E14-1 $F_\mathrm{S}/F_\mathrm{E} = M_\mathrm{S}{r_\mathrm{E}}^2/M_\mathrm{E}{r_\mathrm{S}}^2$, since everything else cancels out in the expression. Then

$$\frac{F_\mathrm{S}}{F_\mathrm{E}} = \frac{(1.99\times10^{30}\mathrm{kg})(3.84\times10^{8}\mathrm{m})^2}{(5.98\times10^{24})(1.50\times10^{11}\mathrm{m})^2} = 2.18$$

E14-2 Consider the force from the Sun and the force from the Earth. $F_\mathrm{S}/F_\mathrm{E} = M_\mathrm{S}{r_\mathrm{E}}^2/M_\mathrm{E}{r_\mathrm{S}}^2$, since everything else cancels out in the expression. We want the ratio to be one; we are also constrained because $r_\mathrm{E} + r_\mathrm{S} = R$ is the distance from the Sun to the Earth. Then

$$\begin{aligned} M_\mathrm{E}\left(R - r_\mathrm{E}\right)^2 &= M_\mathrm{S}{r_\mathrm{E}}^2, \\ R - r_\mathrm{E} &= \sqrt{\frac{M_\mathrm{S}}{M_\mathrm{E}}}r_\mathrm{E}, \\ r_\mathrm{E} &= (1.50\times10^{11}\mathrm{m})/\left(1 + \sqrt{\frac{(1.99\times10^{30}\mathrm{kg})}{(5.98\times10^{24})}}\right) = 2.6\times10^{8}\mathrm{m}. \end{aligned}$$

E14-3 The masses of each object are $m_1 = 20.0\,\mathrm{kg}$ and $m_2 = 7.0\,\mathrm{kg}$; the distance between the centers of the two objects is $15 + 3 = 18\,\mathrm{m}$.

The magnitude of the force from Newton's law of gravitation is then

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(20.0\,\mathrm{kg})(7.0\,\mathrm{kg})}{(18\,\mathrm{m})^2} = 2.9\times10^{-11}\mathrm{N}.$$

E14-4 (a) The magnitude of the force from Newton's law of gravitation is

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(12.7\,\mathrm{kg})(9.85\times10^{-3}\,\mathrm{kg})}{(0.108\,\mathrm{m})^2} = 7.15\times10^{-10}\mathrm{N}.$$

(b) The torque is $\tau = 2(0.262\,\mathrm{m})(7.15\times10^{-10}\mathrm{N}) = 3.75\times10^{-10}\mathrm{N}\cdot\mathrm{m}$.

E14-5 The force of gravity on an object near the surface of the earth is given by

$$F = \frac{GMm}{(r_e + y)^2},$$

where M is the mass of the Earth, m is the mass of the object, r_e is the radius of the Earth, and y is the height above the surface of the Earth. Expand the expression since $y \ll r_e$. We'll use a Taylor expansion, where $F(r_e + y) \approx F(r_e) + y\partial F/\partial r_e$;

$$F \approx \frac{GMm}{r_e^2} - 2y\frac{GMm}{r_e^3}$$

Since we are interested in the difference between the force at the top and the bottom, we really want

$$\Delta F = 2y\frac{GMm}{r_e^3} = 2\frac{y}{r_e}\frac{GMm}{r_e^2} = 2\frac{y}{r_e}W,$$

where in the last part we substituted for the weight, which is the same as the force of gravity,

$$W = \frac{GMm}{r_e^2}.$$

Finally,

$$\Delta F = 2(411\,\mathrm{m})/(6.37\times10^{6}\mathrm{m})(120\text{ lb}) = 0.015\text{ lb}.$$

E14-6 $g \propto 1/r^2$, so $g_1/g_2 = r_2^2/r_1^2$. Then

$$r_2 = \sqrt{(9.81\,\mathrm{m/s^2})/(7.35\,\mathrm{m/s^2})}(6.37\times10^6\mathrm{m}) = 7.36\times10^6\,\mathrm{m}.$$

That's 990 kilometers above the surface of the Earth.

E14-7 (a) $a = GM/r^2$, or

$$a = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})}{(10.0\times10^3\mathrm{m})^2} = 1.33\times10^{12}\mathrm{m/s^2}.$$

(b) $v = \sqrt{2ax} = \sqrt{2(1.33\times10^{12}\mathrm{m/s^2})(1.2\,\mathrm{m/s})} = 1.79\times10^6\mathrm{m/s}$.

E14-8 (a) $g_0 = GM/r^2$, or

$$g_0 = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(7.36\times10^{22}\mathrm{kg})}{(1.74\times10^6\mathrm{m})^2} = 1.62\,\mathrm{m/s^2}.$$

(b) $W_\mathrm{m} = W_\mathrm{e}(g_\mathrm{m}/g_\mathrm{e})$ so

$$W_\mathrm{m} = (100\,\mathrm{N})(1.62\,\mathrm{m/s^2}/9.81\,\mathrm{m/s^2}) = 16.5\,\mathrm{N}.$$

(c) Invert $g = GM/r^2$;

$$r = \sqrt{GM/g} = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})/(1.62\,\mathrm{m/s^2})} = 1.57\times10^7\mathrm{m}.$$

That's 2.46 Earth radii, or 1.46 Earth radii above the surface of the Earth.

$\boxed{\textbf{E14-9}}$ The object fell through $y = -10.0\,\mathrm{m}$; the time required to fall would then be

$$t = \sqrt{-2y/g} = \sqrt{-2(-10.0\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 1.43\,\mathrm{s}.$$

We are interested in the *error*, that means taking the total derivative of $y = -\frac{1}{2}gt^2$. and getting

$$\delta y = -\frac{1}{2}\delta g\, t^2 - gt\,\delta t.$$

$\delta y = 0$ so $-\frac{1}{2}\delta g\, t = g\delta\, t$, which can be rearranged as

$$\delta t = -\frac{\delta g\, t}{2g}$$

The percentage error in t needs to be $\delta t/t = 0.1\,\%/2 = 0.05\,\%$. The absolute error is then $\delta t = (0.05\,\%)(1.43\,\mathrm{s}) = 0.7$ ms.

E14-10 Treat mass which is inside a spherical shell as being located at the center of that shell. Ignore any contributions from shells farther away from the center than the point in question.
(a) $F = G(M_1 + M_2)m/a^2$.
(b) $F = G(M_1)m/b^2$.
(c) $F = 0$.

$\boxed{\textbf{E14-11}}$ For a sphere of uniform density and radius $R > r$,

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3},$$

where M is the total mass.

