.484\,\text{s}) = 2.07\,\text{s}^{-1}$.
(c) $\omega = 2\pi f = 13.0$ rad/s.
(d) $k = m\omega^2 = (0.512\,\text{kg})(13.0\text{ rad/s})^2 = 86.5\,\text{N/m}$.
(e) $v_\text{m} = \omega x_\text{m} = (13.0\text{ rad/s})(0.347\,\text{m}) = 4.51\,\text{m/s}$.
(f) $F_\text{m} = ma_\text{m} = (0.512\,\text{kg})(13.0\text{ rad/s})^2(0.347\,\text{m}) = 30.0\,\text{N}$.

E17-3 $a_\text{m} = (2\pi f)^2 x_\text{m}$. Then

$$f = \sqrt{(9.81\,\text{m/s}^2)/(1.20\times10^{-6}\text{m})}/(2\pi) = 455\text{ Hz}.$$

E17-4 (a) $\omega = (2\pi)/(0.645\,\text{s}) = 9.74$ rad/s. $k = m\omega^2 = (5.22\,\text{kg})(9.74\text{ rad/s})^2 = 495\,\text{N/m}$.
(b) $x_\text{m} = v_\text{m}/\omega = (0.153\,\text{m/s})/(9.74\text{ rad/s}) = 1.57\times10^{-2}\text{m}$.
(c) $f = 1/(0.645\,\text{s}) = 1.55$ Hz.

E17-5 (a) The amplitude is half of the distance between the extremes of the motion, so $A = (2.00\text{ mm})/2 = 1.00$ mm.
(b) The maximum blade speed is given by $v_\text{m} = \omega x_\text{m}$. The blade oscillates with a frequency of 120 Hz, so $\omega = 2\pi f = 2\pi(120\,\text{s}^{-1}) = 754$ rad/s, and then $v_\text{m} = (754\text{ rad/s})(0.001\,\text{m}) = 0.754\,\text{m/s}$.
(c) Similarly, $a_\text{m} = \omega^2 x_\text{m}$, $a_\text{m} = (754\text{ rad/s})^2(0.001\,\text{m}) = 568\,\text{m/s}^2$.

E17-6 (a) $k = m\omega^2 = (1460\,\text{kg}/4)(2\pi 2.95/\text{s})^2 = 1.25\times10^5\text{N/m}$
(b) $f = \sqrt{k/m}/2\pi = \sqrt{(1.25\times10^5\text{N/m})/(1830\,\text{kg}/4)}/2\pi = 2.63/\text{s}$.

E17-7 (a) $x = (6.12\,\text{m})\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 3.27\,\text{m}$.
(b) $v = -(6.12\,\text{m})(8.38/\text{s})\sin[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 43.4\,\text{m/s}$.
(c) $a = -(6.12\,\text{m})(8.38/\text{s})^2\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = -229\,\text{m/s}^2$.
(d) $f = (8.38\text{ rad/s})/2\pi = 1.33/\text{s}$.
(e) $T = 1/f = 0.750\,\text{s}$.

E17-8 $k = (50.0\text{ lb})/(4.00\text{ in}) = 12.5$ lb/in.

$$mg = \frac{(32\text{ ft/s}^2)(12\text{ in/ft})(12.5\text{ lb/in})}{[2\pi(2.00/\text{s})]^2} = 30.4\text{ lb}.$$

E17-9 If the drive wheel rotates at 193 rev/min then

$$\omega = (193\text{ rev/min})(2\pi\text{ rad/rev})(1/60\text{ s/min}) = 20.2\text{ rad/s},$$

then $v_\text{m} = \omega x_\text{m} = (20.2\text{ rad/s})(0.3825\,\text{m}) = 7.73\,\text{m/s}$.

E17-10 $k = (0.325\,\text{kg})(9.81\,\text{m/s}^2)/(1.80\times10^{-2}\,\text{m}) = 177\,\text{N/m}.$.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(2.14\,\text{kg})}{(177\,\text{N/m})}} = 0.691\,\text{s}.$$

E17-11 For the tides $\omega = 2\pi/(12.5\text{ h})$. Half the maximum occurs when $\cos\omega t = 1/2$, or $\omega t = \pi/3$. Then $t = (12.5\text{ h})/6 = 2.08$ h.

E17-12 The two will separate if the (maximum) acceleration exceeds g.
(a) Since $\omega = 2\pi/T = 2\pi/(1.18\,\text{s}) = 5.32$ rad/s the maximum amplitude is

$$x_\text{m} = (9.81\,\text{m/s}^2)/(5.32\text{ rad/s})^2 = 0.347\,\text{m}.$$

(b) In this case $\omega = \sqrt{(9.81\,\text{m/s}^2)/(0.0512\,\text{m})} = 13.8$ rad/s. Then $f = (13.8\text{ rad/s})/2\pi = 2.20/\text{s}$.

E17-13 (a) $a_x/x = -\omega^2$. Then

$$\omega = \sqrt{-(-123\,\text{m/s})/(0.112\,m)} = 33.1\text{ rad/s},$$

so $f = (33.1\text{ rad/s})/2\pi = 5.27/\text{s}$.
(b) $m = k/\omega^2 = (456\,\text{N/m})/(33.1\text{ rad/s})^2 = 0.416\,\text{kg}$.
(c) $x = x_\text{m}\cos\omega t$; $v = -x_\text{m}\omega\sin\omega t$. Combining,

$$x^2 + (v/\omega)^2 = {x_\text{m}}^2\cos^2\omega t + {x_\text{m}}^2\sin^2\omega t = {x_\text{m}}^2.$$

Consequently,

$$x_\text{m} = \sqrt{(0.112\,\text{m})^2 + (-13.6\,\text{m/s})^2/(33.1\text{ rad/s})^2} = 0.426\,\text{m}.$$

E17-14 $x_1 = x_\text{m}\cos\omega t$, $x_2 = x_\text{m}\cos(\omega t + \phi)$. The crossing happens when $x_1 = x_\text{m}/2$, or when $\omega t = \pi/3$ (and other values!). The same constraint happens for x_2, except that it is moving in the other direction. The closest value is $\omega t + \phi = 2\pi/3$, or $\phi = \pi/3$.

E17-15 (a) The net force on the three cars is zero before the cable breaks. There are three forces on the cars: the weight, W, a normal force, N, and the upward force from the cable, F. Then

$$F = W\sin\theta = 3mg\sin\theta.$$

This force is from the elastic properties of the cable, so

$$k = \frac{F}{x} = \frac{3mg\sin\theta}{x}$$

The frequency of oscillation of the remaining two cars after the bottom car is released is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = \frac{1}{2\pi}\sqrt{\frac{3mg\sin\theta}{2mx}} = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}}.$$

Numerically, the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}} = \frac{1}{2\pi}\sqrt{\frac{3(9.81\,\text{m/s}^2)\sin(26^\circ)}{2(0.142\,\text{m})}} = 1.07\text{ Hz}.$$

(b) Each car contributes equally to the stretching of the cable, so one car causes the cable to stretch $14.2/3 = 4.73$ cm. The amplitude is then 4.73 cm.

