E13-1 If the projectile had *not* experienced air drag it would have risen to a height y_2, but because of air drag 68 kJ of mechanical energy was dissipated so it only rose to a height y_1. In either case the initial velocity, and hence initial kinetic energy, was the same; and the velocity at the highest point was zero. Then $W = \Delta U$, so the potential energy would have been 68 kJ greater, and

$$\Delta y = \Delta U/mg = (68\times 10^3\mathrm{J})/(9.4\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 740\,\mathrm{m}$$

is how much higher it would have gone without air friction.

E13-2 (a) The road incline is $\theta = \arctan(0.08) = 4.57°$. The frictional forces are the same; the car is now moving with a vertical upward speed of $(15\,\mathrm{m/s})\sin(4.57°) = 1.20\,\mathrm{m/s}$. The additional power required to drive up the hill is then $\Delta P = mgv_y = (1700\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m/s}) = 20000\,\mathrm{W}$. The total power required is 36000 W.

(b) The car will "coast" if the power generated by rolling downhill is equal to 16000 W, or

$$v_y = (16000\,\mathrm{W})/[(1700\,\mathrm{kg})(9.81\,\mathrm{m/s^2})] = 0.959\,\mathrm{m/s},$$

down. Then the incline is

$$\theta = \arcsin(0.959\,\mathrm{m/s}/15\,\mathrm{m/s}) = 3.67°.$$

This corresponds to a downward grade of $\tan(3.67°) = 6.4\,\%$.

E13-3 Apply energy conservation:

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}ky^2 = 0,$$

so

$$v = \sqrt{-2(9.81\,\mathrm{m/s^2})(-0.084\,\mathrm{m}) - (262\,\mathrm{N/m})(-0.084\,\mathrm{m})^2/(1.25\,\mathrm{kg})} = 0.41\,\mathrm{m/s}.$$

E13-4 The car climbs a vertical distance of $(225\,\mathrm{m})\sin(10°) = 39.1\,\mathrm{m}$ in coming to a stop. The change in energy of the car is then

$$\Delta E = -\frac{1}{2}\frac{(16400\,\mathrm{N})}{(9.81\,\mathrm{m/s^2})}(31.4\,\mathrm{m/s})^2 + (16400\,\mathrm{N})(39.1\,\mathrm{m}) = -1.83\times 10^5\mathrm{J}.$$

E13-5 (a) Applying conservation of energy to the points where the ball was dropped and where it entered the oil,

$$\begin{aligned}
\frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} &= \frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i},\\
\frac{1}{2}{v_\mathrm{f}}^2 + g(0) &= \frac{1}{2}(0)^2 + gy_\mathrm{i},\\
v_\mathrm{f} &= \sqrt{2gy_\mathrm{i}},\\
&= \sqrt{2(9.81\,\mathrm{m/s^2})(0.76\,\mathrm{m})} = 3.9\,\mathrm{m/s}.
\end{aligned}$$

(b) The change in internal energy of the ball + oil can be found by considering the points where the ball was released and where the ball reached the bottom of the container.

$$\begin{aligned}
\Delta E &= K_\mathrm{f} + U_\mathrm{f} - K_\mathrm{i} - U_\mathrm{i},\\
&= \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} - \frac{1}{2}m(0)^2 - mgy_\mathrm{i},\\
&= \frac{1}{2}(12.2\times 10^{-3}\mathrm{kg})(1.48\mathrm{m/s})^2 - (12.2\times 10^{-3}\mathrm{kg})(9.81\mathrm{m/s^2})(-0.55\mathrm{m} - 0.76\mathrm{m}),\\
&= -0.143\,\mathrm{J}
\end{aligned}$$

E13-6 (a) $U_\mathrm{i} = (25.3\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(12.2\,\mathrm{m}) = 3030\,\mathrm{J}$.
(b) $K_\mathrm{f} = \frac{1}{2}(25.3\,\mathrm{kg})(5.56\,\mathrm{m/s})^2 = 391\,\mathrm{J}$.
(c) $\Delta E_\mathrm{int} = 3030\,\mathrm{J} - 391\,\mathrm{J} = 2640\,\mathrm{J}$.