The force of gravity on the object of mass m is then

$$F = \frac{GMm}{r^2}\frac{r^3}{R^3} = \frac{GMmr}{R^3}.$$

g is the free-fall acceleration of the object, and is the gravitational force divided by the mass, so

$$g = \frac{GMr}{R^3} = \frac{GM}{R^2}\frac{r}{R} = \frac{GM}{R^2}\frac{R-D}{R}.$$

Since R is the distance from the center to the surface, and D is the distance of the object beneath the surface, then $r = R - D$ is the distance from the center to the object. The first fraction is the free-fall acceleration on the surface, so

$$g = \frac{GM}{R^2}\frac{R-D}{R} = g_s\frac{R-D}{R} = g_s\left(1 - \frac{D}{R}\right)$$

E14-12 The work required to move the object is $GM_{\mathrm{S}}m/r$, where r is the gravitational radius. But if this equals mc^2 we can write

$$\begin{aligned} mc^2 &= GM_{\mathrm{S}}m/r, \\ r &= GM_{\mathrm{S}}/c^2. \end{aligned}$$

For the sun, $r = (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(1.99\times10^{30}\mathrm{kg})/(3.00\times10^8\mathrm{m/s})^2 = 1.47\times10^3\mathrm{m}$. That's $2.1\times10^{-6}R_{\mathrm{S}}$.

E14-13 The distance from the center is

$$r = (80000)(3.00\times10^8\mathrm{m/s})(3.16\times10^7\mathrm{s}) = 7.6\times10^{20}\mathrm{m}.$$

The mass of the galaxy is

$$M = (1.4\times10^{11})(1.99\times10^{30}\mathrm{kg}) = 2.8\times10^{41}\mathrm{kg}.$$

The escape velocity is

$$v = \sqrt{2GM/r} = \sqrt{2(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(2.8\times10^{41}\mathrm{kg})/(7.6\times10^{20}\mathrm{m})} = 2.2\times10^5\mathrm{m/s}.$$

E14-14 Staying in a circular orbit requires the centripetal force be equal to the gravitational force, so

$$m{v_{\mathrm{orb}}}^2/r = GMm/r^2,$$

or $m{v_{\mathrm{orb}}}^2 = GMm/r$. But $-GMm/r$ is the gravitational potential energy; to escape one requires a kinetic energy

$$m{v_{\mathrm{esc}}}^2/2 = GMm/r = m{v_{\mathrm{orb}}}^2,$$

which has solution $v_{\mathrm{esc}} = \sqrt{2}v_{\mathrm{orb}}$.

E14-15 (a) Near the surface of the Earth the total energy is

$$E = K + U = \frac{1}{2}m\left(2\sqrt{gR_\mathrm{E}}\right)^2 - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}$$

but

$$g = \frac{GM}{R_\mathrm{E}{}^2},$$

so the total energy is

$$\begin{aligned} E &= 2mgR_\mathrm{E} - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ &= 2m\left(\frac{GM}{R_\mathrm{E}{}^2}\right)R_\mathrm{E} - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ &= \frac{GM_\mathrm{E}m}{R_\mathrm{E}} \end{aligned}$$

This is a positive number, so the rocket will escape.

(b) Far from earth there is no gravitational potential energy, so

$$\frac{1}{2}mv^2 = \frac{GM_\mathrm{E}m}{R_\mathrm{E}} = \frac{GM_\mathrm{E}}{R_\mathrm{E}{}^2}mR_\mathrm{E} = gmR\mathrm{E},$$

with solution $v = \sqrt{2gR_\mathrm{E}}$.

E14-16 The rotational acceleration of the sun is related to the galactic acceleration of free fall by

$$4\pi^2 mr/T^2 = GNm^2/r^2,$$

where N is the number of "sun" sized stars of mass m, r is the size of the galaxy, T is period of revolution of the sun. Then

$$N = \frac{4\pi^2 r^3}{GmT^2} = \frac{4\pi^2(2.2\times10^{20}\mathrm{m})^3}{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(2.0\times10^{30}\mathrm{kg})(7.9\times10^{15}\mathrm{s})^2} = 5.1\times10^{10}.$$

E14-17 Energy conservation is $K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$, but at the highest point $K_\mathrm{f} = 0$, so

$$\begin{aligned} U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i}, \\ -\frac{GM_\mathrm{E}m}{R} &= \frac{1}{2}mv_0^2 - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ \frac{1}{R} &= \frac{1}{R_\mathrm{E}} - \frac{1}{2GM_\mathrm{E}}v_0^2, \\ \frac{1}{R} &= \frac{1}{(6.37\times10^6\mathrm{m})} - \frac{(9.42\times10^3\mathrm{m/s})^2)}{2(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})}, \\ R &= 2.19\times10^7\ \mathrm{m}. \end{aligned}$$

The distance above the Earth's surface is $2.19\times10^7\ \mathrm{m} - 6.37\times10^6\mathrm{m} = 1.55\times10^6\ \mathrm{m}$.

E14-18 (a) Free-fall acceleration is $g = GM/r^2$. Escape speed is $v = \sqrt{2GM/r}$. Then $v = \sqrt{2gr} = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})} = 2.02\times10^3\mathrm{m/s}$.

(b) $U_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i}$. But $U/m = -g_0 r_0^2/r$, so

$$\frac{1}{r_\mathrm{f}} = \frac{1}{(1.569\times10^6\mathrm{m})} - \frac{(1.01\times10^3\mathrm{m/s})^2}{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2} = \frac{1}{2.09\times10^6\mathrm{m}}.$$

That's 523 km above the surface.

(c) $K_\mathrm{f} = U_\mathrm{i} - U_\mathrm{f}$. But $U/m = -g_0 r_0^2/r$, so

$$v = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2[1/(1.569\times10^6\mathrm{m}) - 1/(2.569\times10^6\mathrm{m})]} = 1260\,\mathrm{m/s}.$$

(d) $M = gr^2/G$, or

$$M = (1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2/(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2}) = 4.8\times10^{22}\mathrm{kg}.$$

E14-19 (a) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(4.67\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 3.34\times10^7\mathrm{m/s}.$$

(b) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(1.26\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 5.49\times10^7\mathrm{m/s}.$$

E14-20 Call the particles 1 and 2. Then conservation of momentum requires the particle to have the same momentum of the same magnitude, $p = mv_1 = Mv_2$. The momentum of the particles is given by

$$\begin{aligned}
\frac{1}{2m}p^2 + \frac{1}{2M}p^2 &= \frac{GMm}{d}, \\
\frac{m+M}{mM}p^2 &= 2GMm/d, \\
p &= mM\sqrt{2G/d(m+M)}.
\end{aligned}$$

Then $v_\mathrm{rel} = |v_1| + |v_2|$ is equal to

$$\begin{aligned}
v_\mathrm{rel} &= mM\sqrt{2G/d(m+M)}\left(\frac{1}{m} + \frac{1}{M}\right), \\
&= mM\sqrt{2G/d(m+M)}\left(\frac{m+M}{mM}\right), \\
&= \sqrt{2G(m+M)/d}
\end{aligned}$$

E14-21 The maximum speed is $mv^2 = Gm^2/d$, or $v = \sqrt{Gm/d}$.

