E15-1 The pressure in the syringe is

$$p = \frac{(42.3\,\mathrm{N})}{\pi(1.12\times 10^{-2}\mathrm{m}/s)^2} = 4.29\times 10^5\mathrm{Pa}.$$

E15-2 The total mass of fluid is

$$m = (0.5\times10^{-3}\mathrm{m}^3)(2600\,\mathrm{kg/m}^3)+(0.25\times10^{-3}\mathrm{m}^3)(1000\,\mathrm{kg/m}^3)+(0.4\times10^{-3}\mathrm{m}^3)(800\,\mathrm{kg/m}^3) = 1.87\,\mathrm{kg}.$$

The weight is $(18.7\,\mathrm{kg})(9.8\,\mathrm{m/s}^2) = 18\,\mathrm{N}$.

E15-3 $F = A\Delta p$, so

$$F = (3.43\,\mathrm{m})(2.08\,\mathrm{m})(1.00\text{ atm} - 0.962\text{ atm})(1.01\times 10^5\mathrm{Pa/\ atm}) = 2.74\times 10^4\mathrm{N}.$$

E15-4 $B\Delta V/V = -\Delta p$; $V = L^3$; $\Delta V \approx L^2\Delta L/3$. Then

$$\Delta p = (140\times 10^9\mathrm{Pa})\frac{(5\times 10^{-3}\mathrm{m})}{3(0.85\,\mathrm{m})} = 2.74\times 10^9\mathrm{Pa}.$$

E15-5 There is an inward force F_1 pushing the lid closed from the pressure of the air outside the box; there is an outward force F_2 pushing the lid open from the pressure of the air inside the box. To lift the lid we need to exert an additional outward force F_3 to get a net force of zero.

The magnitude of the inward force is $F_1 = P_{\mathrm{out}}A$, where A is the area of the lid and P_{out} is the pressure outside the box. The magnitude of the outward force F_2 is $F_2 = P_{\mathrm{in}}A$. We are told $F_3 = 108$ lb. Combining,

$$\begin{aligned} F_2 &= F_1 - F_3, \\ P_{\mathrm{in}}A &= P_{\mathrm{out}}A - F_3, \\ P_{\mathrm{in}} &= P_{\mathrm{out}} - F_3/A, \end{aligned}$$

so $P_{\mathrm{in}} = (15\text{ lb/in}^2 - (108\text{ lb})/(12\text{ in}^2) = 6.0\text{ lb/in}^2$.

E15-6 $h = \Delta p/\rho g$, so

$$h = \frac{(0.05\text{ atm})(1.01\times 10^5\mathrm{Pa/\ atm})}{(1000\,\mathrm{kg/m}^3)(9.8\,\mathrm{m/s}^2)} = 0.52\,\mathrm{m}.$$

E15-7 $\Delta p = (1060\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(1.83\,\mathrm{m}) = 1.90\times 10^4\mathrm{Pa}$.

E15-8 $\Delta p = (1024\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(118\,\mathrm{m}) = 1.19\times 10^6\mathrm{Pa}$. Add this to p_0; the total pressure is then $1.29\times 10^6\mathrm{Pa}$.

E15-9 The pressure differential assuming we don't have a sewage pump:

$$p_2 - p_1 = -\rho g\,(y_2 - y_1) = (926\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)(8.16\,\mathrm{m} - 2.08\,\mathrm{m}) = 5.52\times 10^4\mathrm{Pa}.$$

We need to overcome this pressure difference with the pump.

E15-10 (a) $p = (1.00\text{ atm})e^{-5.00/8.55} = 0.557$ atm.
(b) $h = (8.55\text{ km})\ln(1.00/0.500) = 5.93$ km.

E15-11 The mercury will rise a distance a on one side and fall a distance a on the other so that the difference in mercury height will be $2a$. Since the masses of the "excess" mercury and the water will be proportional, we have $2a\rho_{\rm m} = d\rho_{\rm w}$, so

$$a = \frac{(0.112\text{m})(1000\,\text{kg/m}^3)}{2(13600\,\text{kg/m}^3)} = 4.12\times 10^{-3}\text{m}.$$

E15-12 (a) The pressure (due to the water alone) at the bottom of the pool is

$$P = (62.45 \text{ lb/ft}^3)(8.0 \text{ ft}) = 500 \text{ lb/ft}^2.$$

The force on the bottom is

$$F = (500 \text{ lb/ft}^2)(80 \text{ ft})(30 \text{ ft}) = 1.2\times 10^6 \text{ lb}.$$

The average pressure on the side is half the pressure on the bottom, so

$$F = (250 \text{ lb/ft}^2)(80 \text{ ft})(8.0 \text{ ft}) = 1.6\times 10^5 \text{ lb}.$$

The average pressure on the end is half the pressure on the bottom, so

$$F = (250 \text{ lb/ft}^2)(30 \text{ ft})(8.0 \text{ ft}) = 6.0\times 10^4 \text{ lb}.$$

(b) No, since that additional pressure acts on both sides.