E13-7 (a) At atmospheric entry the kinetic energy is

$$K = \frac{1}{2}(7.9\times 10^4\mathrm{kg})(8.0\times 10^3\mathrm{m/s})^2 = 2.5\times 10^{12}\mathrm{J}.$$

The gravitational potential energy is

$$U = (7.9\times 10^4\mathrm{kg})(9.8\,\mathrm{m/s^2})(1.6\times 10^5\mathrm{m}) = 1.2\times 10^{11}\mathrm{J}.$$

The total energy is $2.6\times 10^{12}\mathrm{J}$.
(b) At touch down the kinetic energy is

$$K = \frac{1}{2}(7.9\times 10^4\mathrm{kg})(9.8\times 10^1\mathrm{m/s})^2 = 3.8\times 10^8\mathrm{J}.$$

E13-8 $\Delta E/\Delta t = (68\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(59\,\mathrm{m/s}) = 39000\,\mathrm{J/s}$.

E13-9 Let m be the mass of the water under consideration. Then the percentage of the potential energy "lost" which appears as kinetic energy is

$$\frac{K_\mathrm{f} - K_\mathrm{i}}{U_\mathrm{i} - U_\mathrm{f}}.$$

Then

$$\begin{aligned}
\frac{K_\mathrm{f} - K_\mathrm{i}}{U_\mathrm{i} - U_\mathrm{f}} &= \frac{1}{2}m\left({v_\mathrm{f}}^2 - {v_\mathrm{i}}^2\right)/\left(mgy_\mathrm{i} - mgy_\mathrm{f}\right), \\
&= \frac{{v_\mathrm{f}}^2 - {v_\mathrm{i}}^2}{-2g\Delta y}, \\
&= \frac{(13\,\mathrm{m/s})^2 - (3.2\,\mathrm{m/s})^2}{-2(9.81\,\mathrm{m/s^2})(-15\,\mathrm{m})}, \\
&= 54\,\%.
\end{aligned}$$

The rest of the energy would have been converted to sound and thermal energy.

E13-10 The change in energy is

$$\Delta E = \frac{1}{2}(524\,\mathrm{kg})(62.6\,\mathrm{m/s})^2 - (524\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(292\,\mathrm{m}) = 4.74\times 10^5\mathrm{J}.$$

E13-11 $U_\mathrm{f} = K_\mathrm{i} - (34.6\,\mathrm{J})$. Then

$$h = \frac{1}{2}\frac{(7.81\,\mathrm{m/s})^2}{(9.81\,\mathrm{m/s^2})} - \frac{(34.6\,\mathrm{J})}{(4.26\,\mathrm{kg})(9.81\,\mathrm{m/s^2})} = 2.28\,\mathrm{m};$$

which means the distance along the incline is $(2.28\,\mathrm{m})/\sin(33.0^\circ) = 4.19\,\mathrm{m}$.

E13-12 (a) $K_\text{f} = U_\text{i} - U_\text{f}$, so

$$v_\text{f} = \sqrt{2(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})]} = 48.7\,\text{m/s}.$$

That's a quick 175 km/h; but the speed at the bottom of the valley is 40% of the speed of sound!

(b) $\Delta E = U_\text{f} - U_\text{i}$, so

$$\Delta E = (54.4\,\text{kg})(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})] = -6.46\times10^4\text{J};$$

which means the internal energy of the snow and skis increased by 6.46×10^4J.

E13-13 The final potential energy is 15% less than the initial kinetic plus potential energy of the ball, so

$$0.85(K_\text{i} + U_\text{i}) = U_\text{f}.$$

But $U_\text{i} = U_\text{f}$, so $K_\text{i} = 0.15U_\text{f}/0.85$, and then

$$v_\text{i} = \sqrt{\frac{0.15}{0.85}2gh} = \sqrt{2(0.176)(9.81\,\text{m/s}^2)(12.4\,\text{m})} = 6.54\,\text{m/s}.$$

E13-14 Focus on the potential energy. After the nth bounce the ball will have a potential energy at the top of the bounce of $U_n = 0.9U_{n-1}$. Since $U \propto h$, one can write $h_n = (0.9)^n h_0$. Solving for n,

$$n = \log(h_n/h_0)/\log(0.9) = \log(3\text{ ft}/6\text{ ft})/\log(0.9) = 6.58,$$

which must be rounded up to 7.

E13-15 Let m be the mass of the ball and M be the mass of the block.