E14-22 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00\ \mathrm{y})(1.52)^{3/2} = 1.87\ \mathrm{y}.$$

$\boxed{\textbf{E14-23}}$ We can use Eq. 14-23 to find the mass of Mars; all we need to do is rearrange to solve for M—

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(9.4\times10^6\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.75\times10^4\text{s})^2} = 6.5\times10^{23}\text{kg}.$$

E14-24 Use $GM/r^2 = 4\pi^2 r/T^2$, so $M = 4\pi^2 r^3/GT^2$, and

$$M = \frac{4\pi^2(3.82\times10^8\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86400\,\text{s})^2} = 5.93\times10^{24}\text{kg}.$$

E14-25 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00\text{ month})(1/2)^{3/2} = 0.354\text{ month}.$$

E14-26 Geosynchronous orbit was found in Sample Problem 14-8 to be 4.22×10^7m. The latitude is given by

$$\theta = \arccos(6.37\times10^6\text{m}/4.22\times10^7\text{m}) = 81.3°.$$

$\boxed{\textbf{E14-27}}$ (b) Make the assumption that the altitude of the satellite is so low that the radius of the orbit is effectively the radius of the moon. Then

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\text{kg})}\right)(1.74\times10^6\text{m})^3 = 4.24\times10^7\text{s}^2. \end{aligned}$$

So $T = 6.5\times10^3$s.

(a) The speed of the satellite is the circumference divided by the period, or

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.74\times10^6\text{m})}{(6.5\times10^3\text{s})} = 1.68\times10^3\text{m/s}.$$

E14-28 The total energy is $-GMm/2a$. Then

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

so

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

E14-29 $r_\text{a} = a(1+e)$, so from Ex. 14-28,

$$v_\text{a} = \sqrt{GM\left(\frac{2}{a(1+e)} - \frac{1}{a}\right)};$$

$r_\text{p} = a(1-e)$, so from Ex. 14-28,

$$v_\text{p} = \sqrt{GM\left(\frac{2}{a(1-e)} - \frac{1}{a}\right)};$$

Dividing one expression by the other,

$$v_\text{p} = v_\text{a}\sqrt{\frac{2/(1-e)-1}{2/(1+e)-1}} = (3.72\text{ km/s})\sqrt{\frac{2/0.12-1}{2/1.88-1}} = 58.3\text{ km/s}.$$

E14-30 (a) Convert.

$$G = \left(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{1.99\times10^{30}\text{kg}}{M_\text{S}}\right)\left(\frac{3.156\times10^7\text{s}}{\text{y}}\right)^2\left(\frac{\text{AU}}{1.496\times10^{11}\text{m}}\right)^3,$$

which is $G = 39.49\,\text{AU}^3/{M_\text{S}}^2\cdot\text{y}^2$.

(b) Here is a hint: $4\pi^2 = 39.48$. Kepler's law then looks like

$$T^2 = \left(\frac{{M_\text{S}}^2\cdot\text{y}^2}{\text{AU}^3}\right)\frac{r^3}{M}.$$

$\boxed{\textbf{E14-31}}$ Kepler's third law states $T^2 \propto r^3$, where r is the mean distance from the *Sun* and T is the period of revolution. Newton was in a position to find the acceleration of the Moon toward the Earth by assuming the Moon moved in a circular orbit, since $a_c = v^2/r = 4\pi^2 r/T^2$. But this means that, because of Kepler's law, $a_c \propto r/T^2 \propto 1/r^2$.

E14-32 (a) The force of attraction between the two bodies is

$$F = \frac{GMm}{(r+R)^2}.$$

The centripetal acceleration for the body of mass m is

$$\begin{aligned} r\omega^2 &= \frac{GM}{(r+R)^2}, \\ \omega^2 &= \frac{GM}{r^3(1+R/r)^2}, \\ T^2 &= \frac{4\pi^2}{GM}r^3(1+R/r)^2. \end{aligned}$$

(b) Note that $r = Md/(m+M)$ and $R = md/(m+M)$. Then $R/r = m/M$, so the correction is $(1 + 5.94\times10^{24}/1.99\times10^{30})^2 = 1.000006$ for the Earth/Sun system and 1.025 for the Earth/Moon system.

E14-33 (a) Use the results of Exercise 14-32. The center of mass is located a distance $r = 2md/(m+2m) = 2d/3$ from the star of mass m and a distance $R = d/3$ from the star of mass $2m$. The period of revolution is then given by

$$T^2 = \frac{4\pi^2}{G(2m)}\left(\frac{2}{3}d\right)^3\left(1+\frac{d/3}{2d/3}\right)^2 = \frac{4\pi^2}{3Gm}d^3.$$

(b) Use $L_m = mr^2\omega$, then

$$\frac{L_m}{L_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

(c) Use $K = I\omega^2/2 = mr^2\omega^2/2$. Then

$$\frac{K_m}{K_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

E14-34 Since we don't know which direction the orbit will be, we will assume that the satellite on the surface of the Earth starts with zero kinetic energy. Then $E_\mathrm{i} = U_\mathrm{i}$.

$\Delta U = U_\mathrm{f} - U_\mathrm{i}$ to get the satellite up to the specified altitude. $\Delta K = K_\mathrm{f} = -U_\mathrm{f}/2$. We want to know if $\Delta U - \Delta K$ is positive (more energy to get it up) or negative (more energy to put it in orbit). Then we are interested in

$$\Delta U - \Delta K = 3U_\mathrm{f}/2 - U_\mathrm{i} = GMm\left(\frac{1}{r_\mathrm{i}} - \frac{3}{2r_\mathrm{f}}\right).$$

The "break-even" point is when $r_\mathrm{f} = 3r_\mathrm{i}/2 = 3(6400\ \mathrm{km})/2 = 9600\ \mathrm{km}$, which is 3200 km above the Earth.

(a) More energy to put it in orbit.
(b) Same energy for both.
(c) More energy to get it up.