E15-13 (a) Equation 15-8 can be used to find the height y_2 of the atmosphere if the density is constant. The pressure at the top of the atmosphere would be $p_2 = 0$, and the height of the bottom y_1 would be zero. Then

$$y_2 = (1.01\times 10^5\text{Pa})/\left[(1.21\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\right] = 8.51\times 10^3\text{m}.$$

(b) We have to go back to Eq. 15-7 for an atmosphere which has a density which varies linearly with altitude. Linear variation of density means

$$\rho = \rho_0 \left(1 - \frac{y}{y_{\rm max}}\right)$$

Substitute this into Eq. 15-7,

$$\begin{aligned}
p_2 - p_1 &= -\int_0^{y_{\rm max}} \rho g\, dy, \\
&= -\int_0^{y_{\rm max}} \rho_0 g \left(1 - \frac{y}{y_{\rm max}}\right) dy, \\
&= -\rho_0 g \left(y - \frac{y^2}{2y_{\rm max}}\right)\Bigg|_0^{y_{\rm max}}, \\
&= -\rho g y_{\rm max}/2.
\end{aligned}$$

In this case we have $y_{\rm max} = 2p_1/(\rho g)$, so the answer is twice that in part (a), or 17 km.

E15-14 $\Delta P = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(112\,\text{m}) = 1.1\times 10^6\text{Pa}$. The force required is then $F = (1.1\times 10^6\text{Pa})(1.22\,\text{m})(0.590\,\text{m}) = 7.9\times 10^5\text{N}$.

E15-15 (a) Choose *any* infinitesimally small spherical region where equal volumes of the two fluids are in contact. The denser fluid will have the larger mass. We can treat the system as being a sphere of uniform mass with a hemisphere of additional mass being superimposed in the region of higher density. The orientation of this hemisphere is the only variable when calculating the potential energy. The center of mass of this hemisphere will be a low as possible only when the surface is horizontal. So all two-fluid interfaces will be horizontal.

(b) If there exists a region where the interface is not horizontal then there will be two different values for $\Delta p = \rho g h$, depending on the path taken. This means that there will be a horizontal pressure gradient, and the fluid will flow along that gradient until the horizontal pressure gradient is equalized.

E15-16 The mass of liquid originally in the first vessel is $m_1 = \rho A h_1$; the center of gravity is at $h_1/2$, so the potential energy of the liquid in the first vessel is originally $U_1 = \rho g A h_1^2/2$. A similar expression exists for the liquid in the second vessel. Since the two vessels have the same cross sectional area the final height in both containers will be $h_\mathrm{f} = (h_1 + h_2)/2$. The final potential energy of the liquid in *each* container will be $U_\mathrm{f} = \rho g A (h_1 + h_2)^2/8$. The work done by gravity is then

$$\begin{aligned} W &= U_1 + U_2 - 2U_\mathrm{f}, \\ &= \frac{\rho g A}{4}\left[2h_1^2 + 2h_2^2 - (h_1^2 + 2h_1h_2 + h_2^2)\right], \\ &= \frac{\rho g A}{4}(h_1 - h_2)^2. \end{aligned}$$

E15-17 There are *three* force on the block: gravity ($W = mg$), a buoyant force $B_0 = m_\mathrm{w} g$, and a tension T_0. When the container is at rest all three forces balance, so $B_0 - W - T_0 = 0$. The tension in this case is $T_0 = (m_\mathrm{w} - m)g$.