E17-16 Let the height of one side over the equilibrium position be x. The net restoring force on the liquid is $2\rho Axg$, where A is the cross sectional area of the tube and g is the acceleration of free-fall. This corresponds to a spring constant of $k = 2\rho Ag$. The mass of the fluid is $m = \rho AL$. The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{2L}{g}}.$$

E17-17 (a) There are two forces on the log. The weight, $W = mg$, and the buoyant force B. We'll assume the log is cylindrical. If x is the length of the log beneath the surface and A the cross sectional area of the log, then $V = Ax$ is the volume of the displaced water. Furthermore, $m_\text{w} = \rho_\text{w} V$ is the mass of the displaced water and $B = m_\text{w} g$ is then the buoyant force on the log. Combining,

$$B = \rho_\text{w} Agx,$$

where ρ_w is the density of water. This certainly looks similar to an elastic spring force law, with $k = \rho_\text{w} Ag$. We would then expect the motion to be simple harmonic.

(b) The period of the oscillation would be

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\rho_\text{w} Ag}},$$

where m is the total mass of the log and lead. We are told the log is in equilibrium when $x = L = 2.56$ m. This would give us the *weight* of the log, since $W = B$ is the condition for the log to float. Then

$$m = \frac{B}{g} = \frac{\rho_\text{w} AgL}{g} = \rho AL.$$

From this we can write the period of the motion as

$$T = 2\pi\sqrt{\frac{\rho AL}{\rho_\text{w} Ag}} = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{(2.56\,\text{m})}{(9.81\,\text{m/s}^2)}} = 3.21\,\text{s}.$$

E17-18 (a) $k = 2(1.18\,\text{J})/(0.0984\,\text{m})^2 = 244\,\text{N/m}$.

(b) $m = 2(1.18\,\text{J})/(1.22\,\text{m/s})^2 = 1.59\,\text{kg}$.

(c) $f = [(1.22\,\text{m/s})/(0.0984\,\text{m})]/(2\pi) = 1.97/\text{s}$.

E17-19 (a) Equate the kinetic energy of the object just after it leaves the slingshot with the potential energy of the stretched slingshot.

$$k = \frac{mv^2}{x^2} = \frac{(0.130\,\text{kg})(11.2\times 10^3\text{m/s})^2}{(1.53\,\text{m})^2} = 6.97\times 10^6\text{N/m}.$$

(b) $N = (6.97\times 10^6\,\text{N/m})(1.53\,\text{m})/(220\,\text{N}) = 4.85\times 10^4$ people.

E17-20 (a) $E = k{x_\text{m}}^2/2$, $U = kx^2/2 = k(x_\text{m}/2)^2/2 = E/4$. $K = E - U = 3E/4$. The the energy is 25% potential and 75% kinetic.

(b) If $U = E/2$ then $kx^2/2 = k{x_\text{m}}^2/4$, or $x = x_\text{m}/\sqrt{2}$.

E17-21 (a) $a_{\rm m} = \omega^2 x_{\rm m}$ so

$$\omega = \sqrt{\frac{a_{\rm m}}{x_{\rm m}}} = \sqrt{\frac{(7.93\times10^3\,{\rm m/s^2})}{(1.86\times10^{-3}\,{\rm m})}} = 2.06\times10^3\ {\rm rad/s}$$

The period of the motion is then

$$T = \frac{2\pi}{\omega} = 3.05\times10^{-3}{\rm s}.$$

(b) The maximum speed of the particle is found by

$$v_{\rm m} = \omega x_{\rm m} = (2.06\times10^3\ {\rm rad/s})(1.86\times10^{-3}\,{\rm m}) = 3.83\,{\rm m/s}.$$

(c) The mechanical energy is given by Eq. 17-15, except that we will focus on when $v_x = v_{\rm m}$, because then $x = 0$ and

$$E = \frac{1}{2}m{v_{\rm m}}^2 = \frac{1}{2}(12.3\,{\rm kg})(3.83\,{\rm m/s})^2 = 90.2\,{\rm J}.$$

E17-22 (a) $f = \sqrt{k/m}/2\pi = \sqrt{(988\,{\rm N/m})/(5.13\,{\rm kg})}/2\pi = 2.21/{\rm s}$.
(b) $U_{\rm i} = (988\,{\rm N/m})(0.535\,{\rm m})^2/2 = 141\,{\rm J}$.
(c) $K_{\rm i} = (5.13\,{\rm kg})(11.2\,{\rm m/s})^2/2 = 322\,{\rm J}$.
(d) $x_{\rm m} = \sqrt{2E/k} = \sqrt{2(322\,{\rm J} + 141\,{\rm J})/(988\,{\rm N/m})} = 0.968\,{\rm m}$.

E17-23 (a) $\omega = \sqrt{(538\,{\rm N/m})/(1.26\,{\rm kg})} = 20.7\ {\rm rad/s}$.

$$x_{\rm m} = \sqrt{(0.263\,{\rm m})^2 + (3.72\,{\rm m/s})^2/(20.7\ {\rm rad/s})^2} = 0.319\,{\rm m}.$$

(b) $\phi = \arctan\{-(-3.72\,{\rm m/s})/[(20.7\ {\rm rad/s})(0.263\,{\rm m})]\} = 34.3°$.

E17-24 Before doing anything else apply conservation of momentum. If v_0 is the speed of the bullet just before hitting the block and v_1 is the speed of the bullet/block system just after the two begin moving as one, then $v_1 = mv_0/(m+M)$, where m is the mass of the bullet and M is the mass of the block.

For this system $\omega = \sqrt{k/(m+M)}$.

(a) The total energy of the oscillation is $\frac{1}{2}(m+M)v_1^2$, so the amplitude is

$$x_{\rm m} = \sqrt{\frac{m+M}{k}}v_1 = \sqrt{\frac{m+M}{k}}\frac{mv_0}{m+M} = mv_0\sqrt{\frac{1}{k(m+M)}}.$$

The numerical value is

$$x_{\rm m} = (0.050\,{\rm kg})(150\,{\rm m/s})\sqrt{\frac{1}{(500\,{\rm N/m})(0.050\,{\rm kg} + 4.00\,{\rm kg})}} = 0.167\,{\rm m}.$$

(b) The fraction of the energy is

$$\frac{(m+M)v_1^2}{mv_0^2} = \frac{m+M}{m}\left(\frac{m}{m+M}\right)^2 = \frac{m}{m+M} = \frac{(0.050\,{\rm kg})}{(0.050\,{\rm kg} + 4.00\,{\rm kg})} = 1.23\times10^{-2}.$$

E17-25 $L = (9.82\,{\rm m/s^2})(1.00\,{\rm s}/2\pi)^2 = 0.249\,{\rm m}$.

E17-26 $T = (180\,\mathrm{s})/(72.0)$. Then

$$g = \left(\frac{2\pi(72.0)}{(180\,\mathrm{s})}\right)^2 (1.53\,\mathrm{m}) = 9.66\,\mathrm{m/s}.$$

E17-27 We are interested in the value of θ_m which will make the second term 2% of the first term. We want to solve

$$0.02 = \frac{1}{2^2}\sin^2\frac{\theta_\mathrm{m}}{2},$$

which has solution

$$\sin\frac{\theta_\mathrm{m}}{2} = \sqrt{0.08}$$

or $\theta_m = 33°$.