E14-35 (a) The approximate force of gravity on a 2000 kg pickup truck on Eros will be

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\ \mathrm{kg})(2000\,\mathrm{kg})}{(7000\,\mathrm{m})^2} = 13.6\,\mathrm{N}.$$

(b) Use

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\ \mathrm{kg})}{(7000\,\mathrm{m})}} = 6.9\,\mathrm{m/s}.$$

E14-36 (a) $U = -GMm/r$. The variation is then

$$\begin{aligned}\Delta U &= (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})(5.98\times10^{24}\mathrm{kg})\left(\frac{1}{(1.47\times10^{11}\mathrm{m})} - \frac{1}{(1.52\times10^{11}\mathrm{m})}\right)\\ &= 1.78\times10^{32}\mathrm{J}.\end{aligned}$$

(b) $\Delta K + \Delta U = \Delta E = 0$, so $|\Delta K| = 1.78\times10^{32}\mathrm{J}$.
(c) $\Delta E = 0$.
(d) Since $\Delta l = 0$ and $l = mvr$, we have

$$v_\mathrm{p} - v_\mathrm{a} = v_\mathrm{p}\left(1 - \frac{r_\mathrm{p}}{r_\mathrm{a}}\right) = v_\mathrm{p}\left(1 - \frac{(1.47\times10^{11}\mathrm{m})}{(1.52\times10^{11}\mathrm{m})}\right) = 3.29\times10^{-2}v_\mathrm{p}.$$

But $v_\mathrm{p} \approx v_\mathrm{av} = 2\pi(1.5\times10^{11}\mathrm{m})/(3.16\times10^{7}\mathrm{s}) = 2.98\times10^{4}\mathrm{m/s}$. Then $\Delta v = 981\,\mathrm{m/s}$.

E14-37 Draw a triangle. The angle made by Chicago, Earth center, satellite is 47.5°. The distance from Earth center to satellite is 4.22×10^{7}m. The distance from Earth center to Chicago is 6.37×10^{6}m. Applying the cosine law we find the distance from Chicago to the satellite is

$$\sqrt{(4.22\times10^{7}\mathrm{m})^2 + (6.37\times10^{6}\mathrm{m})^2 - 2(4.22\times10^{7}\mathrm{m})(6.37\times10^{6}\mathrm{m})\cos(47.5^\circ)} = 3.82\times10^{7}\mathrm{m}.$$

Applying the sine law we find the angle made by Earth center, Chicago, satellite to be

$$\arcsin\left(\frac{(4.22\times10^{7}\mathrm{m})}{(3.82\times10^{7}\mathrm{m})}\sin(47.5^\circ)\right) = 126^\circ.$$

That's 36° above the horizontal.

E14-38 (a) The new orbit is an ellipse with eccentricity given by $r = a'(1+e)$. Then

$$e = r/a' - 1 = (6.64\times10^6\text{m})/(6.52\times10^6\text{m}) - 1 = 0.0184.$$

The distance at P' is given by $r_{P'} = a'(1-e)$. The potential energy at P' is

$$U_{P'} = U_P \frac{1+e}{1-e} = 2(-9.76\times10^{10}\text{J})\frac{1+0.0184}{1-0.0184} = -2.03\times10^{11}\text{J}.$$

The kinetic energy at P' is then

$$K_{P'} = (-9.94\times10^{10}\text{J}) - (-2.03\times10^{11}\text{J}) = 1.04\times10^{11}\text{J}.$$

That would mean $v = \sqrt{2(1.04\times10^{11}\text{J})/(3250\text{kg})} = 8000\,\text{m/s}$.

(b) The average speed is

$$v = \frac{2\pi(6.52\times10^6\text{m})}{(5240\,\text{s})} = 7820\,\text{m/s}.$$

$\boxed{\textbf{E14-39}}$ (a) The Starshine satellite was approximately 275 km above the surface of the Earth on 1 January 2000. We can find the orbital period from Eq. 14-23,

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})}\right)(6.65\times10^6\text{m})^3 = 2.91\times10^7\text{s}^2, \end{aligned}$$

so $T = 5.39\times10^3$s.

(b) Equation 14-25 gives the total energy of the system of a satellite of mass m in a circular orbit of radius r around a stationary body of mass $M \gg m$,

$$E = -\frac{GMm}{2r}.$$

We want the rate of change of this with respect to time, so

$$\frac{dE}{dt} = \frac{GMm}{2r^2}\frac{dr}{dt}$$

We can estimate the value of dr/dt from the diagram. I'll choose February 1 and December 1 as my two reference points.

$$\left.\frac{dr}{dt}\right|_{t=t_0} \approx \frac{\Delta r}{\Delta t} = \frac{(240\text{ km}) - (300\text{ km})}{(62\text{ days})} \approx -1\text{ km/day}$$

The rate of energy loss is then

$$\frac{dE}{dt} = \frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})(39\,\text{kg})}{2(6.65\times10^6\text{m})^2}\frac{-1000\,\text{m}}{8.64\times10^4\,\text{s}} = -2.0\,\text{J/s}.$$

$\boxed{\textbf{P14-1}}$ The object on the top experiences a force down from gravity W_1 and a force down from the tension in the rope T. The object on the bottom experiences a force down from gravity W_2 and a force up from the tension in the rope.

In either case, the magnitude of W_i is

$$W_i = \frac{GMm}{r_i^2}$$

where r_i is the distance of the ith object from the center of the Earth. While the objects fall they have the same acceleration, and since they have the same mass we can quickly write

$$\frac{GMm}{r_1^2} + T = \frac{GMm}{r_2^2} - T,$$

or

$$\begin{aligned} T &= \frac{GMm}{2r_2^2} - \frac{GMm}{2r_1^2}, \\ &= \frac{GMm}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right), \\ &= \frac{GMm}{2}\frac{r_2^2 - r_1^2}{r_1^2 r_2^2}. \end{aligned}$$

Now $r_1 \approx r_2 \approx R$ in the denominator, but $r_2 = r_1 + l$, so $r_2^2 - r_1^2 \approx 2Rl$ in the numerator. Then

$$T \approx \frac{GMml}{R^3}.$$

P14-2 For a planet of uniform density, $g = GM/r^2 = G(4\pi\rho r^3/3)/r^2 = 4\pi G\rho r/3$. Then if ρ is doubled while r is halved we find that g will be unchanged.

P14-3 (a) $F = GMm/r^2$, $a = F/m = GM/r$.

(b) The acceleration of the Earth toward the center of mass is $a_{\mathrm{E}} = F/M = Gm/r^2$. The relative acceleration is then $GM/r + Gm/r = G(m + M)/r$. Only if $M \gg m$ can we assume that a is independent of m relative to the Earth.

P14-4 (a) $g = GM/r^2$, $\delta g = -(2GM/r^3)\delta r$. In this case $\delta r = h$ and $M = 4\pi\rho r^3/3$. Then

$$\delta W = m\,\delta g = 8\pi G\rho m h/3.$$

(b) $\Delta W/W = \Delta g/g = 2h/r$. Then an error of one part in a million will occur when h is one part in two million of r, or 3.2 meters.

$\boxed{\textbf{P14-5}}$ (a) The magnitude of the gravitational force from the Moon on a particle at A is

$$F_A = \frac{GMm}{(r-R)^2},$$

where the denominator is the distance from the center of the moon to point A.

(b) At the center of the Earth the gravitational force of the moon on a particle of mass m is $F_C = GMm/r^2$.