When the container accelerates upward we now have $B - W - T = ma$. Note that neither the tension *nor* the buoyant force stay the same; the buoyant force increases according to $B = m_\mathrm{w}(g+a)$. The new tension is then

$$T = m_\mathrm{w}(g + a) - mg - ma = (m_\mathrm{w} - m)(g + a) = T_0(1 + a/g).$$

E15-18 (a) $F_1/d_1^2 = F_2/d_2^2$, so

$$F_2 = (18.6\ \mathrm{kN})(3.72\ \mathrm{cm})^2/(51.3\ \mathrm{cm})^2 = 97.8\,\mathrm{N}.$$

(b) $F_2 h_2 = F_1 h_1$, so

$$h_2 = (1.65\,\mathrm{m})(18.6\ \mathrm{kN})/(97.8\,\mathrm{N}) = 314\,\mathrm{m}.$$

E15-19 (a) 35.6 kN; the boat doesn't get heavier or lighter just because it is in different water!

(b) Yes.

$$\Delta V = \frac{(35.6\times10^3\mathrm{N})}{(9.81\,m/\mathrm{s}^2)}\left(\frac{1}{(1024\,\mathrm{kg/m^3})} - \frac{1}{(1000\,\mathrm{kg/m^3})}\right) = -8.51\times10^{-2}\mathrm{m}^3.$$

E15-20 (a) $\rho_2 = \rho_1(V_1/V_2) = (1000\,\mathrm{kg/m^3})(0.646) = 646\,\mathrm{kg/m^3}$.

(b) $\rho_2 = \rho_1(V_1/V_2) = (646\,\mathrm{kg/m^3})(0.918)^{-1} = 704\,\mathrm{kg/m^3}$.

E15-21 The can has a volume of 1200 cm^3, so it can displace that much water. This would provide a buoyant force of

$$B = \rho V g = (998\text{kg/m}^3)(1200\times10^{-6}\text{m}^3)(9.81\,\text{m/s}^2) = 11.7\,\text{N}.$$

This force can then support a total mass of $(11.7\,\text{N})/(9.81\,\text{m/s}^2) = 1.20\,\text{kg}$. If 130 g belong to the can, then the can will be able to carry 1.07 kg of lead.

E15-22 $\rho_2 = \rho_1(V_1/V_2) = (0.98\text{ g/cm}^3)(2/3)^{-1} = 1.47\text{ g/cm}^3$.

E15-23 Let the object have a mass m. The buoyant force of the air on the object is then

$$B_\text{o} = \frac{\rho_\text{a}}{\rho_\text{o}} mg.$$

There is also a buoyant force on the brass, equal to

$$B_\text{b} = \frac{\rho_\text{a}}{\rho_\text{b}} mg.$$

The fractional error in the weighing is then

$$\frac{B_\text{o} - B_\text{b}}{mg} = \frac{(0.0012\text{ g/cm}^3)}{(3.4\text{ g/cm}^3)} - \frac{(0.0012\text{ g/cm}^3)}{(8.0\text{ g/cm}^3)} = 2.0\times10^{-4}$$

E15-24 The volume of iron is

$$V_\text{i} = (6130\,\text{N})/(9.81\,\text{m/s}^2)(7870\,\text{kg/m}^3) = 7.94\times10^{-2}\text{m}^3.$$

The buoyant force of water is $6130\,\text{N} - 3970\,\text{N} = 2160\,\text{N}$. This corresponds to a volume of

$$V_\text{w} = (2160\,\text{N})/(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3) = 2.20\times10^{-1}\text{m}^3.$$

The volume of air is then $2.20\times10^{-1}\text{m}^3 - 7.94\times10^{-2}\text{m}^3 = 1.41\times10^{-1}\text{m}^3$.

E15-25 (a) The pressure on the top surface is $p = p_0 + \rho g L/2$. The downward force is

$$\begin{aligned} F_\text{t} &= (p_0 + \rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + (944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 3.84\times10^4\text{N}. \end{aligned}$$

(b) The pressure on the bottom surface is $p = p_0 + 3\rho g L/2$. The upward force is

$$\begin{aligned} F_\text{b} &= (p_0 + 3\rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + 3(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 4.05\times10^4\text{N}. \end{aligned}$$

(c) The tension in the wire is given by $T = W + F_\text{t} - F_\text{b}$, or

$$T = (4450\,\text{N}) + (3.84\times10^4\text{N}) - (4.05\times10^4\text{N}) = 2350\,\text{N}.$$

(d) The buoyant force is

$$B = L^3\rho g = (0.608^3)(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2) = 2080\,\text{N}.$$

