E12-1 (a) Integrate.

$$U(x) = -\int_{\infty}^{x} G\frac{m_1 m_2}{x^2}\,dx = -G\frac{m_1 m_2}{x}.$$

(b) $W = U(x) - U(x+d)$, so

$$W = Gm_1m_2\left(\frac{1}{x} - \frac{1}{x+d}\right) = Gm_1m_2\frac{d}{x(x+d)}.$$

E12-2 If $d << x$ then $x(x+d) \approx x^2$, so

$$W \approx G\frac{m_1m_2}{x^2}d.$$

E12-3 Start with Eq. 12-6.

$$\begin{aligned} U(x) - U(x_0) &= -\int_{x_0}^{x} F_x(x)dx, \\ &= -\int_{x_0}^{x} \left(-\alpha x e^{-\beta x^2}\right) dx, \\ &= \left.\frac{-\alpha}{2\beta}e^{-\beta x^2}\right|_{x_0}^{x}. \end{aligned}$$

Finishing the integration,

$$U(x) = U(x_0) + \frac{\alpha}{2\beta}\left(e^{-\beta x_0^2} - e^{-\beta x^2}\right).$$

If we *choose* $x_0 = \infty$ *and* $U(x_0) = 0$ we would be left with

$$U(x) = -\frac{\alpha}{2\beta}e^{-\beta x^2}.$$

E12-4 $\Delta K = -\Delta U$ so $\Delta K = mg\Delta y$. The power output is then

$$P = (58\%)\frac{(1000\,\text{kg/m}^3)(73,800\,\text{m}^3)}{(60\,\text{s})}(9.81\,\text{m/s}^2)(96.3\,\text{m}) = 6.74\times10^8\text{W}.$$

E12-5 $\Delta U = -\Delta K$, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Then

$$k = \frac{mv^2}{x^2} = \frac{(2.38\,\text{kg})(10.0\times10^3\,/\text{s})}{(1.47\,\text{m})^2} = 1.10\times10^8\text{N/m}.$$

Wow.

E12-6 $\Delta U_\text{g} + \Delta U_\text{s} = 0$, since $K = 0$ when the man jumps and when the man stops. Then $\Delta U_\text{s} = -mg\Delta y = (220\text{ lb})(40.4\text{ ft}) = 8900\text{ ft}\cdot\text{lb}$.

E12-7 Apply Eq. 12-15,

$$\begin{aligned} K_\text{f} + U_\text{f} &= K_\text{i} + U_\text{i}, \\ \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i}, \\ \frac{1}{2}{v_\text{f}}^2 + g(-r) &= \frac{1}{2}(0)^2 + g(0). \end{aligned}$$

Rearranging,

$$v_\text{f} = \sqrt{-2g(-r)} = \sqrt{-2(9.81\,\text{m/s}^2)(-0.236\,\text{m})} = 2.15\,\text{m/s}.$$

E12-8 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.40\,\text{kg})(150\,\text{m/s})^2 = 2.70\times10^4\text{J}$.

(b) Assuming that the ground is zero, $U = mgy = (2.40\,\text{kg})(9.81\,\text{m/s}^2)(125\,\text{m}) = 2.94\times10^3\text{J}$.

(c) $K_\text{f} = K_\text{i} + U_\text{i}$ since $U_\text{f} = 0$. Then

$$v_\text{f} = \sqrt{2\frac{(2.70\times10^4\text{J}) + (2.94\times10^3\text{J})}{(2.40\,\text{kg})}} = 158\,\text{m/s}.$$

Only (a) and (b) depend on the mass.

E12-9 (a) Since $\Delta y = 0$, then $\Delta U = 0$ and $\Delta K = 0$. Consequently, at B, $v = v_0$.

(b) At C $K_C = K_A + U_A - U_C$, or

$$\frac{1}{2}m{v_C}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg\frac{h}{2},$$

or

$$v_B = \sqrt{{v_0}^2 + 2g\frac{h}{2}} = \sqrt{{v_0}^2 + gh}.$$

(c) At D $K_D = K_A + U_A - U_D$, or

$$\frac{1}{2}m{v_D}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg(0),$$

or

$$v_B = \sqrt{{v_0}^2 + 2gh}.$$

E12-10 From the slope of the graph, $k = (0.4\,\text{N})/(0.04\,\text{m}) = 10\,\text{N/m}$.