The kinetic energy of the ball just before colliding with the block is given by $K_1 = U_0$, so $v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.687\,\text{m})} = 3.67\,\text{m/s}$.

Momentum is conserved, so if v_2 and v_3 are velocities of the ball and block after the collision then $mv_1 = mv_2 + Mv_3$. Kinetic energy is not conserved, instead

$$\frac{1}{2}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{2}mv_2^2 + \frac{1}{2}Mv_3^2.$$

Combine the energy and momentum expressions to eliminate v_3:

$$\begin{aligned} mv_1^2 &= 2mv_2^2 + 2M\left(\frac{m}{M}(v_1 - v_2)\right)^2, \\ Mv_1^2 &= 2Mv_2^2 + 2mv_1^2 - 4mv_1v_2 + 2mv_2^2, \end{aligned}$$

which can be formed into a quadratic. The solution for v_2 is

$$v_2 = \frac{2m \pm \sqrt{2(M^2 - mM)}}{2(M+m)}v_1 = (0.600 \pm 1.95)\,\text{m/s}.$$

The corresponding solutions for v_3 are then found from the momentum expression to be $v_3 = 0.981\,\text{m/s}$ and $v_3 = 0.219$. Since it is unlikely that the ball passed through the block we can toss out the second set of answers.

E13-16 $E_\text{f} = K_\text{f} + U_\text{f} = 3mgh$, or

$$v_\text{f} = \sqrt{2(9.81\,\text{m/s}^2)2(0.18\,\text{m})} = 2.66\,\text{m/s}.$$

E13-17 We can find the kinetic energy of the center of mass of the woman when her feet leave the ground by considering energy conservation and her highest point. Then

$$\begin{aligned}\frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} &= \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f},\\ \frac{1}{2}mv_\mathrm{i} &= mg\Delta y,\\ &= (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.90\,\mathrm{m}) = 162\,\mathrm{J}.\end{aligned}$$

(a) During the jumping phase her potential energy changed by

$$\Delta U = mg\Delta y = (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.50\,\mathrm{m}) = 270\,\mathrm{J}$$

while she was moving up. Then

$$F_\mathrm{ext} = \frac{\Delta K + \Delta U}{\Delta s} = \frac{(162\,\mathrm{J}) + (270\,\mathrm{J})}{(0.5\,\mathrm{m})} = 864\,\mathrm{N}.$$

(b) Her fastest speed was when her feet left the ground,

$$v = \frac{2K}{m} = \frac{2(162\,\mathrm{J})}{(55.0\,\mathrm{kg})} = 2.42\,\mathrm{m/s}.$$

E13-18 (b) The ice skater needs to lose $\frac{1}{2}(116\,\mathrm{kg})(3.24\,\mathrm{m/s})^2 = 609\,\mathrm{J}$ of internal energy.
(a) The average force exerted on the rail is $F = (609\,\mathrm{J})/(0.340\,\mathrm{m}) = 1790\,\mathrm{N}$.

E13-19 12.6 km/h is equal to 3.50 m/s; the initial kinetic energy of the car is

$$\frac{1}{2}(2340\,\mathrm{kg})(3.50\,\mathrm{m/s})^2 = 1.43\times 10^4\mathrm{J}.$$

(a) The force exerted on the car is $F = (1.43\times 10^4\mathrm{J})/(0.64\,\mathrm{m}) = 2.24\times 10^4\mathrm{N}$.
(b) The change increase in internal energy of the car is

$$\Delta E_\mathrm{int} = (2.24\times 10^4\mathrm{N})(0.640\,\mathrm{m} - 0.083\,\mathrm{m}) = 1.25\times 10^4\mathrm{J}.$$

E13-20 Note that $v_n^2 = {v'_n}^2 - 2\vec{\mathbf{v}}'_n \cdot \vec{\mathbf{v}}_\mathrm{cm} + {v_\mathrm{cm}}^2$. Then

$$\begin{aligned}K &= \sum_n \frac{1}{2}\left(m_n {v'_n}^2 - 2m_n\vec{\mathbf{v}}'_n \cdot \vec{\mathbf{v}}_\mathrm{cm} + m_n {v_\mathrm{cm}}^2\right),\\ &= \sum_n \frac{1}{2}m_n {v'_n}^2 - \left(\sum_n m_n\vec{\mathbf{v}}'_n\right)\cdot \vec{\mathbf{v}}_\mathrm{cm} + \left(\sum_n \frac{1}{2}m_n\right){v_\mathrm{cm}}^2,\\ &= K_\mathrm{int} - \left(\sum_n m_n\vec{\mathbf{v}}'_n\right)\cdot \vec{\mathbf{v}}_\mathrm{cm} + K_\mathrm{cm}.\end{aligned}$$

The middle term vanishes because of the definition of velocities relative to the center of mass.