E15-26 The fish has the (average) density of water if

$$\rho_{\rm w} = \frac{m_{\rm f}}{V_{\rm c} + V_{\rm a}}$$

or

$$V_{\rm a} = \frac{m_{\rm f}}{\rho_{\rm w}} - V_{\rm c}.$$

We want the fraction $V_{\rm a}/(V_{\rm c} + V_{\rm a})$, so

$$\begin{aligned} \frac{V_{\rm a}}{V_{\rm c} + V_{\rm a}} &= 1 - \rho_{\rm w}\frac{V_{\rm c}}{m_{\rm f}}, \\ &= 1 - \rho_{\rm w}/\rho_{\rm c} = 1 - (1.024\ {\rm g/cm^3})/(1.08\ {\rm g/cm^3}) = 5.19\times10^{-2}. \end{aligned}$$

E15-27 There are *three* force on the dirigible: gravity ($W = m_{\rm g}g$), a buoyant force $B = m_{\rm a}g$, and a tension T. Since these forces must balance we have $T = B - W$. The masses are related to the densities, so we can write

$$T = (\rho_{\rm a} - \rho_{\rm g})Vg = (1.21\,{\rm kg/m^3} - 0.796\,{\rm kg/m^3})(1.17\times10^6{\rm m^3})(9.81{\rm m/s^2}) = 4.75\times10^6\,{\rm N}.$$

E15-28 $\Delta m = \Delta\rho V$, so

$$\Delta m = [(0.160\,kg/{\rm m^3}) - (0.0810\,{\rm kg/m^3})](5000\,{\rm m^3}) = 395\,{\rm kg}.$$

E15-29 The volume of one log is $\pi(1.05/2\ {\rm ft})^2(5.80\ {\rm ft}) = 5.02\ {\rm ft^3}$. The weight of the log is $(47.3\ {\rm lb/ft^3})(5.02\ {\rm ft^3}) = 237$ lb. Each log if completely submerged will displace a weight of water $(62.4\ {\rm lb/ft^3})(5.02\ {\rm ft^3}) = 313$ lb. So each log can support at most 313 lb − 237 lb = 76 lb. The three children have a total weight of 247 lb, so that will require 3.25 logs. Round up to four.

E15-30 (a) The ice will hold up the automobile if

$$\rho_{\rm w} > \frac{m_{\rm a} + m_{\rm i}}{V_{\rm i}} = \frac{m_{\rm a}}{At} + \rho_{\rm i}.$$

Then

$$A = \frac{(1120\,{\rm kg})}{(0.305\,{\rm m})[(1000\,{\rm kg/m^3}) - (917\,{\rm kg/m^3})]} = 44.2\,{\rm m^2}.$$

E15-31 If there were *no* water vapor pressure above the barometer then the height of the water would be $y_1 = p/(\rho g)$, where $p = p_0$ is the atmospheric pressure. If there is water vapor where there should be a vacuum, then p is the difference, and we would have $y_2 = (p_0 - p_{\rm v})/(\rho g)$. The relative error is

$$\begin{aligned} (y_1 - y_2)/y_1 &= [p_0/(\rho g) - (p_0 - p_{\rm v})/(\rho g)]\,/\,[p_0/(\rho g)]\,, \\ &= p_{\rm v}/p_0 = (3169\,{\rm Pa})/(1.01\times10^5{\rm Pa}) = 3.14\,\%. \end{aligned}$$

E15-32 $\rho = (1.01\times10^5{\rm Pa})/(9.81\,{\rm m/s^2})(14\,{\rm m}) = 740\,{\rm kg/m^3}$.

E15-33 $h = (90)(1.01\times10^5{\rm Pa})/(8.60\,{\rm m/s^2})(1.36\times10^4{\rm kg/m^3}) = 78\,{\rm m}$.

E15-34 $\Delta U = 2(4.5\times10^{-2}{\rm N/m})4\pi(2.1\times10^{-2}{\rm m})^2 = 5.0\times10^{-4}{\rm J}$.

E15-35 The force required is just the surface tension times the circumference of the circular patch. Then

$$F = (0./072\,\mathrm{N/m})2\pi(0.12\,\mathrm{m}) = 5.43\times10^{-2}\mathrm{N}.$$

E15-36 $\Delta U = 2(2.5\times10^{-2}\mathrm{N/m})4\pi(1.4\times10^{-2}\mathrm{m})^2 = 1.23\times10^{-4}\mathrm{J}.$

P15-1 (a) One can replace the two hemispheres with an open flat end with two hemispheres with a closed flat end. Then the area of relevance is the area of the flat end, or πR^2. The net force from the pressure difference is $\Delta pA = \Delta p\pi R^2$; this much force must be applied to pull the hemispheres apart.