(b) How large is the third term at this angle?

$$\frac{3^2}{2^2 4^2}\sin^4\frac{\theta_m}{2} = \frac{3^2}{2^2}\left(\frac{1}{2^2}\sin^2\frac{\theta_m}{2}\right)^2 = \frac{9}{4}(0.02)^2$$

or 0.0009, which is very small.

E17-28 Since $T \propto \sqrt{1/g}$ we have

$$T_\mathrm{p} = T_\mathrm{e}\sqrt{g_\mathrm{e}/g_\mathrm{p}} = (1.00\,\mathrm{s})\sqrt{\frac{(9.78\,\mathrm{m/s^2})}{(9.834\,\mathrm{m/s^2})}} = 0.997\,\mathrm{s}.$$

E17-29 Let the period of the clock in Paris be T_1. In a day of length $D_1 = 24$ hours it will undergo $n = D/T_1$ oscillations. In Cayenne the period is T_2. n oscillations should occur in 24 hours, but since the clock runs slow, D_2 is 24 hours + 2.5 minutes elapse. So

$$T_2 = D_2/n = (D_2/D_1)T_1 = [(1442.5\text{ min})/(1440.0\text{ min})]T_1 = 1.0017T_1.$$

Since the ratio of the periods is $(T_2/T_1) = \sqrt{(g_1/g_2)}$, the g_2 in Cayenne is

$$g_2 = g_1(T_1/T_2)^2 = (9.81\,\mathrm{m/s^2})/(1.0017)^2 = 9.78\,\mathrm{m/s^2}.$$

E17-30 (a) Take the differential of

$$g = \left(\frac{2\pi(100)}{T}\right)^2 (10\,\mathrm{m}) = \frac{4\pi^2\times 10^5\,\mathrm{m}}{T^2},$$

so $\delta g = (-8\pi^2\times 10^5\,\mathrm{m}/T^3)\delta T$. Note that T is not the period here, it is the time for 100 oscillations! The relative error is then

$$\frac{\delta g}{g} = -2\frac{\delta T}{T}.$$

If $\delta g/g = 0.1\%$ then $\delta T/T = 0.05\%$.

(b) For $g \approx 10\,\mathrm{m/s^2}$ we have

$$T \approx 2\pi(100)\sqrt{(10\,\mathrm{m})/(10\,\mathrm{m/s^2})} = 628\,\mathrm{s}.$$

Then $\delta T \approx (0.0005)(987\,\mathrm{s}) \approx 300$ ms.

E17-31 $T = 2\pi\sqrt{(17.3\,\text{m})/(9.81\,\text{m/s}^2)} = 8.34\,\text{s}$.

E17-32 The spring will extend until the force from the spring balances the weight, or when $Mg = kh$. The frequency of this system is then

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{Mg/h}{M}} = \frac{1}{2\pi}\sqrt{\frac{g}{h}},$$

which is the frequency of a pendulum of length h. The mass of the bob is irrelevant.

E17-33 The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}},$$

where d is the distance from the pivot about which the hoop oscillates and the center of mass of the hoop.

The rotational inertia I is about an axis through the pivot, so we apply the parallel axis theorem. Then

$$I = Md^2 + I_{\text{cm}} = Md^2 + Mr^2.$$

But d is r, since the pivot point is on the rim of the hoop. So $I = 2Md^2$, and the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{2Md^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2d}} = \frac{1}{2\pi}\sqrt{\frac{(9.81\,\text{m/s}^2)}{2(0.653\,\text{m})}} = 0.436\text{ Hz}.$$

(b) Note the above expression looks like the simple pendulum equation if we replace $2d$ with l. Then the equivalent length of the simple pendulum is $2(0.653\,\text{m}) = 1.31\,\text{m}$.

E17-34 Apply Eq. 17-21:

$$I = \frac{T^2\kappa}{4\pi^2} = \frac{(48.7\,\text{s}/20.0)^2(0.513\,\text{N}\cdot\text{m})}{4\pi^2} = 7.70\times10^{-2}\text{kg}\cdot\text{m}^2.$$

E17-35 $\kappa = (0.192\,\text{N}\cdot\text{m})/(0.850\text{ rad}) = 0.226\,\text{N}\cdot\text{m}$. $I = \frac{2}{5}(95.2\,\text{kg})(0.148\,\text{m})^2 = 0.834\,\text{kg}\cdot\text{m}^2$. Then

$$T = 2\pi\sqrt{I/\kappa} = 2\pi\sqrt{(0.834\,\text{kg}\cdot\text{m}^2)/(0.226\,\text{N}\cdot\text{m})} = 12.1\,\text{s}.$$

E17-36 x is d in Eq. 17-29. Since the hole is drilled off center we apply the principle axis theorem to find the rotational inertia:

$$I = \frac{1}{12}ML^2 + Mx^2.$$

Then

$$\begin{aligned}\frac{1}{12}ML^2 + Mx^2 &= \frac{T^2Mgx}{4\pi^2},\\ \frac{1}{12}(1.00\,\text{m})^2 + x^2 &= \frac{(2.50\,\text{s})^2(9.81\,\text{m/s}^2)}{4\pi^2}x,\\ (8.33\times10^{-2}\text{m}^2) - (1.55\,\text{m})x + x^2 &= 0.\end{aligned}$$

This has solutions $x = 1.49\,\text{m}$ and $x = 0.0557\,\text{m}$. Use the latter.

E17-37 For a stick of length L which can pivot about the end, $I = \frac{1}{3}ML^2$. The center of mass of such a stick is located $d = L/2$ away from the end.

The frequency of oscillation of such a stick is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{Mg\,(L/2)}{\frac{1}{3}ML^2}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{3g}{2L}}. \end{aligned}$$

This means that f is proportional to $\sqrt{1/L}$, regardless of the mass or density of the stick. The ratio of the frequency of two such sticks is then $f_2/f_1 = \sqrt{L_1/L_2}$, which in our case gives

$$f_2 = f_1\sqrt{L_2/L_1} = f_1\sqrt{(L_1)/(2L_1/3)} = 1.22 f_1.$$

E17-38 The rotational inertia of the pipe section about the cylindrical axis is

$$I_{\rm cm} = \frac{M}{2}\left[r_1^2 + r_2^2\right] = \frac{M}{2}\left[(0.102\,\mathrm{m})^2 + (0.1084\,\mathrm{m})^2\right] = (1.11\times 10^{-2}\mathrm{m}^2)M$$

(a) The total rotational inertia about the pivot axis is

$$I = 2I_{\rm cm} + M(0.102\,\mathrm{m})^2 + M(0.3188\,\mathrm{m})^2 = (0.134\,\mathrm{m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.134\,\mathrm{m}^2)M}{M(9.81\,\mathrm{m/s}^2)(0.2104\,\mathrm{m})}} = 1.60\,\mathrm{s}$$

(b) The rotational inertia of the pipe section about a diameter is

$$I_{\rm cm} = \frac{M}{4}\left[r_1^2 + r_2^2\right] = \frac{M}{4}\left[(0.102\,\mathrm{m})^2 + (0.1084\,\mathrm{m})^2\right] = (5.54\times 10^{-3}\mathrm{m}^2)M$$

The total rotational inertia about the pivot axis is now

$$I = M(1.11\times 10^{-2}\mathrm{m}^2) + M(0.102\,\mathrm{m})^2 + M(5.54\times 10^{-3}\mathrm{m}^2) + M(0.3188\,\mathrm{m})^2 = (0.129\,\mathrm{m}^2)M$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.129\,\mathrm{m}^2)M}{M(9.81\,\mathrm{m/s}^2)(0.2104\,\mathrm{m})}} = 1.57\,\mathrm{s}.$$

The percentage difference with part (a) is $(0.03\,\mathrm{s})/(1.60\,\mathrm{s}) = 1.9\%$.