E13-21 Momentum conservation requires $mv_0 = mv + MV$, where the sign indicates the direction. We are assuming one dimensional collisions. Energy conservation requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 + E.$$

Combining,

$$\begin{aligned}\frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v_0 - \frac{m}{M}v\right)^2 + E,\\ Mv_0^2 &= Mv^2 + m\left(v_0 - v\right)^2 + 2(M/m)E.\end{aligned}$$

Arrange this as a quadratic in v,

$$(M+m)\,v^2 - (2mv_0)\,v + \left(2(M/m)E + mv_0^2 - Mv_0^2\right) = 0.$$

This will only have real solutions if the discriminant ($b^2 - 4ac$) is greater than or equal to zero. Then

$$(2mv_0)^2 \geq 4\,(M+m)\left(2(M/m)E + mv_0^2 - Mv_0^2\right)$$

is the condition for the minimum v_0. Solving the equality condition,

$$4m^2v_0^2 = 4(M+m)\left(2(M/m)E + (m-M)v_0^2\right),$$

or $M^2v_0^2 = 2(M+m)(M/m)E$. One last rearrangement, and $v_0 = \sqrt{2(M+m)E/(mM)}$.

P13-1 (a) The initial kinetic energy will equal the potential energy at the highest point *plus* the amount of energy which is dissipated because of air drag.

$$\begin{aligned}mgh + fh &= \frac{1}{2}mv_0^2,\\ h &= \frac{v_0^2}{2(g+f/m)} = \frac{v_0^2}{2g(1+f/w)}.\end{aligned}$$

(b) The final kinetic energy when the stone lands will be equal to the initial kinetic energy *minus* twice the energy dissipated on the way up, so

$$\begin{aligned}\frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - 2fh,\\ &= \frac{1}{2}mv_0^2 - 2f\frac{v_0^2}{2g(1+f/w)},\\ &= \left(\frac{m}{2} - \frac{f}{g(1+f/w)}\right)v_0^2,\\ v^2 &= \left(1 - \frac{2f}{w+f}\right)v_0^2,\\ v &= \left(\frac{w-f}{w+f}\right)^{1/2} v_0.\end{aligned}$$

P13-2 The object starts with $U = (0.234\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.05\,\mathrm{m}) = 2.41\,\mathrm{J}$. It will move back and forth across the flat portion $(2.41\,\mathrm{J})/(0.688\,\mathrm{J}) = 3.50$ times, which means it will come to a rest at the center of the flat part while attempting one last right to left journey.

P13-3 (a) The work done on the block block because of friction is

$$(0.210)(2.41\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.83\,\mathrm{m}) = 9.09\,\mathrm{J}.$$

The energy dissipated because of friction is $(9.09\,\mathrm{J})/0.83 = 11.0\,\mathrm{J}$.

The speed of the block just after the bullet comes to a rest is

$$v = \sqrt{2K/m} = \sqrt{2(1.10\,\mathrm{J})/(2.41\,\mathrm{kg})} = 3.02\,\mathrm{m/s}.$$

(b) The initial speed of the bullet is

$$v_0 = \frac{M+m}{m}v = \frac{(2.41\,\mathrm{kg}) + (0.00454\,\mathrm{kg})}{(0.00454\,\mathrm{kg})}(3.02\,\mathrm{m/s}) = 1.60\times 10^3\,\mathrm{m/s}.$$

P13-4 The energy stored in the spring after compression is $\frac{1}{2}(193\,\mathrm{N/m})(0.0416\,\mathrm{m})^2 = 0.167\,\mathrm{J}$. Since 117 mJ was dissipated by friction, the kinetic energy of the block before colliding with the spring was $0.284\,\mathrm{J}$. The speed of the block was then

$$v = \sqrt{2(0.284\,\mathrm{J})/(1.34\,\mathrm{kg})} = 0.651\,\mathrm{m/s}.$$

P13-5 (a) Using Newton's second law, $F = ma$, so for circular motion around the proton

$$\frac{mv^2}{r} = F = k\frac{e^2}{r^2}.$$

The kinetic energy of the electron in an orbit is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}k\frac{e^2}{r}.$$