(c) Now we want to know the difference between these two expressions:

$$\begin{aligned} F_A - F_C &= \frac{GMm}{(r-R)^2} - \frac{GMm}{r^2}, \\ &= GMm\left(\frac{r^2}{r^2(r-R)^2} - \frac{(r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{r^2 - (r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{R(2r-R)}{r^2(r-R)^2}\right). \end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r - R) \approx r$. The force difference simplifies to

$$F_T = GMm\frac{R(2r)}{r^2(r)^2} = \frac{2GMmR}{r^3}$$

(d) Repeat part (c) except we want $r + R$ instead of $r - R$. Then

$$\begin{aligned}
F_A - F_C &= \frac{GMm}{(r+R)^2} - \frac{GMm}{r^2}, \\
&= GMm\left(\frac{r^2}{r^2(r+R)^2} - \frac{(r+R)^2}{r^2(r+R)^2}\right), \\
&= GMm\left(\frac{r^2-(r+R)^2}{r^2(r+R)^2}\right), \\
&= GMm\left(\frac{-R(2r+R)}{r^2(r+R)^2}\right).
\end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r + R) \approx r$. The force difference simplifies to

$$F_T = GMm\frac{-R(2r)}{r^2(r)^2} = -\frac{2GMmR}{r^3}$$

The negative sign indicates that this "apparent" force points *away* from the moon, not toward it.

(e) Consider the directions: the water is effectively attracted to the moon when closer, but repelled when farther.

P14-6 $F_{\text{net}} = mr{\omega_{\text{s}}}^2$, where ω_{s} is the rotational speed of the ship. But since the ship is moving relative to the earth with a speed v, we can write $\omega_{\text{s}} = \omega \pm v/r$, where the sign is positive if the ship is sailing east. Then $F_{\text{net}} = mr(\omega \pm v/r)^2$.

The scale measures a force W which is given by $mg - F_{\text{net}}$, or

$$W = mg - mr(\omega \pm v/r)^2.$$

Note that $W_0 = m(g - r\omega^2)$. Then

$$\begin{aligned}
W &= W_0\frac{g - r(\omega \pm v/r)^2}{g - r\omega^2}, \\
&\approx W_0\left(1 \pm \frac{2\omega v}{1 - r\omega^2}\right), \\
&\approx W_0(1 \pm 2v\omega/g).
\end{aligned}$$

P14-7 (a) $a = GM/r^2 - r\omega^2$. ω is the rotational speed of the Earth. Since Frank observes $a = g/2$ we have

$$\begin{aligned}
g/2 &= GM/r^2 - r\omega^2, \\
r^2 &= (2GM - 2r^3\omega^2)/g, \\
r &= \sqrt{2(GM - r^3\omega^2)/g}
\end{aligned}$$

Note that

$$GM = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg}) = 3.99\times10^{14}\text{m}^3/\text{s}^2$$

while

$$r^3\omega^2 = (6.37\times10^{6}\text{m})^3(2\pi/86400\,\text{s})^2 = 1.37\times10^{12}\text{m}^3/\text{s}^2.$$

Consequently, $r^3\omega^2$ can be treated as a perturbation of GM bear the Earth. Solving iteratively,

$$\begin{aligned}
r_0 &= \sqrt{2[(3.99\times10^{14}\mathrm{m^3/s^2}) - (6.37\times10^6\mathrm{m})^3(2\pi/86400\,\mathrm{s})^2]/(9.81\,\mathrm{m/s^2})} = 9.00\times10^6\mathrm{m},\\
r_1 &= \sqrt{2[(3.99\times10^{14}\mathrm{m^3/s^2}) - (9.00\times10^6\mathrm{m})^3(2\pi/86400\,\mathrm{s})^2]/(9.81\,\mathrm{m/s^2})} = 8.98\times10^6\mathrm{m},
\end{aligned}$$

which is close enough for me. Then $h = 8.98\times10^6\mathrm{m} - 6.37\times10^6\mathrm{m} = 2610$ km.

(b) $\Delta E = E_\mathrm{f} - E_\mathrm{i} = U_\mathrm{f}/2 - U_\mathrm{i}$. Then

$$\Delta E = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})(100\,\mathrm{kg})\left(\frac{1}{(6.37\times10^6\mathrm{m})} - \frac{1}{2(8.98\times10^6\mathrm{m})}\right) = 4.0\times10^9\mathrm{J}.$$

P14-8 (a) Equate centripetal force with the force of gravity.

$$\begin{aligned}
\frac{4\pi^2 mr}{T^2} &= \frac{GMm}{r^2},\\
\frac{4\pi^2}{T^2} &= \frac{G(4/3)\pi r^3\rho}{r^3},\\
T &= \sqrt{\frac{3\pi}{G\rho}}
\end{aligned}$$

(b) $T = \sqrt{3\pi/(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(3.0\times10^3\mathrm{kg/m^3})} = 6800\,\mathrm{s}$.

P14-9 (a) One can find δg by pretending the Earth is not there, but the material in the hole is. Concentrate on the vertical component of the resulting force of attraction. Then

$$\delta g = \frac{GM}{r^2}\frac{d}{r},$$

where r is the straight line distance from the prospector to the center of the hole and M is the mass of material that would fill the hole. A few substitutions later,

$$\delta g = \frac{4\pi G\rho R^3 d}{3(\sqrt{d^2+x^2})^3}.$$

(b) Directly above the hole $x = 0$, so a ratio of the two readings gives

$$\frac{(10.0\text{ milligals})}{(14.0\text{ milligals})} = \left(\frac{d^2}{d^2+(150\,\mathrm{m})^2}\right)^{3/2}$$

or

$$(0.800)(d^2 + 2.25\times10^4\mathrm{m^2}) = d^2,$$

which has solution $d = 300\,\mathrm{m}$. Then

$$R^3 = \frac{3(14.0\times10^{-5}\mathrm{m/s^2})(300\,\mathrm{m})^2}{4\pi(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(2800\,\mathrm{kg/m^3})},$$

so $R = 250\,\mathrm{m}$. The top of the cave is then $300\,\mathrm{m} - 250\,\mathrm{m} = 50\,\mathrm{m}$ beneath the surface.

(b) All of the formulae stay the same *except* replace ρ with the difference between rock and water. d doesn't change, but R will now be given by

$$R^3 = \frac{3(14.0\times10^{-5}\mathrm{m/s^2})(300\,\mathrm{m})^2}{4\pi(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1800\,\mathrm{kg/m^3})},$$

so $R = 292\,\mathrm{m}$, and then the cave is located $300\,\mathrm{m} - 292\,\mathrm{m} = 7\,\mathrm{m}$ beneath the surface.