E17-39

E17-40

E17-41 (a) Since effectively $x = y$, the path is a diagonal line.

(b) The path will be an ellipse which is symmetric about the line $x = y$.

(c) Since $\cos(\omega t + 90^\circ) = -\sin(\omega t)$, the path is a circle.

E17-42 (a)

(b) Take two time derivatives and multiply by m,

$$\vec{\mathbf{F}} = -mA\omega^2\left(\hat{\mathbf{i}}\cos\omega t + 9\hat{\mathbf{j}}\cos 3\omega t\right).$$

(c) $U = -\int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$, so

$$U = \frac{1}{2}mA^2\omega^2\left(\cos^2\omega t + 9\cos^2 3\omega t\right).$$

(d) $K = \frac{1}{2}mv^2$, so

$$K = \frac{1}{2}mA^2\omega^2\left(\sin^2\omega t + 9\sin^2 3\omega t\right);$$

And then $E = K + U = 5mA^2\omega^2$.

(e) Yes; the period is $2\pi/\omega$.

E17-43 The ω which describes the angular velocity in uniform circular motion is effectively the same ω which describes the angular frequency of the corresponding simple harmonic motion. Since $\omega = \sqrt{k/m}$, we can find the effective force constant k from knowledge of the Moon's mass and the period of revolution.

The moon orbits with a period of T, so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(27.3\times 24\times 3600\,\mathrm{s})} = 2.66\times10^{-6}\mathrm{rad/s}.$$

This can be used to find the value of the effective force constant k from

$$k = m\omega^2 = (7.36\times10^{22}\mathrm{kg})(2.66\times10^{-6}\mathrm{rad/s})^2 = 5.21\times10^{11}\mathrm{N/m}.$$

E17-44 (a) We want to know when $e^{-bt/2m} = 1/3$, or

$$t = \frac{2m}{b}\ln 3 = \frac{2(1.52\,\mathrm{kg})}{(0.227\,\mathrm{kg/s})}\ln 3 = 14.7\,\mathrm{s}$$

(b) The (angular) frequency is

$$\omega' = \sqrt{\left(\frac{(8.13\,\mathrm{N/m})}{(1.52\,\mathrm{kg})}\right) - \left(\frac{(0.227\,\mathrm{kg/s})}{2(1.52\,\mathrm{kg})}\right)^2} = 2.31\ \mathrm{rad/s}.$$

The number of oscillations is then

$$(14.7\,\mathrm{s})(2.31\ \mathrm{rad/s})/2\pi = 5.40$$

E17-45 The first derivative of Eq. 17-39 is

$$\begin{aligned}\frac{dx}{dt} &= x_\mathrm{m}(-b/2m)e^{-bt/2m}\cos(\omega' t+\phi) + x_\mathrm{m}e^{-bt/2m}(-\omega')\sin(\omega' t+\phi),\\ &= -x_\mathrm{m}e^{-bt/2m}\left((b/2m)\cos(\omega' t+\phi) + \omega'\sin(\omega' t+\phi)\right)\end{aligned}$$

The second derivative is quite a bit messier;

$$\begin{aligned}\frac{d^2}{dx^2} &= -x_\mathrm{m}(-b/2m)e^{-bt/2m}\left((b/2m)\cos(\omega' t+\phi) + \omega'\sin(\omega' t+\phi)\right)\\ &\quad -x_\mathrm{m}e^{-bt/2m}\left((b/2m)(-\omega')\sin(\omega' t+\phi) + (\omega')^2\cos(\omega' t+\phi)\right),\\ &= x_\mathrm{m}e^{-bt/2m}\left((\omega' b/m)\sin(\omega' t+\phi) + (b^2/4m^2 - \omega'^2)\cos(\omega' t+\phi)\right).\end{aligned}$$

Substitute these three expressions into Eq. 17-38. There are, however, some fairly obvious simplifications. Every one of the terms above has a factor of $x_{\rm m}$, and every term above has a factor of $e^{-bt/2m}$, so simultaneously with the substitution we will cancel out those factors. Then Eq. 17-38 becomes

$$m\left[(\omega' b/m)\sin(\omega' t+\phi)+(b^2/4m^2-\omega'^2)\cos(\omega' t+\phi)\right]$$
$$-b\left[(b/2m)\cos(\omega' t+\phi)+\omega'\sin(\omega' t+\phi)\right]+k\cos(\omega' t+\phi)=0$$

Now we collect terms with cosine and terms with sine,

$$(\omega' b-\omega' b)\sin(\omega' t+\phi)+\left(mb^2/4m^2-\omega'^2-b^2/2m+k\right)\cos(\omega' t+\phi)=0.$$

The coefficient for the sine term is identically zero; furthermore, because the cosine term must then vanish regardless of the value of t, the coefficient for the sine term must also vanish. Then

$$\left(mb^2/4m^2-m\omega'^2-b^2/2m+k\right)=0,$$

or

$$\omega'^2=\frac{k}{m}-\frac{b^2}{4m^2}.$$

If this condition is met, then Eq. 17-39 is indeed a solution of Eq. 17-38.

E17-46 (a) Four complete cycles requires a time $t_4=8\pi/\omega'$. The amplitude decays to 3/4 the original value in this time, so $0.75=e^{-bt_4/2m}$, or

$$\ln(4/3)=\frac{8\pi b}{2m\omega'}.$$

It is probably reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega'\approx\omega$. We'll do it the hard way anyway. Then

$$\begin{aligned}\omega'^2&=\left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\ \frac{k}{m}-\left(\frac{b}{2m}\right)^2&=\left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\ \frac{k}{m}&=(7630)\left(\frac{b}{2m}\right)^2\end{aligned}$$

Numerically, then,

$$b=\sqrt{\frac{4(1.91\,\mathrm{kg})(12.6\,\mathrm{N/m})}{(7630)}}=0.112\,\mathrm{kg/s}.$$

(b)

E17-47 (a) Use Eqs. 17-43 and 17-44. At resonance $\omega''=\omega$, so

$$G=\sqrt{b^2\omega^2}=b\omega,$$

and then $x_{\rm m}=F_{\rm m}/b\omega$.

(b) $v_{\rm m}=\omega x_{\rm m}=F_{\rm m}/b$.