(b) $F = \pi(0.9)(1.01\times10^5\mathrm{Pa})(0.305\,\mathrm{m})^2 = 2.6\times10^4\mathrm{N}.$

P15-2 The pressure required is 4×10^9Pa. This will happen at a depth

$$h = \frac{(4\times10^9\mathrm{Pa})}{(9.8\,\mathrm{m/s^2})(3100\,\mathrm{kg/m^3})} = 1.3\times10^5\mathrm{m}.$$

P15-3 (a) The resultant force on the wall will be

$$\begin{aligned} F &= \int\int P\,dx\,dy, \\ &= \int(-\rho g y)W\,dy, \\ &= \rho g D^2 W/2. \end{aligned}$$

(b) The torque will is given by $\tau = F(D-y)$ (the distance is from the bottom) so if we generalize,

$$\begin{aligned} \tau &= \int\int Py\,dx\,dy, \\ &= \int\left(-\rho g(D-y)\right)yW\,dy, \\ &= \rho g D^3 W/6. \end{aligned}$$

(c) Dividing to find the location of the equivalent resultant force,

$$d = \tau/F = (\rho g D^3 W/6)/(\rho g D^2 W/2) = D/3,$$

this distance being measured from the bottom.

P15-4 $p = \rho g y = \rho g(3.6\,\mathrm{m})$; the force on the bottom is $F = pA = \rho g(3.6\,\mathrm{m})\pi(0.60\,\mathrm{m})^2 = 1.296\pi\rho g$. The volume of liquid is

$$V = (1.8\,\mathrm{m})\left[\pi(0.60\,\mathrm{m}) + 4.6\times10^{-4}\mathrm{m}^2\right] = 2.037\,\mathrm{m}^3$$

The weight is $W = \rho g(2.037\,\mathrm{m}^3)$. The ratio is 2.000.

P15-5 The pressure at b is $\rho_\mathrm{c}(3.2\times10^4\mathrm{m})+\rho_\mathrm{m}y$. The pressure at a is $\rho_\mathrm{c}(3.8\times10^4\mathrm{m}+d)+\rho_\mathrm{m}(y-d)$. Set these quantities equal to each other:

$$\begin{aligned} \rho_\mathrm{c}(3.8\times10^4\mathrm{m} + d) + \rho_\mathrm{m}(y-d) &= \rho_\mathrm{c}(3.2\times10^4\mathrm{m}) + \rho_\mathrm{m}y, \\ \rho_\mathrm{c}(6\times10^3\mathrm{m} + d) &= \rho_\mathrm{m}d, \\ d &= \rho_\mathrm{c}(6\times10^3\mathrm{m})/(\rho_\mathrm{m} - \rho_\mathrm{c}), \\ &= (2900\,\mathrm{kg/m^3})(6\times10^3\mathrm{m})/(400\,\mathrm{kg/m^3}) = 4.35\times10^4\mathrm{m}. \end{aligned}$$

P15-6 (a) The pressure (difference) at a depth y is $\Delta p = \rho g y$. Since $\rho = m/V$, then

$$\Delta\rho \approx -\frac{m}{V}\frac{\Delta V}{V} = \rho_{\rm s}\frac{\Delta p}{B}.$$

Then

$$\rho \approx \rho_{\rm s} + \Delta\rho = \rho_{\rm s} + \frac{{\rho_{\rm s}}^2 g y}{B}.$$

(b) $\Delta\rho/\rho_{\rm s} = \rho_{\rm s} g y/B$, so

$$\Delta\rho/\rho \approx (1000\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(4200\,\mathrm{m})/(2.2\times10^9\mathrm{Pa}) = 1.9\,\%.$$

P15-7 (a) Use Eq. 15-10, $p = (p_0/\rho_0)\rho$, then Eq. 15-13 will look like

$$(p_0/\rho_0)\rho = (p_0/\rho_0)\rho_0 e^{-h/a}.$$

(b) The upward velocity of the rocket as a function of time is given by $v = a_r t$. The height of the rocket above the ground is given by $y = \frac{1}{2}a_r t^2$. Combining,

$$v = a_r\sqrt{\frac{2y}{a_r}} = \sqrt{2ya_r}.$$

Put this into the expression for drag, along with the equation for density variation with altitude;

$$D = CA\rho v^2 = CA\rho_0 e^{-y/a} 2ya_r.$$

Now take the derivative with respect to y,

$$dD/dy = (-1/a)CA\rho_0 e^{-y/a}(2ya_r) + CA\rho_0 e^{-y/a}(2a_r).$$

This will vanish when $y = a$, regardless of the acceleration a_r.