(a) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\text{f}}^2 = \frac{1}{2}k{x_\text{i}}^2$, or

$$v_\text{f} = \sqrt{\frac{(10\,\text{N/m})}{(0.00380\,\text{kg})}}(0.0550\,\text{m}) = 2.82\,\text{m/s}.$$

(b) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\text{f}}^2 = \frac{1}{2}k({x_\text{i}}^2 - {x_\text{f}}^2)$, or

$$v_\text{f} = \sqrt{\frac{(10\,\text{N/m})}{(0.00380\,\text{kg})}\left[(0.0550\,\text{m})^2 - (0.0150\,\text{m})^2\right]} = 2.71\,\text{m/s}.$$

E12-11 (a) The force constant of the spring is

$$k = F/x = mg/x = (7.94\,\text{kg})(9.81\,\text{m/s}^2)/(0.102\,\text{m}) = 764\,\text{N/m}.$$

(b) The potential energy stored in the spring is given by Eq. 12-8,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(764\,\text{N/m})(0.286\,\text{m} + 0.102\,\text{m})^2 = 57.5\,\text{J}.$$

(c) Conservation of energy,

$$\begin{aligned} K_\text{f} + U_\text{f} &= K_\text{i} + U_\text{i}, \\ \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} + \frac{1}{2}k{x_\text{f}}^2 &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i} + \frac{1}{2}k{x_\text{i}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\text{i}}^2. \end{aligned}$$

Rearranging,

$$h = \frac{k}{2mg}{x_\text{i}}^2 = \frac{(764\,\text{N/m})}{2(7.94\,\text{kg})(9.81\,\text{m/s}^2)}(0.388\,\text{m})^2 = 0.738\,\text{m}.$$

E12-12 The annual mass of water is $m = (1000\,\text{kg/m}^3)(8\times10^{12}\text{m}^2)(0.75\,\text{m})$. The change in potential energy each year is then $\Delta U = -mgy$, where $y = -500\,\text{m}$. The power available is then

$$P = \frac{1}{3}(1000\,\text{kg/m}^3)(8\times10^{12}\text{m}^2)\frac{(0.75\,\text{m})}{(3.15\times10^7\text{m})}(500\,\text{m}) = 3.2\times10^7\text{W}.$$

E12-13 (a) From kinematics, $v = -gt$, so $K = \frac{1}{2}mg^2t^2$ and $U = U_0 - K = mgh - \frac{1}{2}mg^2t^2$.
(b) $U = mgy$ so $K = U_0 - U = mg(h-y)$.

E12-14 The potential energy is the same in both cases. Consequently, $mg_{\text{E}}\Delta y_{\text{E}} = mg_{\text{M}}\Delta y_{\text{M}}$, and then

$$y_{\text{M}} = (2.05\,\text{m} - 1.10\,\text{m})(9.81\,\text{m/s}^2)/(1.67\,\text{m/s}^2) + 1.10\,\text{m} = 6.68\,\text{m}.$$

E12-15 The working is identical to Ex. 12-11,

$$\begin{aligned} K_{\text{f}} + U_{\text{f}} &= K_{\text{i}} + U_{\text{i}}, \\ \frac{1}{2}m{v_{\text{f}}}^2 + mgy_{\text{f}} + \frac{1}{2}k{x_{\text{f}}}^2 &= \frac{1}{2}m{v_{\text{i}}}^2 + mgy_{\text{i}} + \frac{1}{2}k{x_{\text{i}}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_{\text{i}}}^2, \end{aligned}$$

so

$$h = \frac{k}{2mg}{x_{\text{i}}}^2 = \frac{(2080\,\text{N/m})}{2(1.93\,\text{kg})(9.81\,\text{m/s}^2)}(0.187\,\text{m})^2 = 1.92\,\text{m}.$$

The distance up the incline is given by a trig relation,

$$d = h/\sin\theta = (1.92\,\text{m})/\sin(27^\circ) = 4.23\,\text{m}.$$

E12-16 The vertical position of the pendulum is $y = -l\cos\theta$, where θ is measured from the downward vertical and l is the length of the string. The total mechanical energy of the pendulum is

$$E = \frac{1}{2}m{v_{\text{b}}}^2$$

if we set $U = 0$ at the bottom of the path and v_{b} is the speed at the bottom. In this case $U = mg(l+y)$.
(a) $K = E - U = \frac{1}{2}m{v_{\text{b}}}^2 - mgl(1-\cos\theta)$. Then

$$v = \sqrt{(8.12\,\text{m/s})^2 - 2(9.81\,\text{m/s}^2)(3.82\,\text{kg})(1-\cos 58.0^\circ)} = 5.54\,\text{m/s}.$$

(b) $U = E - K$, but at highest point $K = 0$. Then

$$\theta = \arccos\left(1 - \frac{1}{2}\frac{(8.12\,\text{m/s})^2}{(9.81\,\text{m/s}^2)(3.82\,\text{kg})}\right) = 83.1^\circ.$$

(c) $E = \frac{1}{2}(1.33\,\text{kg})(8.12\,\text{m/s})^2 = 43.8\,\text{J}$.