E17-48 We need the first two derivatives of

$$x = \frac{F_\mathrm{m}}{G}\cos(\omega''t - \beta)$$

The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_\mathrm{m}}{G}(-\omega'')\sin(\omega''t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_\mathrm{m}}{G}(\omega'')^2\cos(\omega''t - \beta),$$

We'll substitute this into Eq. 17-42,

$$m\left(-\frac{F_\mathrm{m}}{G}(\omega'')^2\cos(\omega''t - \beta)\right)$$
$$+b\left(\frac{F_\mathrm{m}}{G}(-\omega'')\sin(\omega''t - \beta)\right) + k\frac{F_\mathrm{m}}{G}\cos(\omega''t - \beta) = F_\mathrm{m}\cos\omega''t.$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega'')^2\right)\cos(\omega''t - \beta) - (b\omega'')\sin(\omega''t - \beta) = G\cos\omega''t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega''t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega'')^2, \\ A\sin\alpha_1 &= b\omega'', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega'')^2\right)^2 + (b\omega'')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega''t - \beta) = G\cos\omega''t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 = \beta$.

E17-49 The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_\mathrm{m}}{G}(-\omega''')\sin(\omega'''t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_\mathrm{m}}{G}(\omega''')^2\cos(\omega'''t - \beta),$$

We'll substitute this into Eq. 17-42,

$$m\left(-\frac{F_\mathrm{m}}{G}(\omega''')^2\cos(\omega'''t - \beta)\right)$$
$$+b\left(\frac{F_\mathrm{m}}{G}(-\omega''')\sin(\omega'''t - \beta)\right) + k\frac{F_\mathrm{m}}{G}\cos(\omega'''t - \beta) = F_\mathrm{m}\cos\omega''t.$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega''')^2\right)\cos(\omega'''t - \beta) - (b\omega''')\sin(\omega'''t - \beta) = G\cos\omega''t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega'''t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega''')^2, \\ A\sin\alpha_1 &= b\omega''', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega''')^2\right)^2 + (b\omega''')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega'''t - \beta) = G\cos\omega''t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 + \omega'''t - \beta = \omega''t$ and $\alpha_1 = \beta$ and $\omega''' = \omega''$.

E17-50 Actually, Eq. 17-39 is *not* a solution to Eq. 17-42 by itself, this is a wording mistake in the exercise. Instead, Eq. 17-39 can be added to *any* solution of Eq. 17-42 and the result will still be a solution.

Let x_n be *any* solution to Eq. 17-42 (such as Eq. 17-43.) Let x_h be given by Eq. 17-39. Then

$$x = x_\mathrm{n} + x_\mathrm{h}.$$

Take the first two time derivatives of this expression.

$$\begin{aligned} \frac{dx}{dt} &= \frac{dx_\mathrm{n}}{dt} + \frac{dx_\mathrm{h}}{dt}, \\ \frac{d^2x}{dt^2} &= \frac{d^2x_\mathrm{n}}{dt^2} + \frac{d^2x_\mathrm{h}}{dt^2} \end{aligned}$$

Substitute these three expressions into Eq. 17-42.

$$m\left(\frac{d^2x_\mathrm{n}}{dt^2} + \frac{d^2x_\mathrm{h}}{dt^2}\right) + b\left(\frac{dx_\mathrm{n}}{dt} + \frac{dx_\mathrm{h}}{dt}\right) + k\left(x_\mathrm{n} + x_\mathrm{h}\right) = F_\mathrm{m}\cos\omega''t.$$

Rearrange and regroup.

$$\left(m\frac{d^2x_\mathrm{n}}{dt^2} + b\frac{dx_\mathrm{n}}{dt} + kx_\mathrm{n}\right) + \left(m\frac{d^2x_\mathrm{h}}{dt^2} + b\frac{dx_\mathrm{h}}{dt} + kx_\mathrm{h}\right) = F_\mathrm{m}\cos\omega''t.$$

Consider the second term on the left. The parenthetical expression is just Eq. 17-38, the damped harmonic oscillator equation. It is given in the text (and proved in Ex. 17-45) the x_h is a solution, so this term is identically zero. What remains is Eq. 17-42; and we took as a given that x_n was a solution.

(b) The "add-on" solution of x_h represents the transient motion that will die away with time.

E17-51 The time between "bumps" is the solution to

$$\begin{aligned} vt &= x, \\ t &= \frac{(13\text{ ft})}{(10\text{ mi/hr})}\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = 0.886\,\text{s} \end{aligned}$$

The angular frequency is

$$\omega = \frac{2\pi}{T} = 7.09\text{ rad/s}$$

This is the driving frequency, and the problem states that at this frequency the up-down bounce oscillation is at a maximum. This occurs when the driving frequency is approximately equal to the natural frequency of oscillation. The force constant for the car is k, and this is related to the natural angular frequency by

$$k = m\omega^2 = \frac{W}{g}\omega^2,$$

where $W = (2200 + 4 \times 180)$ lb= 2920 lb is the weight of the car and occupants. Then

$$k = \frac{(2920\text{ lb})}{(32\text{ ft/s}^2)}(7.09\text{ rad/s})^2 = 4590\text{ lb/ft}$$

When the four people get out of the car there is less downward force on the car springs. The important relationship is

$$\Delta F = k\Delta x.$$

In this case $\Delta F = 720$ lb, the weight of the four people who got out of the car. Δx is the distance the car will rise when the people get out. So

$$\Delta x = \frac{\Delta F}{k} = \frac{(720\text{ lb})}{4590\text{ lb/ft}} = 0.157\text{ ft} \approx 2\text{ in.}$$

E17-52 The derivative is easy enough to find,

$$\frac{dx}{dt} = \frac{F_\text{m}}{G}(-\omega'')\sin(\omega'' t - \beta),$$

The velocity amplitude is

$$\begin{aligned} v_\text{m} &= \frac{F_\text{m}}{G}\omega'', \\ &= \frac{F_\text{m}}{\frac{1}{\omega''}\sqrt{m^2(\omega''^2 - \omega^2)^2 + b^2\omega''^2}}, \\ &= \frac{F_\text{m}}{\sqrt{(m\omega'' - k/\omega'')^2 + b^2}}. \end{aligned}$$

Note that this *is exactly* a maximum when $\omega'' = \omega$.