The change in kinetic energy is

$$\Delta K = \frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(b) The potential energy difference is

$$\Delta U = -\int_{r_1}^{r_2}\frac{ke^2}{r^2}dr = -ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(c) The total energy change is

$$\Delta E = \Delta K + \Delta U = -\frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

P13-6 (a) The initial energy of the system is $(4000\text{ lb})(12\text{f}t) = 48,000\text{ ft}\cdot\text{lb}$. The safety device removes $(1000\text{ lb})(12\text{f}t) = 12,000\text{ ft}\cdot\text{lb}$ before the elevator hits the spring, so the elevator has a kinetic energy of $36,000\text{ ft}\cdot\text{lb}$ when it hits the spring. The speed of the elevator when it hits the spring is

$$v = \sqrt{\frac{2(36,000\text{ ft}\cdot\text{lb})(32.0\text{ ft/s}^2)}{(4000\text{ lb})}} = 24.0\text{ ft/s}.$$

(b) Assuming the safety clamp remains in effect while the elevator is in contact with the spring then the distance compressed will be found from

$$36,000\text{ ft}\cdot\text{lb} = \frac{1}{2}(10,000\text{ lb/ft})y^2 - (4000\text{ lb})y + (1000\text{ lb})y.$$

This is a quadratic expression in y which can be simplified to look like

$$5y^2 - 3y - 36 = 0,$$

which has solutions $y = (0.3 \pm 2.7)$ ft. Only $y = 3.00$ ft makes sense here.

(c) The distance through which the elevator will bounce back up is found from

$$33,000 \text{ ft} = (4000 \text{ lb})y - (1000 \text{ lb})y,$$

where y is measured from the most compressed point of the spring. Then $y = 11$ ft, or the elevator bounces back up 8 feet.

(d) The elevator will bounce until it has traveled a total distance so that the safety device dissipates all of the original energy, or 48 ft.

P13-7 The net force on the top block while it is being pulled is

$$11.0\,\text{N} - F_f = 11.0\,\text{N} - (0.35)(2.5\,\text{kg})(9.81\,\text{m/s}^2) = 2.42\,\text{N}.$$

This means it is accelerating at $(2.42\,\text{N})/(2.5\,\text{kg}) = 0.968\,\text{m/s}^2$. That acceleration will last a time $t = \sqrt{2(0.30\,\text{m})/(0.968\,\text{m/s}^2)} = 0.787\,\text{s}$. The speed of the top block after the force stops pulling is then $(0.968\,\text{m/s}^2)(0.787\,\text{s}) = 0.762\,\text{m/s}$. The force on the bottom block is F_f, so the acceleration of the bottom block is

$$(0.35)(2.5\,\text{kg})(9.81\,\text{m/s}^2)/(10.0\,\text{kg}) = 0.858\,\text{m/s}^2,$$

and the speed after the force stops pulling on the top block is $(0.858\,\text{m/s}^2)(0.787\,\text{s}) = 0.675\,\text{m/s}$.

(a) $W = Fs = (11.0\,\text{N})(0.30\,\text{m}) = 3.3\,\text{J}$ of energy were delivered to the system, but after the force stops pulling only

$$\frac{1}{2}(2.5\,\text{kg})(0.762\,\text{m/s})^2 + \frac{1}{2}(10.0\,\text{kg})(0.675\,\text{m/s})^2 = 3.004\,\text{J}$$

were present as kinetic energy. So 0.296 J is "missing" and would be now present as internal energy.

(b) The impulse received by the two block system is then $J = (11.0\,\text{N})(0.787\,\text{s}) = 8.66\,\text{N}\cdot\text{s}$. This impulse causes a change in momentum, so the speed of the two block system after the external force stops pulling and both blocks move as one is $(8.66\,\text{N}\cdot\text{s})(12.5\,\text{kg}) = 0.693\,\text{m/s}$. The final kinetic energy is

$$\frac{1}{2}(12.5\,\text{kg})(0.693\,\text{m/s})^2 = 3.002\,\text{J};$$

this means that 0.002 J are dissipated.

P13-8 Hmm.