P15-8 (a) Consider a slice of cross section A a depth h beneath the surface. The net force on the fluid above the slice will be

$$F_{\rm net} = ma = \rho h A g,$$

Since the weight of the fluid above the slice is

$$W = mg = \rho h A g,$$

then the upward force on the bottom of the fluid at the slice must be

$$W + F_{\rm net} = \rho h A(g + a),$$

so the pressure is $p = F/A = \rho h(g + a)$.

(b) Change a to $-a$.

(c) The pressure is zero (ignores atmospheric contributions.)

P15-9 (a) Consider a portion of the liquid at the surface. The net force on this portion is $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = ma\hat{\mathbf{i}}$. The force of gravity on this portion is $\vec{\mathbf{W}} = -mg\hat{\mathbf{j}}$. There must then be a buoyant force on the portion with direction $\vec{\mathbf{B}} = \vec{\mathbf{F}} - \vec{\mathbf{W}} = m(a\hat{\mathbf{i}} + g\hat{\mathbf{j}})$. The buoyant force makes an angle $\theta = \arctan(a/g)$ with the vertical. The buoyant force must be perpendicular to the surface of the fluid; there are no pressure-related forces which are parallel to the surface. Consequently, the surface must make an angle $\theta = \arctan(a/g)$ with the *horizontal*.

(b) It will still vary as $\rho g h$; the derivation on page 334 is still valid for vertical displacements.

P15-10 $dp/dr = -\rho g$, but now $g = 4\pi G\rho r/3$. Then

$$\begin{aligned}\int_0^p dp &= -\frac{4}{3}\pi G\rho^2 \int_R^r r\,dr,\\ p &= \frac{2}{3}\pi G\rho^2\left(R^2 - r^2\right).\end{aligned}$$

P15-11 We can start with Eq. 15-11, except that we'll write our distance in terms of r instead if y. Into this we can substitute our expression for g,

$$g = g_0\frac{R^2}{r^2}.$$

Substituting, then integrating,

$$\begin{aligned}\frac{dp}{p} &= -\frac{g\rho_0}{p_0}dr,\\ \frac{dp}{p} &= -\frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2},\\ \int_{p_0}^{p}\frac{dp}{p} &= -\int_R^r \frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2},\\ \ln\frac{p}{p_0} &= \frac{g_0\rho_0 R^2}{p_0}\left(\frac{1}{r} - \frac{1}{R}\right)\end{aligned}$$

If $k = g_0\rho_0 R^2/p_0$, then

$$p = p_0 e^{k(1/r - 1/R)}.$$

P15-12 (a) The net force on a small volume of the fluid is $dF = r\omega^2\,dm$ directed toward the center. For radial displacements, then, $dF/dr = -r\omega^2 dm/dr$ or $dp/dr = -r\omega^2\rho$.

(b) Integrating outward,

$$p = p_c + \int_0^r \rho\omega^2 r\,dr = p_c + \frac{1}{2}\rho r^2\omega^2.$$

(c) Do part (d) first.

(d) It will still vary as ρgh; the derivation on page 334 is still valid for vertical displacements.

(c) The pressure anywhere in the liquid is then given by

$$p = p_0 + \frac{1}{2}\rho r^2\omega^2 - \rho g y,$$

where p_0 is the pressure on the surface, y is measured from the bottom of the paraboloid, and r is measured from the center. The surface is defined by $p = p_0$, so

$$\frac{1}{2}\rho r^2\omega^2 - \rho g y = 0,$$

or $y = r^2\omega^2/2g$.

P15-13 The total mass of the shell is $m = \rho_w\pi d_o{}^3/3$, or it wouldn't barely float. The mass of iron in the shell is $m = \rho_i\pi(d_o{}^3 - d_i{}^3)/3$, so

$$d_i{}^3 = \frac{\rho_i - \rho_w}{\rho_i}d_o{}^3,$$

so

$$d_i = \sqrt[3]{\frac{(7870\,\text{kg/m}^3) - (1000\,\text{kg/m}^3)}{(7870\,\text{kg/m}^3)}}(0.587\,\text{m}) = 0.561\,\text{m}.$$

P15-14 The wood will displace a volume of water equal to $(3.67\,\text{kg})/(594\,\text{kg/m}^3)(0.883) = 5.45\times 10^{-3}\text{m}^3$ in either case. That corresponds to a mass of $(1000\,\text{kg/m}^3)(5.45\times10^{-3}\text{m}^3) = 5.45\,\text{kg}$ that can be supported.