E17-53 The reduced mass is

$$m = (1.13\,\text{kg})(3.24\,\text{kg})/(1.12\,\text{kg} + 3.24\,\text{kg}) = 0.840\,\text{kg}.$$

The period of oscillation is

$$T = 2\pi\sqrt{(0.840\,\text{kg})/(252\,\text{N/m})} = 0.363\,\text{s}$$

E17-54

E17-55 Start by multiplying the kinetic energy expression by $(m_1+m_2)/(m_1+m_2)$.

$$\begin{aligned} K &= \frac{(m_1+m_2)}{2(m_1+m_2)}\left(m_1v_1^2+m_2v_2^2\right), \\ &= \frac{1}{2(m_1+m_2)}\left(m_1^2v_1^2+m_1m_2(v_1^2+v_2^2)+m_2^2v_2^2\right), \end{aligned}$$

and then add $2m_1m_2v_1v_2 - 2m_1m_2v_1v_2$,

$$\begin{aligned} K &= \frac{1}{2(m_1+m_2)}\left(m_1^2v_1^2+2m_1m_2v_1v_2+m_2^2v_2^2+m_1m_2(v_1^2+v_2^2-2v_1v_2)\right), \\ &= \frac{1}{2(m_1+m_2)}\left((m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2\right). \end{aligned}$$

But $m_1v_2 + m_2v_2 = 0$ by conservation of momentum, so

$$\begin{aligned} K &= \frac{(m_1m_2)}{2(m_1+m_2)}\left(v_1-v_2\right)^2, \\ &= \frac{m}{2}(v_1-v_2)^2. \end{aligned}$$

P17-1 The mass of one silver atom is $(0.108\,\text{kg})/(6.02\times10^{23}) = 1.79\times10^{-25}\text{kg}$. The effective spring constant is

$$k = (1.79\times10^{-25}\text{kg})4\pi^2(10.0\times10^{12}/\text{s})^2 = 7.07\times10^2\text{N/m}.$$

P17-2 (a) Rearrange Eq. 17-8 except replace m with the total mass, or $m+M$. Then $(M+m)/k = T^2/(4\pi^2)$, or

$$M = (k/4\pi^2)T^2 - m.$$

(b) When $M = 0$ we have

$$m = [(605.6\,\text{N/m})/(4\pi^2)](0.90149\,\text{s})^2 = 12.467\,\text{kg}.$$

(c) $M = [(605.6\,\text{N/m})/(4\pi^2)](2.08832\,\text{s})^2 - (12.467\,\text{kg}) = 54.432\,\text{kg}.$

P17-3 The maximum static friction is $F_f \leq \mu_s N$. Then

$$F_f = \mu_s N = \mu_s W = \mu_s mg$$

is the maximum available force to accelerate the upper block. So the maximum acceleration is

$$a_\text{m} = \frac{F_f}{m} = \mu_s g$$

The maximum possible amplitude of the oscillation is then given by

$$x_\text{m} = \frac{a_\text{m}}{\omega^2} = \frac{\mu_s g}{k/(m+M)},$$

where in the last part we substituted the total mass of the two blocks because both blocks are oscillating. Now we put in numbers, and find

$$x_\text{m} = \frac{(0.42)(1.22\,\text{kg}+8.73\,\text{kg})(9.81\,\text{m/s}^2)}{(344\,\text{N/m})} = 0.119\,\text{m}.$$

P17-4 (a) Equilibrium occurs when $F = 0$, or $b/r^3 = a/r^2$. This happens when $r = b/a$.

(b) $dF/dr = 2a/r^3 - 3b/r^4$. At $r = b/a$ this becomes

$$dF/dr = 2a^4/b^3 - 3a^4/b^3 = -a^4/b^3,$$

which corresponds to a force constant of a^4/b^3.

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{mb^3}/a^2$, where m is the reduced mass.

P17-5 Each spring helps to restore the block. The net force on the block is then of magnitude $F_1 + F_2 = k_1x + k_2x = (k_1 + k_2)x = kx$. We can then write the frequency as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}, \\ &= \sqrt{\frac{1}{4\pi^2}\frac{k_1}{m} + \frac{1}{4\pi^2}\frac{k_2}{m}}, \\ &= \sqrt{f_1^2 + f_2^2}. \end{aligned}$$

P17-6 The tension in the two spring is the same, so $k_1x_1 = k_2x_2$, where x_i is the extension of the ith spring. The total extension is $x_1 + x_2$, so the effective spring constant of the combination is

$$\frac{F}{x} = \frac{F}{x_1 + x_2} = \frac{1}{x_1/F + x_2/F} = \frac{1}{1/k_1 + 1/k_2} = \frac{k_1k_2}{k_1 + k_2}.$$

The period is then

$$T = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1 + k_2)m}}$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1 + k_2)m}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{m/k_1 + m/k_2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{1/\omega_1^2 + 1/\omega_2^2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}}, \\ &= \frac{f_1f_2}{\sqrt{f_1^2 + f_2^2}}. \end{aligned}$$

P17-7 (a) When a spring is stretched the tension is the same everywhere in the spring. The stretching, however, is distributed over the entire length of the spring, so that the relative amount of stretch is proportional to the length of the spring under consideration. Half a spring, half the extension. But $k = -F/x$, so half the extension means twice the spring constant.

In short, cutting the spring in half will create two stiffer springs with twice the spring constant, so $k = 7.20$ N/cm for each spring.

(b) The two spring halves now support a mass M. We can view this as each spring is holding one-half of the total mass, so in effect

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M/2}}$$

or, solving for M,

$$M = \frac{2k}{4\pi^2 f^2} = \frac{2(720\,\text{N/m})}{4\pi^2(2.87\,\text{s}^{-1})^2} = 4.43\,\text{kg}.$$

P17-8 Treat the spring a being composed of N massless springlets each with a point mass m_s/N at the end. The spring constant for each springlet will be kN. An expression for the conservation of energy is then

$$\frac{m}{2}v^2 + \frac{m_\text{s}}{2N}\sum_1^N {v_n}^2 + \frac{Nk}{2}\sum_1^N {x_n}^2 = E.$$

Since the spring stretches proportionally along the length then we conclude that each springlet compresses the same amount, and then $x_n = A/N \sin\omega t$ could describe the *change* in length of each springlet. The energy conservation expression becomes

$$\frac{m}{2}v^2 + \frac{m_\text{s}}{2N}\sum_1^N {v_n}^2 + \frac{k}{2}A^2\sin^2\omega t = E.$$

$v = A\omega\cos\omega t$. The hard part to sort out is the v_n, since the displacements for all springlets to one side of the nth must be added to get the net displacement. Then

$$v_n = n\frac{A}{N}\omega\cos\omega t,$$

and the energy expression becomes

$$\left(\frac{m}{2} + \frac{m_\text{s}}{2N^3}\sum_1^N n^2\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

Replace the sum with an integral, then

$$\frac{1}{N^3}\int_0^N n^2\,dn = \frac{1}{3},$$

and the energy expression becomes

$$\frac{1}{2}\left(m + \frac{m_\text{s}}{3}\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

This will only be constant if

$$\omega^2 = \left(m + \frac{m_\text{s}}{3}\right)/k,$$

or $T = 2\pi\sqrt{(m + m_\text{s}/3)/k}$.