E12-17 The equilibrium position is when $F = ky = mg$. Then $\Delta U_{\text{g}} = -mgy$ and $\Delta U_{\text{s}} = \frac{1}{2}(ky)y = \frac{1}{2}mgy$. So $2\Delta U_{\text{s}} = -\Delta U_{\text{g}}$.

E12-18 Let the spring get compressed a distance x. If the object fell from a height $h = 0.436\,\text{m}$, then conservation of energy gives $\frac{1}{2}kx^2 = mg(x+h)$. Solving for x,

$$x = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}h}$$

only the positive answer is of interest, so

$$x = \frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})} \pm \sqrt{\left(\frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})}\right)^2 + 2\frac{(2.14\,\text{kg})(9.81\,\text{m/s}^2)}{(1860\,\text{N/m})}(0.436\,\text{m})} = 0.111\,\text{m}.$$

E12-19 The horizontal distance traveled by the marble is $R = vt_\text{f}$, where t_f is the time of flight and v is the speed of the marble when it leaves the gun. We find *that* speed using energy conservation principles applied to the spring just before it is released and just after the marble leaves the gun.

$$\begin{aligned} K_\text{i} + U_\text{i} &= K_\text{f} + U_\text{f}, \\ 0 + \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 + 0. \end{aligned}$$

$K_\text{i} = 0$ because the marble isn't moving originally, and $U_\text{f} = 0$ because the spring is no longer compressed. Substituting R into this,

$$\frac{1}{2}kx^2 = \frac{1}{2}m\left(\frac{R}{t_\text{f}}\right)^2.$$

We have two values for the compression, x_1 and x_2, and two ranges, R_1 and R_2. We can put both pairs into the above equation and get two expressions; if we divide one expression by the other we get

$$\left(\frac{x_2}{x_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^2.$$

We can easily take the square root of both sides, then

$$\frac{x_2}{x_1} = \frac{R_2}{R_1}.$$

R_1 was Bobby's try, and was equal to $2.20 - 0.27 = 1.93\,\text{m}$. $x_1 = 1.1$ cm was his compression. If Rhoda wants to score, she wants $R_2 = 2.2\,\text{m}$, then

$$x_2 = \frac{2.2\,\text{m}}{1.93\,\text{m}} 1.1 \text{ cm} = 1.25 \text{ cm}.$$

E12-20 Conservation of energy— $U_1 + K_1 = U_2 + K_2$— but $U_1 = mgh$, $K_1 = 0$, and $U_2 = 0$, so $K_2 = \frac{1}{2}mv^2 = mgh$ at the bottom of the swing.

The net force on Tarzan at the bottom of the swing is $F = mv^2/r$, but this net force is equal to the tension T minus the weight $W = mg$. Then $2mgh/r = T - mg$. Rearranging,

$$T = (180 \text{ lb})\left(\frac{2(8.5 \text{ ft})}{(50 \text{ ft})} + 1\right) = 241 \text{ lb}.$$

This isn't enough to break the vine, but it is close.

E12-21 Let point 1 be the start position of the first mass, point 2 be the collision point, and point 3 be the highest point in the swing after the collision. Then $U_1 = K_2$, or $\frac{1}{2}m_1{v_1}^2 = m_1gd$, where v_1 is the speed of m_1 just before it collides with m_2. Then $v_1 = \sqrt{2gd}$.

After the collision the speed of both objects is, by momentum conservation, $v_2 = m_1v_1/(m_1+m_2)$.

Then, by energy conservation, $U_3 = K_2'$, or $\frac{1}{2}(m_1+m_2){v_2}^2 = (m_1+m_2)gy$, where y is the height to which the combined masses rise.