(a) The mass of lead is $5.45\,\text{kg} - 3.67\,\text{kg} = 1.78\,\text{kg}$.

(b) When the lead is submerged beneath the water it displaces water, which affects the "apparent" mass of the lead. The true weight of the lead is mg, the buoyant force is $(\rho_\text{w}/\rho_\text{l})mg$, so the apparent weight is $(1-\rho_\text{w}/\rho_\text{l})mg$. This means the apparent mass of the submerged lead is $(1-\rho_\text{w}/\rho_\text{l})m$. This apparent mass is 1.78 kg, so the true mass is

$$m = \frac{(11400\,\text{kg/m}^3)}{(11400\,\text{kg/m}^3)-(1000\,\text{kg})}(1.78\,\text{kg}) = 1.95\,\text{kg}.$$

P15-15 We initially have

$$\frac{1}{4} = \frac{\rho_\text{o}}{\rho_\text{mercury}}.$$

When water is poured over the object the simple relation no longer works.

Once the water is over the object there are two buoyant forces: one from mercury, F_1, and one from the water, F_2. Following a derivation which is similar to Sample Problem 15-3, we have

$$F_1 = \rho_1 V_1 g \text{ and } F_2 = \rho_2 V_2 g$$

where ρ_1 is the density of mercury, V_1 the volume of the object which is in the mercury, ρ_2 is the density of water, and V_2 is the volume of the object which is in the water. We also have

$$F_1 + F_2 = \rho_\text{o} V_\text{o} g \text{ and } V_1 + V_2 = V_\text{o}$$

as expressions for the net force on the object (zero) and the total volume of the object. Combining these four expressions,

$$\rho_1 V_1 + \rho_2 V_2 = \rho_\text{o} V_\text{o},$$

or

$$\begin{aligned}\rho_1 V_1 + \rho_2 (V_\text{o} - V_1) &= \rho_\text{o} V_\text{o},\\ (\rho_1 - \rho_2) V_1 &= (\rho_\text{o} - \rho_2) V_\text{o},\\ \frac{V_1}{V_\text{o}} &= \frac{\rho_\text{o} - \rho_2}{\rho_1 - \rho_2}.\end{aligned}$$

The left hand side is the fraction that is submerged in the mercury, so we just need to substitute our result for the density of the material from the beginning to solve the problem. The fraction submerged after adding water is then

$$\begin{aligned}\frac{V_1}{V_\text{o}} &= \frac{\rho_\text{o} - \rho_2}{\rho_1 - \rho_2},\\ &= \frac{\rho_1/4 - \rho_2}{\rho_1 - \rho_2},\\ &= \frac{(13600\,\text{kg/m}^3)/4 - (998\,\text{kg/m}^3)}{(13600\,\text{kg/m}^3) - (998\,\text{kg/m}^3)} = 0.191.\end{aligned}$$

P15-16 (a) The car floats if it displaces a mass of water equal to the mass of the car. Then $V = (1820\,\text{kg})/(1000\,\text{kg/m}^3) = 1.82\,\text{m}^3$.

(b) The car has a total volume of $4.87\,\text{m}^3 + 0.750\,\text{m}^3 + 0.810\,\text{m}^3 = 6.43\,\text{m}^3$. It will sink if the total mass inside the car (car + water) is then $(6.43\,\text{m}^3)(1000\,\text{kg/m}^3) = 6430\,\text{kg}$. So the mass of the water in the car is $6430\,\text{kg} - 1820\,\text{kg} = 4610\,\text{kg}$ when it sinks. That's a volume of $(4610\,\text{kg})/(1000\,\text{kg/m}^3) = 4.16\,\text{m}^3$.

P15-17 When the beaker is half filled with water it has a total mass exactly equal to the maximum amount of water it can displace. The total mass of the beaker is the mass of the beaker plus the mass of the water inside the beaker. Then

$$\rho_\mathrm{w}(m_\mathrm{g}/\rho_\mathrm{g} + V_\mathrm{b}) = m_\mathrm{g} + \rho_\mathrm{w} V_\mathrm{b}/2,$$

where $m_\mathrm{g}/\rho_\mathrm{g}$ is the volume of the glass which makes up the beaker. Rearrange,

$$\rho_\mathrm{g} = \frac{m_\mathrm{g}}{m_\mathrm{g}/\rho_\mathrm{w} - V_\mathrm{b}/2} = \frac{(0.390\,\mathrm{kg})}{(0.390\,\mathrm{kg})/(1000\,\mathrm{kg/m^3}) - (5.00\times10^{-4}\,\mathrm{m^3})/2} = 2790\,\mathrm{kg/m^3}.$$