P14-10 $g = GM/r^2$, where M is the mass enclosed in within the sphere of radius r. Then $dg = (G/r^2)dM - 2(GM/r^3)dr$, so that g is locally constant if $dM/dr = 2M/r$. Expanding,

$$\begin{aligned} 4\pi r^2 \rho_1 &= 8\pi r^2 \rho/3, \\ \rho_1 &= 2\rho/3. \end{aligned}$$

$\boxed{\textbf{P14-11}}$ The force of gravity on the small sphere of mass m is equal to the force of gravity from a solid lead sphere minus the force which would have been contributed by the smaller lead sphere which would have filled the hole. So we need to know about the size and mass of the lead which was removed to make the hole.

The density of the lead is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

The hole has a radius of $R/2$, so if the density is constant the mass of the hole will be

$$M_h = \rho V = \left(\frac{M}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

The "hole" is closer to the small sphere; the center of the hole is $d - R/2$ away. The force of the whole lead sphere minus the force of the "hole" lead sphere is

$$\frac{GMm}{d^2} - \frac{G(M/8)m}{(d - R/2)^2}$$

P14-12 (a) Use $v = \omega\sqrt{R^2 - r^2}$, where $\omega = \sqrt{GM_E/R^3}$. Then

$$\begin{aligned} T &= \int_0^T dt = \int_R^0 \frac{dr}{dr/dt} = \int_R^0 \frac{dr}{v}, \\ &= \int_R^0 \frac{dr}{\omega\sqrt{R^2 - r^2}}, \\ &= \frac{\pi}{2\omega} \end{aligned}$$

Knowing that

$$\omega = \sqrt{\frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})}{(6.37\times10^6\mathrm{m})^3}} = 1.24\times10^{-3}/\mathrm{s},$$

we can find $T = 1260\,\mathrm{s} = 21$ min.

(b) Same time, 21 minutes. To do a complete journey would require four times this, or $2\pi/\omega$. That's 84 minutes!

(c) The answers are the same.

P14-13 (a) $g = GM/r^2$ and $M = 1.93\times10^{24}\mathrm{kg} + 4.01\times10^{24}\mathrm{kg} + 3.94\times10^{22}\mathrm{kg} = 5.98\times10^{24}\mathrm{kg}$ so

$$g = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})/(6.37\times10^6\mathrm{m})^2 = 9.83\,\mathrm{m/s^2}.$$

(b) Now $M = 1.93\times10^{24}\mathrm{kg} + 4.01\times10^{24}\mathrm{kg} = 5.94\times10^{24}\mathrm{kg}$ so

$$g = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.94\times10^{24}\mathrm{kg})/(6.345\times10^6\mathrm{m})^2 = 9.84\,\mathrm{m/s^2}.$$

(c) For a uniform body, $g = 4\pi G\rho r/3 = GMr/R^3$, so

$$g = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})(6.345\times10^6\mathrm{m})/(6.37\times10^6\mathrm{m})^3 = 9.79\,\mathrm{m/s^2}.$$

P14-14 (a) Use $g = GM/r^2$, then

$$g = (6.67\times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(1.93\times 10^{24}\text{kg})/(3.490\times 10^6\text{m})^2 = 106\,\text{m/s}^2.$$

The variation with depth is linear if core has uniform density.

(b) In the mantle we have $g = G(M_\text{c} + M)/r^2$, where M is the amount of the mass of the mantle which is enclosed in the sphere of radius r. The density of the core is

$$\rho_\text{c} = \frac{3(1.93\times 10^{24}\text{kg})}{4\pi(3.490\times 10^6\text{m})^3} = 1.084\times 10^4\text{kg/m}^3.$$

The density of the mantle is harder to find,

$$\rho_\text{c} = \frac{3(4.01\times 10^{24}\text{kg})}{4\pi[(6.345\times 10^6\text{m})^3 - (3.490\times 10^6\text{m})^3} = 4.496\times 10^3\text{kg/m}^3.$$

We can pretend that the core is made up of a point mass at the center and the rest has a density equal to that of the mantle. That point mass would be

$$M_\text{p} = \frac{4\pi(3.490\times 10^6\text{m})^3(1.084\times 10^4\text{kg/m}^3 - 4.496\times 10^3\text{kg/m}^3)}{3} = 1.130\times 10^{24}\text{kg}.$$

Then

$$g = GM_\text{p}/r^2 + 4\pi G\rho_\text{m} r/3.$$

Find dg/dr, and set equal to zero. This happens when

$$2M_\text{p}/r^3 = 4\pi\rho_\text{m}/3,$$

or $r = 4.93\times 10^6$m. Then $g = 9.29\,\text{m/s}^2$. Since this is less than the value at the end points it must be a minimum.

P14-15 (a) We will use part of the hint, but we will integrate instead of assuming the bit about g_av; doing it this way will become important for later chapters. Consider a small horizontal slice of the column of thickness dr. The weight of the material above the slice exerts a force $F(r)$ on the top of the slice; there is a force of gravity on the slice given by

$$dF = \frac{GM(r)\,dm}{r^2},$$

where $M(r)$ is the mass contained in the sphere of radius r,

$$M(r) = \frac{4}{3}\pi r^3\rho.$$

Lastly, the mass of the slice dm is related to the thickness and cross sectional area by $dm = \rho A\,dr$. Then

$$dF = \frac{4\pi GA\rho^2}{3} r\,dr.$$

Integrate both sides of this expression. On the left the limits are 0 to F_center, on the right the limits are R to 0; we need to throw in an extra negative sign because the force increases as r decreases. Then

$$F = \frac{2}{3}\pi GA\rho^2 R^2.$$

Divide both sides by A to get the compressive stress.

(b) Put in the numbers!

$$S = \frac{2}{3}\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(4000\,\mathrm{kg/m}^3)^2(3.0\times10^5\mathrm{m})^2 = 2.0\times10^8\mathrm{N/m}^2.$$

(c) Rearrange, and then put in numbers;

$$R = \sqrt{\frac{3(4.0\times10^7\mathrm{N/m}^2)}{2\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(3000\,\mathrm{kg/m}^3)^2}} = 1.8\times10^5\,\mathrm{m}.$$

P14-16 The two mass increments each exert a vertical and a horizontal force on the particle, but the horizontal components will cancel. The vertical component is proportional to the sine of the angle, so that

$$dF = \frac{2Gm\,dm}{r^2}\frac{y}{r} = \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r},$$

where $r^2 = x^2 + y^2$. We will eventually integrate from 0 to ∞, so

$$\begin{aligned}
F &= \int_0^\infty \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r},\\
&= 2Gm\lambda y\int_0^\infty \frac{dx}{(x^2+y^2)^{3/2}},\\
&= \frac{2Gm\lambda}{y}.
\end{aligned}$$

P14-17 For any arbitrary point P the cross sectional area which is perpendicular to the axis $dA' = r^2 d\Omega$ is not equal to the projection dA onto the surface of the sphere. It depends on the angle that the axis makes with the normal, according to $dA' = \cos\theta dA$. Fortunately, the angle made at point 1 is identical to the angle made at point 2, so we can write

$$\begin{aligned}
d\Omega_1 &= d\Omega_2,\\
dA_1/r_1^2 &= dA_2/r_2^2
\end{aligned}$$

But the mass of the shell contained in dA is proportional to dA, so

$$\begin{aligned}
r_1^2 dm_1 &= r_2^2 dm_2,\\
Gm\,dm_1/r_1^2 &= Gm\,dm_2/r_2^2.
\end{aligned}$$

Consequently, the force on an object at point P is balanced by both cones.