P17-9 (a) Apply conservation of energy. When $x = x_{\rm m}$ $v = 0$, so

$$\begin{aligned}\frac{1}{2}k{x_{\rm m}}^2 &= \frac{1}{2}m[v(0)]^2 + \frac{1}{2}k[x(0)]^2,\\ {x_{\rm m}}^2 &= \frac{m}{k}[v(0)]^2 + [x(0)]^2,\\ x_{\rm m} &= \sqrt{[v(0)/\omega]^2 + [x(0)]^2}.\end{aligned}$$

(b) When $t = 0$ $x(0) = x_{\rm m}\cos\phi$ and $v(0) = -\omega x_{\rm m}\sin\phi$, so

$$\frac{v(0)}{\omega x(0)} = -\frac{\sin\phi}{\cos\phi} = \tan\phi.$$

P17-10

P17-11 Conservation of momentum for the bullet block collision gives $mv = (m+M)v_{\rm f}$ or

$$v_{\rm f} = \frac{m}{m+M}v.$$

This $v_{\rm f}$ will be equal to the maximum oscillation speed $v_{\rm m}$. The angular frequency for the oscillation is given by

$$\omega = \sqrt{\frac{k}{m+M}}.$$

Then the amplitude for the oscillation is

$$x_{\rm m} = \frac{v_{\rm m}}{\omega} = v\frac{m}{m+M}\sqrt{\frac{m+M}{k}} = \frac{mv}{\sqrt{k(m+M)}}.$$

P17-12 (a) $W = F_{\rm s}$, or $mg = kx$, so $x = mg/k$.

(b) $F = ma$, but $F = W - F_{\rm s} = mg - kx$, and since $ma = m\,d^2x/dt^2$,

$$m\frac{d^2x}{dt^2} + kx = mg.$$

The solution can be verified by direct substitution.

(c) Just look at the answer!

(d) dE/dt is

$$\begin{aligned}mv\frac{dv}{dt} + kx\frac{dx}{dt} - mg\frac{dx}{dt} &= 0,\\ m\frac{dv}{dt} + kx &= mg.\end{aligned}$$

P17-13 The initial energy stored in the spring is $k{x_{\rm m}}^2/2$. When the cylinder passes through the equilibrium point it has a translational velocity $v_{\rm m}$ and a rotational velocity $\omega_{\rm r} = v_{\rm m}/R$, where R is the radius of the cylinder. The total kinetic energy at the equilibrium point is

$$\frac{1}{2}m{v_{\rm m}}^2 + \frac{1}{2}I{\omega_{\rm r}}^2 = \frac{1}{2}\left(m + \frac{1}{2}m\right){v_{\rm m}}^2.$$

Then the kinetic energy is 2/3 translational and 1/3 rotational. The total energy of the system is

$$E = \frac{1}{2}(294\,{\rm N/\,m})(0.239\,{\rm m})^2 = 8.40\,{\rm J}.$$

(a) $K_{\rm t} = (2/3)(8.40\,{\rm J}) = 5.60\,{\rm J}$.
(b) $K_{\rm r} = (1/3)(8.40\,{\rm J}) = 2.80\,{\rm J}$.
(c) The energy expression is

$$\frac{1}{2}\left(\frac{3m}{2}\right)v^2 + \frac{1}{2}kx^2 = E,$$

which leads to a standard expression for the period with $3M/2$ replacing m. Then $T = 2\pi\sqrt{3M/2k}$.

P17-14 (a) Integrate the potential energy expression over one complete period and then divide by the time for one period:

$$\begin{aligned}\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}kx^2\,dt &= \frac{k\omega}{4\pi}\int_0^{2\pi/\omega}{x_{\rm m}}^2\cos^2\omega t\,dt,\\ &= \frac{k\omega}{4\pi}{x_{\rm m}}^2\frac{\pi}{\omega},\\ &= \frac{1}{4}k{x_{\rm m}}^2.\end{aligned}$$

This is half the total energy; since the average total energy is E, then the average kinetic energy must be the other half of the average total energy, or $(1/4)k{x_{\rm m}}^2$.

(b) Integrate over half a cycle and divide by twice the amplitude.

$$\begin{aligned}\frac{1}{2x_{\rm m}}\int_{-x_{\rm m}}^{x_{\rm m}}\frac{1}{2}kx^2\,dx &= \frac{1}{2x_{\rm m}}\frac{1}{3}k{x_{\rm m}}^3,\\ &= \frac{1}{6}k{x_{\rm m}}^2.\end{aligned}$$

This is one-third the total energy. The average kinetic energy must be two-thirds the total energy, or $(1/3)k{x_{\rm m}}^2$.

P17-15 The rotational inertia is

$$I = \frac{1}{2}MR^2 + Md^2 = M\left(\frac{1}{2}(0.144\,{\rm m})^2 + (0.102\,{\rm m})^2\right) = (2.08\times10^{-2}{\rm m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{(2.08\times10^{-2}{\rm m}^2)M}{M(9.81\,{\rm m/s}^2)(0.102\,{\rm m})}} = 0.906\,{\rm s}.$$

P17-16 (a) The rotational inertia of the pendulum about the pivot is

$$(0.488\,{\rm kg})\left(\frac{1}{2}(0.103\,{\rm m})^2 + (0.103\,{\rm m}+0.524\,{\rm m})^2\right) + \frac{1}{3}(0.272\,{\rm kg})(0.524\,{\rm m})^2 = 0.219\,{\rm kg}\cdot{\rm m}^2.$$

(b) The center of mass location is

$$d = \frac{(0.524\,{\rm m})(0.272\,{\rm kg})/2 + (0.103\,{\rm m}+0.524\,{\rm m})(0.488\,{\rm kg})}{(0.272\,{\rm kg}) + (0.488\,{\rm kg})} = 0.496\,{\rm m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{(0.291\,{\rm kg}\cdot{\rm m}^2)/(0.272\,{\rm kg}+0.488\,{\rm kg})(9.81\,{\rm m/s}^2)(0.496\,{\rm m})} = 1.76\,{\rm s}.$$

P17-17 (a) The rotational inertia of a stick about an axis through a point which is a distance d from the center of mass is given by the parallel axis theorem,

$$I = I_{\text{cm}} + md^2 = \frac{1}{12}mL^2 + md^2.$$

The period of oscillation is given by Eq. 17-28,

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}$$

(b) We want to find the minimum period, so we need to take the derivative of T with respect to d. It'll look weird, but

$$\frac{dT}{dd} = \pi\frac{12d^2 - L^2}{\sqrt{12gd^3(L^2 + 12d^2)}}.$$

This will vanish when $12d^2 = L^2$, or when $d = L/\sqrt{12}$.

P17-18 The energy stored in the spring is given by $kx^2/2$, the kinetic energy of the rotating wheel is

$$\frac{1}{2}(MR^2)\left(\frac{v}{r}\right)^2,$$

where v is the tangential velocity of the point of attachment of the spring to the wheel. If $x = x_{\text{m}}\sin\omega t$, then $v = x_{\text{m}}\omega\cos\omega t$, and the energy will only be constant if

$$\omega^2 = \frac{k}{M}\frac{r^2}{R^2}.$$

P17-19 The method of solution is identical to the approach for the simple pendulum on page 381 *except* replace the tension with the normal force of the bowl on the particle. The effective pendulum with have a length R.

P17-20 Let x be the distance from the center of mass to the first pivot point. Then the period is given by

$$T = 2\pi\sqrt{\frac{I + Mx^2}{Mgx}}.$$

Solve this for x by expressing the above equation as a quadratic:

$$\left(\frac{MgT^2}{4\pi^2}\right)x = I + Mx^2,$$

or

$$Mx^2 - \left(\frac{MgT^2}{4\pi^2}\right)x + I = 0$$

There are *two* solutions. One corresponds to the first location, the other the second location. Adding the two solutions together will yield L; in this case the discriminant of the quadratic will drop out, leaving

$$L = x_1 + x_2 = \frac{MgT^2}{M4\pi^2} = \frac{gT^2}{4\pi^2}.$$

Then $g = 4\pi^2L/T^2$.