P15-18 (a) If each atom is a cube then the cube has a side of length

$$l = \sqrt[3]{(6.64\times10^{-27}\,\mathrm{kg})/(145\,\mathrm{kg/m^3})} = 3.58\times10^{-10}\mathrm{m}.$$

Then the atomic surface density is $l^{-2} = (3.58\times10^{-10}\mathrm{m})^{-2} = 7.8\times10^{18}/\mathrm{m}^2$.

(b) The bond surface density is *twice* the atomic surface density. Show this by drawing a square array of atoms and then joining each adjacent pair with a bond. You will need twice as many bonds as there are atoms. Then the energy per bond is

$$\frac{(3.5\times10^{-4}\mathrm{N/m})}{2(7.8\times10^{18}/\mathrm{m^2})(1.6\times10^{-19}\ \mathrm{J/eV})} = 1.4\times10^{-4}\ \mathrm{eV}.$$

P15-19 Pretend the bubble consists of two hemispheres. The force from surface tension holding the hemispheres together is $F = 2\gamma L = 4\pi r\gamma$. The "extra" factor of two occurs because *each* hemisphere has a circumference which "touches" the boundary that is held together by the surface tension of the liquid. The pressure difference between the inside and outside is $\Delta p = F/A$, where A is the area of the flat side of one of the hemispheres, so $\Delta p = (4\pi r\gamma)/(\pi r^2) = 4\gamma/r$.

P15-20 Use the results of Problem 15-19. To get a numerical answer you need to know the surface tension; try $\gamma = 2.5\times10^{-2}\mathrm{N/m}$. The initial pressure inside the bubble is $p_\mathrm{i} = p_0 + 4\gamma/r_\mathrm{i}$. The final pressure inside the bell jar is $p = p_\mathrm{f} - 4\gamma/r_\mathrm{f}$. The initial and final pressure inside the bubble are related by $p_\mathrm{i} {r_\mathrm{i}}^3 = p_\mathrm{f} {r_\mathrm{f}}^3$. Now for numbers:

$$p_\mathrm{i} = (1.00\times10^5\mathrm{Pa}) + 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-3}\mathrm{m}) = 1.001\times10^5\mathrm{Pa}.$$

$$p_\mathrm{f} = (1.0\times10^{-3}\mathrm{m}/1.0\times10^{-2}\mathrm{m})^3(1.001\times10^5\mathrm{Pa}) = 1.001\times10^2\mathrm{Pa}.$$

$$p = (1.001\times10^2\mathrm{Pa}) - 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-2}\mathrm{m}) = 90.1\mathrm{Pa}.$$

P15-21 The force on the liquid in the space between the rod and the cylinder is $F = \gamma L = 2\pi\gamma(R + r)$. This force can support a mass of water $m = F/g$. This mass has a volume $V = m/\rho$. The cross sectional area is $\pi(R^2 - r^2)$, so the height h to which the water rises is

$$\begin{aligned} h &= \frac{2\pi\gamma(R+r)}{\rho g\pi(R^2 - r^2)} = \frac{2\gamma}{\rho g(R-r)}, \\ &= \frac{2(72.8\times10^{-3}\mathrm{N/m})}{(1000\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(4.0\times10^{-3}\mathrm{m})} = 3.71\times10^{-3}\mathrm{m}. \end{aligned}$$

P15-22 (a) Refer to Problem 15-19. The initial pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(3.20\times10^{-2}\text{m}) = 3.25\text{Pa}.$$

(b) The final pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(5.80\times10^{-2}\text{m}) = 1.79\text{Pa}.$$

(c) The work done against the atmosphere is $p\Delta V$, or

$$(1.01\times10^{5}\text{Pa})\frac{4\pi}{3}[(5.80\times10^{-2}\text{m})^3 - (3.20\times10^{-2}\text{m})^3] = 68.7\,\text{J}.$$

(d) The work done in stretching the bubble surface is $\gamma\Delta A$, or

$$(2.60\times10^{-2}\text{N/m})4\pi[(5.80\times10^{-2}\text{m})^2 - (3.20\times10^{-2}\text{m})^2] = 7.65\times10^{-4}\text{J}.$$