(b) Evaluate $\int d\Omega$ for the top and bottom halves of the sphere. Since every $d\Omega$ on the top is balanced by one on the bottom, the net force is zero.

P14-18

P14-19 Assume that the small sphere is always between the two spheres. Then

$$\begin{aligned}
W &= \Delta U_1 + \Delta U_2,\\
&= (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(0.212\,\mathrm{kg})\,[(7.16\,\mathrm{kg}) - (2.53\,\mathrm{kg})]\left[\frac{1}{(0.420\,\mathrm{m})} - \frac{1}{(1.14\,\mathrm{m})}\right],\\
&= 9.85\times10^{-11}\mathrm{J}.
\end{aligned}$$

P14-20 Note that $\frac{1}{2}m{v_{\rm esc}}^2 = -U_0$, where U_0 is the potential energy at the burn-out height. Energy conservation gives

$$\begin{aligned} K &= K_0 + U_0, \\ \frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - \frac{1}{2}m{v_{\rm esc}}^2, \\ v &= \sqrt{v_0^2 - {v_{\rm esc}}^2}. \end{aligned}$$

$\boxed{\textbf{P14-21}}$ (a) The force of one star on the other is given by $F = Gm^2/d^2$, where d is the distance between the stars. The stars revolve around the center of mass, which is halfway between the stars so $r = d/2$ is the radius of the orbit of the stars. If a is the centripetal acceleration of the stars, the period of revolution is then

$$T = \sqrt{\frac{4\pi^2 r}{a}} = \sqrt{\frac{4m\pi^2 r}{F}} = \sqrt{\frac{16\pi^2 r^3}{Gm}}.$$

The numerical value is

$$T = \sqrt{\frac{16\pi^2(1.12\times10^{11}{\rm m})^3}{(6.67\times10^{-11}{\rm N\cdot m^2/kg^2})(3.22\times10^{30}{\rm kg})}} = 3.21\times10^7{\rm s} = 1.02 \text{ y}.$$

(b) The gravitational potential energy per kilogram midway between the stars is

$$-2\frac{Gm}{r} = -2\frac{(6.67\times10^{-11}{\rm N\cdot m^2/kg^2})(3.22\times10^{30}{\rm kg})}{(1.12\times10^{11}{\rm m})} = -3.84\times10^9{\rm J/kg}.$$

An object of mass M at the center between the stars would need $(3.84\times10^9{\rm J/kg})M$ kinetic energy to escape, this corresponds to a speed of

$$v = \sqrt{2K/M} = \sqrt{2(3.84\times10^9{\rm J/kg})} = 8.76\times10^4{\rm m/s}.$$

P14-22 (a) Each differential mass segment on the ring contributes the same amount to the force on the particle,

$$dF = \frac{Gm\,dm}{r^2}\frac{x}{r},$$

where $r^2 = x^2 + R^2$. Since the differential mass segments are all equal distance, the integration is trivial, and the net force is

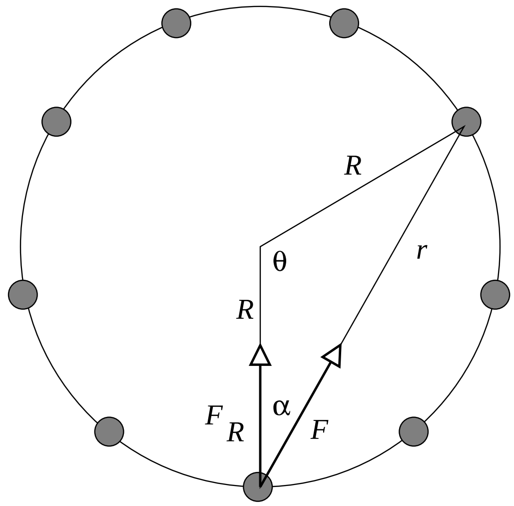
$$F = \frac{GMmx}{(x^2+R^2)^{3/2}}.$$

(b) The potential energy can be found by integrating with respect to x,

$$\Delta U = \int_0^\infty F\,dx = \int_0^\infty \frac{GMmx}{(x^2+R^2)^{3/2}}\,dx = \frac{GMm}{R}.$$

Then the particle of mass m will pass through the center of the ring with a speed $v = \sqrt{2\Delta U/m} = \sqrt{2GM/R}$.

P14-23 (a) Consider the following diagram.



The distance r is given by the cosine law to be

$$r^2 = R^2 + R^2 - 2R^2 \cos\theta = 2R^2(1 - \cos\theta).$$

The force between two particles is then $F = Gm^2/r^2$. Each particle has a symmetric partner, so only the force component directed toward the center contributes. If we call this the R component we have

$$F_R = F\cos\alpha = F\cos(90^\circ - \theta/2) = F\sin(\theta/2).$$

Combining,

$$F_R = \frac{Gm^2}{2R^2}\frac{\sin(\theta/2)}{1 - \cos\theta}.$$

But *each* of the other particles contributes to this force, so

$$F_{\text{net}} = \frac{Gm^2}{2R^2}\sum_i \frac{\sin(\theta_i/2)}{1 - \cos\theta_i}$$

When there are only 9 particles the angles are in steps of 40°; the θ_i are then 40°, 80°, 120°, 160°, 200°, 240°, 280°, and 320°. With a little patience you will find

$$\sum_i \frac{\sin(\theta_i/2)}{1 - \cos\theta_i} = 6.649655,$$

using these angles. Then $F_{\text{net}} = 3.32Gm^2/R^2$.

(b) The rotational period of the ring would need to be

$$T = \sqrt{\frac{4\pi^2 R}{a}} = \sqrt{\frac{4m\pi^2 R}{F}} = \sqrt{\frac{16\pi^2 R^3}{3.32Gm}}.$$

P14-24 The potential energy of the system is $U = -Gm^2/r$. The kinetic energy is mv^2. The total energy is $E = -Gm^2/d$. Then

$$\frac{dr}{dt} = 2\sqrt{Gm(1/r - 1/d)},$$

so the time to come together is

$$T = \int_d^0 \frac{dr}{2\sqrt{Gm(1/r - 1/d)}} = \sqrt{\frac{d^3}{4Gm}} \int_0^1 \sqrt{\frac{x}{1-x}} dx = \frac{\pi}{4}\sqrt{\frac{d^3}{Gm}}.$$

P14-25 (a) $E = U/2$ for each satellite, so the total mechanical energy is $-GMm/r$.