P17-21 In this problem

$$I = (2.50\,\text{kg})\left(\frac{(0.210\,\text{m})^2}{2} + (0.760\,\text{m} + 0.210\,\text{m})^2\right) = 2.41\,\text{kg}/\cdot\text{m}^2.$$

The center of mass is at the center of the disk.

(a) $T = 2\pi\sqrt{(2.41\,\text{kg}/\cdot\text{m}^2)/(2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m})} = 2.00\,\text{s}$.

(b) Replace Mgd with $Mgd + \kappa$ and $2.00\,\text{s}$ with $1.50\,\text{s}$. Then

$$\kappa = \frac{4\pi^2(2.41\,\text{kg}/\cdot\text{m}^2)}{(1.50\,\text{s})^2} - (2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m}) = 18.5\,\text{N}\cdot\text{m/rad}.$$

P17-22 The net force on the bob is toward the center of the circle, and has magnitude $F_{\text{net}} = mv^2/R$. This net force comes from the horizontal component of the tension. There is also a vertical component of the tension of magnitude mg. The tension then has magnitude

$$T = \sqrt{(mg)^2 + (mv^2/R)^2} = m\sqrt{g^2 + v^4/R^2}.$$

It is this tension which is important in finding the restoring force in Eq. 17-22; in effect we want to replace g with $\sqrt{g^2 + v^4/R^2}$ in Eq. 17-24. The frequency will then be

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{\sqrt{g^2 + v^4/R^2}}}.$$

P17-23 (a) Consider an object of mass m at a point P on the axis of the ring. It experiences a gravitational force of attraction to all points on the ring; by symmetry, however, the net force is not directed toward the circumference of the ring, but instead along the axis of the ring. There is then a factor of $\cos\theta$ which will be thrown in to the mix.

The distance from P to *any* point on the ring is $r = \sqrt{R^2 + z^2}$, and θ is the angle between the axis on the line which connects P and *any* point on the circumference. Consequently,

$$\cos\theta = z/r,$$

and then the net force on the star of mass m at P is

$$F = \frac{GMm}{r^2}\cos\theta = \frac{GMmz}{r^3} = \frac{GMmz}{(R^2 + z^2)^{3/2}}.$$

(b) If $z \ll R$ we can apply the binomial expansion to the denominator, and

$$(R^2 + z^2)^{-3/2} = R^{-3}\left(1 + \left(\frac{z}{R}\right)^2\right)^{-3/2} \approx R^{-3}\left(1 - \frac{3}{2}\left(\frac{z}{R}\right)^2\right).$$

Keeping terms only linear in z we have

$$F = \frac{GMm}{R^3}z,$$

which corresponds to a spring constant $k = GMm/R^3$. The frequency of oscillation is then

$$f = \sqrt{k/m}/(2\pi) = \sqrt{GM/R^3}/(2\pi).$$

(c) Using some numbers from the Milky Way galaxy,

$$f = \sqrt{(7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2\times10^{43}\text{kg})/(6\times10^{19}\text{m})^3}/(2\pi) = 1\times10^{-14}\ \text{Hz}.$$

P17-24 (a) The acceleration of the center of mass (point C) is $a = F/M$. The torque about an axis through the center of mass is $tau = FR/2$, since O is $R/2$ away from the center of mass. The angular acceleration of the disk is then

$$\alpha = \tau/I = (FR/2)/(MR^2/2) = F/(MR).$$

Note that the angular acceleration will tend to rotate the disk anti-clockwise. The tangential component to the angular acceleration at P is $a_{\mathrm{T}} = -\alpha R = -F/M$; this is exactly the opposite of the linear acceleration, so P will not (initially) accelerate.

(b) There is no net force at P.

P17-25 The value for k is closest to

$$k \approx (2000\,\mathrm{kg}/4)(9.81\,\mathrm{m/s^2})/(0.10\,\mathrm{m}) = 4.9\times10^4\mathrm{N/m}.$$

One complete oscillation requires a time $t_1 = 2\pi/\omega'$. The amplitude decays to 1/2 the original value in this time, so $0.5 = e^{-bt_1/2m}$, or

$$\ln(2) = \frac{2\pi b}{2m\omega'}.$$

It is *not* reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega' \approx \omega$. Then

$$\begin{aligned} \omega'^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2 \left(\frac{b}{2m}\right)^2, \\ \frac{k}{m} - \left(\frac{b}{2m}\right)^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2 \left(\frac{b}{2m}\right)^2, \\ \frac{k}{m} &= (81.2)\left(\frac{b}{2m}\right)^2 \end{aligned}$$

Then the value for b is

$$b = \sqrt{\frac{4(2000\,\mathrm{kg}/4)(4.9\times10^4\mathrm{N/m})}{(81.2)}} = 1100\,\mathrm{kg/s}.$$

P17-26 $a = d^2x/dt^2 = -A\omega^2\cos\omega t$. Substituting into the non-linear equation,

$$-mA\omega^2\cos\omega t + kA^3\cos^3\omega t = F\cos\omega_{\mathrm{d}}t.$$

Now let $\omega_{\mathrm{d}} = 3\omega$. Then

$$kA^3\cos^3\omega t - mA\omega^2\cos\omega t = F\cos 3\omega t$$

Expand the right hand side as $\cos 3\omega t = 4\cos^3\omega t - 3\cos\omega t$, then

$$kA^3\cos^3\omega t - mA\omega^2\cos\omega t = F(4\cos^3\omega t - 3\cos\omega t)$$

This will only work if $4F = kA^3$ and $3F = mA\omega^2$. Dividing one condition by the other means $4mA\omega^2 = kA^3$, so $A \propto \omega$ and then $F \propto \omega^3 \propto {\omega_{\mathrm{d}}}^3$.

P17-27 (a) Divide the top and the bottom by m_2. Then

$$\frac{m_1 m_2}{m_1 + m_2} = \frac{m_1}{(m_1/m_2) + 1},$$

and in the limit as $m_2 \to \infty$ the value of $(m_1/m_2) \to 0$, so

$$\lim_{m_2 \to \infty} \frac{m_1 m_2}{m_1 + m_2} = \lim_{m_2 \to \infty} \frac{m_1}{(m_1/m_2) + 1} = m_1.$$

(b) m is called the reduced mass because it is always less than either m_1 or m_2. Think about it in terms of

$$m = \frac{m_1}{(m_1/m_2) + 1} = \frac{m_2}{(m_2/m_1) + 1}.$$

Since mass is always positive, the denominator is always greater than or equal to 1. Equality only occurs if one of the masses is infinite. Now $\omega = \sqrt{k/m}$, and since m is always less than m_1, so the existence of a finite wall will cause ω to be larger, and the period to be smaller.

(c) If the bodies have equal mass then $m = m_1/2$. This corresponds to a value of $\omega = \sqrt{2k/m_1}$. In effect, the spring constant is doubled, which is what happens if a spring is cut in half.