Combining,

$$y = \frac{{v_2}^2}{2g} = \frac{{m_1}^2{v_1}^2}{2(m_1+m_2)^2g} = \left(\frac{m_1}{m_1+m_2}\right)^2 d.$$

E12-22 $\Delta K = -\Delta U$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh,$$

where $I = \frac{1}{2}MR^2$ and $\omega = v/R$. Combining,

$$\frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = mgh,$$

so

$$v = \sqrt{\frac{4mgh}{2m+M}} = \sqrt{\frac{4(0.0487\,\text{kg})(9.81\,\text{m/s}^2)(0.540\,\text{m})}{2(0.0487\,\text{kg}) + (0.396\,\text{kg})}} = 1.45\,\text{m/s}.$$

E12-23 There are *three* contributions to the kinetic energy: rotational kinetic energy of the shell (K_s), rotational kinetic energy of the pulley (K_p), and translational kinetic energy of the block (K_b). The conservation of energy statement is then

$$\begin{aligned} K_\text{s,i} + K_\text{p,i} + K_\text{b,i} + U_\text{i} &= K_\text{s,f} + K_\text{p,f} + K_\text{b,f} + U_\text{f}, \\ (0) + (0) + (0) + (0) &= \frac{1}{2}I_\text{s}{\omega_\text{s}}^2 + \frac{1}{2}I_\text{p}{\omega_\text{p}}^2 + \frac{1}{2}m{v_\text{b}}^2 + mgy. \end{aligned}$$

Finally, $y = -h$ and

$$\omega_\text{s}R = \omega_\text{p}r = v_\text{b}.$$

Combine all of this together, and our energy conservation statement will look like this:

$$0 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v_\text{b}}{R}\right)^2 + \frac{1}{2}I_\text{p}\left(\frac{v_\text{b}}{r}\right)^2 + \frac{1}{2}m{v_\text{b}}^2 - mgh$$

which can be fairly easily rearranged into

$${v_\text{b}}^2 = \frac{2mgh}{2M/3 + I_\text{P}/r^2 + m}.$$

E12-24 The angular speed of the flywheel and the speed of the car are related by

$$k = \frac{\omega}{v} = \frac{(1490\text{ rad/s})}{(24.0\,\text{m/s})} = 62.1\text{ rad/m}.$$

The height of the slope is $h = (1500\,\text{m})\sin(5.00°) = 131\,\text{m}$. The rotational inertia of the flywheel is

$$I = \frac{1}{2}\frac{(194\,\text{N})}{(9.81\,\text{m/s}^2)}(0.54\,\text{m})^2 = 2.88\,\text{kg}\cdot\text{m}^2.$$

(a) Energy is conserved as the car moves down the slope: $U_\text{i} = K_\text{f}$, or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}Ik^2v^2,$$

or

$$v = \sqrt{\frac{2mgh}{m + Ik^2}} = \sqrt{\frac{2(822\,\text{kg})(9.81\,\text{m/s}^2)(131\,\text{m})}{(822\,\text{kg}) + (2.88\,\text{kg}\cdot\text{m}^2)(62.1\text{ rad/m})^2}} = 13.3\,\text{m/s},$$

or 47.9 m/s.

(b) The average speed down the slope is $13.3\,\text{m/s}/2 = 6.65\,\text{m/s}$. The time to get to the bottom is $t = (1500\,\text{m})/(6.65\,\text{m/s}) = 226\,\text{s}$. The angular acceleration of the disk is

$$\alpha = \frac{\omega}{t} = \frac{(13.3\,\text{m/s})(62.1\text{ rad/m})}{(226\,\text{s})} = 3.65\text{ rad/s}^2.$$

(c) $P = \tau\omega = I\alpha\omega$, so

$$P = (2.88\,\text{kg}\cdot\text{m}^2)(3.65\text{ rad/s}^2)(13.3\,\text{m/s})(62.1\text{ rad/m}) = 8680\,\text{W}.$$

E12-25 (a) For the solid sphere $I = \frac{2}{5}mr^2$; if it rolls without slipping $\omega = v/r$; conservation of energy means $K_\text{i} = U_\text{f}$. Then

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = mgh.$$

or

$$h = \frac{(5.18\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + \frac{(5.18\,\text{m/s})^2}{5(9.81\,\text{m/s}^2)} = 1.91\,\text{m}.$$

The distance up the incline is $(1.91\,\text{m})/\sin(34.0^\circ) = 3.42\,\text{m}$.

(b) The sphere will travel a distance of 3.42 m with an average speed of $5.18\,\text{m}/2$, so $t = (3.42\,\text{m})/(2.59\,\text{m/s}) = 1.32\,\text{s}$. But wait, it goes up then comes back down, so double this time to get 2.64 s.

(c) The total distance is $6.84\,\text{m}$, so the number of rotations is $(6.84\,\text{m})/(0.0472\,\text{m})/(2\pi) = 23.1$.

E12-26 Conservation of energy means $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$. But $\omega = v/r$ and we are told $h = 3v^2/4g$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mg\frac{3v^2}{4g},$$

or

$$I = 2r^2\left(\frac{3}{4}m - \frac{1}{2}m\right) = \frac{1}{2}mr^2,$$

which could be a solid disk or cylinder.