(b) Now there is no K, so the total mechanical energy is simply $U = -2GMm/r$. The factor of 2 is because there are two satellites.

(c) The wreckage falls vertically onto the Earth.

P14-26 Let $r_{\mathrm{a}} = a(1+e)$ and $r_{\mathrm{p}} = a(1-e)$. Then $r_{\mathrm{a}} + r_{\mathrm{e}} = 2a$ and $r_{\mathrm{a}} - r_{\mathrm{p}} = 2ae$. So the answer is

$$2(0.0167)(1.50\times10^{11}\mathrm{m}) = 5.01\times10^9\mathrm{m},$$

or 7.20 solar radii.

P14-27

P14-28 The net force on an orbiting star is

$$F = Gm\left(\frac{M}{r^2} + m4r^2\right).$$

This is the centripetal force, and is equal to $4\pi^2 mr/T^2$. Combining,

$$\frac{4\pi^2}{T^2} = \frac{G}{4r^3}(4M+m),$$

so $T = 4\pi\sqrt{r^3/[G(4M+m)]}$.

P14-29 (a) $v = \sqrt{GM/r}$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})/(7.01\times10^6\mathrm{m})} = 7.54\times10^3\mathrm{m/s}.$$

(b) $T = 2\pi(7.01\times10^6\mathrm{m})/(7.54\times10^3\mathrm{m/s}) = 5.84\times10^3\mathrm{s}$.

(c) Originally $E_0 = U/2$, or

$$E = -\frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})(220\mathrm{kg})}{2(7.01\times10^6\mathrm{m})} = -6.25\times10^9\mathrm{J}.$$

After 1500 orbits the energy is now $-6.25\times10^9\mathrm{J} - (1500)(1.40\times10^5\mathrm{J}) = -6.46\times10^9\mathrm{J}$. The new distance from the Earth is then

$$r = -\frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times10^{24}\mathrm{kg})(220\mathrm{kg})}{2(-6.46\times10^9\mathrm{J})} = 6.79\times10^6\mathrm{m}.$$

The altitude is now $6790 - 6370 = 420$ km.

(d) $F = (1.40\times10^5\mathrm{J})/(2\pi 7.01\times10^6\mathrm{m}) = 3.2\times10^{-3}\mathrm{N}$.

(e) No.

P14-30 Let the satellite S be directly overhead at some time. The magnitude of the speed is equal to that of a geosynchronous satellite T whose orbit is not inclined, but since there are both parallel and perpendicular components to the motion of S it will appear to move north while "losing ground" compared to T. Eventually, though, it must pass overhead again in 12 hours. When S is as far north as it will go (6 hours) it has a velocity which is parallel to T, but it is located in a region where the required speed to appear fixed is slower. Hence, it will appear to be "gaining ground" against the background stars. Consequently, the motion against the background stars appears to be a figure 8.

P14-31 The net force of gravity on one star because of the other two is

$$F = \frac{2GM^2}{L^2}\cos(30°).$$

The stars orbit about a point $r = L/2\cos(30°)$ from any star. The orbital speed is then found from

$$\frac{Mv^2}{r} = \frac{Mv^2}{L/2\cos(30°)} = \frac{2GM^2}{L^2}\cos(30°),$$

or $v = \sqrt{GM/L}$.

P14-32 A parabolic path will eventually escape; this means that the speed of the comet at any distance is the escape speed for that distance, or $v = \sqrt{2GM/r}$. The angular momentum is constant, and is equal to

$$l = mv_{\mathrm{A}}r_{\mathrm{A}} = m\sqrt{2GMr_{\mathrm{A}}}.$$

For a parabolic path, $r = 2r_{\mathrm{A}}/(1+\cos\theta)$. Combining with Eq. 14-21 and the equation before that one we get

$$\frac{d\theta}{dt} = \frac{\sqrt{2GMr_{\mathrm{A}}}}{4{r_{\mathrm{A}}}^2}(1+\cos\theta)^2.$$

The time required is the integral

$$T = \sqrt{\frac{8{r_{\mathrm{A}}}^3}{GM}}\int_0^{\pi/2}\frac{d\theta}{(1+\cos\theta)^2} = \sqrt{\frac{8{r_{\mathrm{A}}}^3}{GM}}\left(\frac{2}{3}\right).$$

Note that $\sqrt{{r_{\mathrm{A}}}^3/GM}$ is equal to $1/2\pi$ years. Then the time for the comet to move is

$$T = \frac{1}{2\pi}\sqrt{8}\frac{2}{3}\ \mathrm{y} = 0.300\ \mathrm{y}.$$

$\boxed{\textbf{P14-33}}$ There are three forces on loose matter (of mass m_0) sitting on the moon: the force of gravity toward the moon, $F_m = Gmm_0/a^2$, the force of gravity toward the planet, $F_M = GMm_0/(r-a)^2$, and the normal force N of the moon pushing the loose matter away from the center of the moon.

The net force on this loose matter is $F_M + N - F_m$, this value is *exactly* equal to the centripetal force necessary to keep the loose matter moving in a uniform circle. The period of revolution of the loose matter is identical to that of the moon,

$$T = 2\pi\sqrt{r^3/GM},$$

but since the loose matter is actually revolving at a radial distance $r-a$ the centripetal force is

$$F_{\mathrm{c}} = \frac{4\pi^2 m_0(r-a)}{T^2} = \frac{GMm_0(r-a)}{r^3}.$$

Only if the normal force is zero can the loose matter can lift off, and this will happen when $F_\mathrm{c} = F_M - F_m$, or

$$\begin{aligned}
\frac{M(r-a)}{r^3} &= \frac{M}{(r-a)^2} - \frac{m}{a^2}, \\
&= \frac{Ma^2 - m(r-a)^2}{a^2(r-a)^2}, \\
Ma^2(r-a)^3 &= Mr^3a^2 - mr^3(r-a)^2, \\
-3r^2a^3 + 3ra^4 - a^4 &= \frac{m}{M}\left(-r^5 + 2r^4a - r^3a^2\right)
\end{aligned}$$

Let $r = ax$, then x is dimensionless; let $\beta = m/M$, then β is dimensionless. The expression then simplifies to

$$-3x^2 + 3x - 1 = \beta(-x^5 + 2x^4 - x^3).$$

If we assume than x is very large ($r \gg a$) then only the largest term on each side survives. This means $3x^2 \approx \beta x^5$, or $x = (3/\beta)^{1/3}$. In that case, $r = a(3M/m)^{1/3}$. For the Earth's moon $r_\mathrm{c} = 1.1\times 10^7$m, which is only 4,500 km away from the surface of the Earth. It is somewhat interesting to note that the radius r is actually independent of both a and m if the moon has a uniform density!