E12-27 We assume the cannon ball is solid, so the rotational inertia will be $I = (2/5)MR^2$

The normal force on the cannon ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the cannon ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the cannon ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

The time requires for the cannon ball to stop skidding is the time required to change the angular momentum to l, so

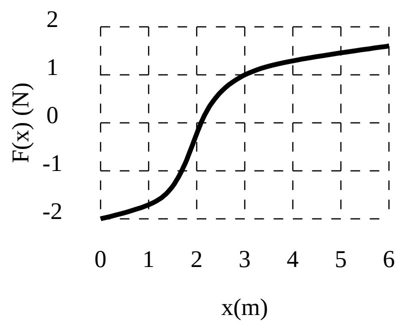
$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

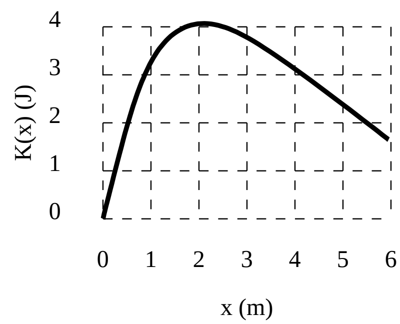
Since we don't know Δt, we can't solve this for v. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k M g} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned} \frac{2v}{5\mu_k g} &= \frac{v_0 - v}{\mu_k g}, \\ 2v &= 5(v_0 - v), \\ v &= 5v_0/7 \end{aligned}$$





E12-28

E12-29 (a) $F = -\Delta U/\Delta x = -[(-17\,\text{J}) - (-3\,\text{J})]/[(4\,\text{m}) - (1\,\text{m})] = 4.7\,\text{N}$.

(b) The total energy is $\frac{1}{2}(2.0\,\text{kg})(-2.0\,\text{m/s})^2 + (-7\,\text{J})$, or $-3\,\text{J}$. The particle is constrained to move between $x = 1\,\text{m}$ and $x = 14\,\text{m}$.

(c) When $x = 7\,\text{m}$ $K = (-3\,\text{J}) - (-17\,\text{J}) = 14\,\text{J}$. The speed is $v = \sqrt{2(14\,\text{J})/(2.0\,\text{kg})} = 3.7\,\text{m/s}$.

E12-30 Energy is conserved, so

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgy,$$

or

$$v = \sqrt{v_0^2 - 2gy},$$

which depends only on y.

E12-31 (a) We can find F_x and F_y from the appropriate derivatives of the potential,

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} = -kx, \\ F_y &= -\frac{\partial U}{\partial y} = -ky. \end{aligned}$$

The force at point (x, y) is then

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = -kx\hat{\mathbf{i}} - ky\hat{\mathbf{j}}.$$

(b) Since the force vector points directly toward the origin there is *no* angular component, and $F_\theta = 0$. Then $F_r = -kr$ where r is the distance from the origin.

(c) A spring which is attached to a point; the spring is free to rotate, perhaps?

E12-32 (a) By symmetry we expect F_x, F_y, and F_z to all have the same form.

$$F_x = -\frac{\partial U}{\partial x} = \frac{-kx}{(x^2+y^2+z^2)^{3/2}},$$

with similar expressions for F_y and F_z. Then

$$\vec{\mathbf{F}} = \frac{-k}{(x^2+y^2+z^2)^{3/2}}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}).$$

(b) In spherical polar coordinates $r^2 = x^2+y^2+z^2$. Then $U = -k/r$ and

$$F_r = -\frac{\partial U}{\partial r} = -\frac{k}{r^2}.$$

E12-33 We'll just do the paths, showing only non-zero terms.
Path 1: $W = \int_0^b(-k_2 a)\,dy) = -k_2ab$.
Path 2: $W = \int_0^a(-k_1 b)\,dx) = -k_1ab$.
Path 3: $W = (\cos\phi\,\sin\phi)\int_0^d(-k_1-k_2)r\,dr = -(k_1+k_2)ab/2$.
These three are only equal if $k_1 = k_2$.

P12-1 (a) We need to integrate an expression like

$$-\int_\infty^z \frac{k}{(z+l)^2}\,dz = \frac{k}{z+l}.$$

The second half is dealt with in a similar manner, yielding

$$U(z) = \frac{k}{z+l} - \frac{k}{z-l}.$$

(b) If $z \gg l$ then we can expand the denominators, then

$$\begin{aligned} U(z) &= \frac{k}{z+l} - \frac{k}{z-l}, \\ &\approx \left(\frac{k}{z} - \frac{kl}{z^2}\right) - \left(\frac{k}{z} + \frac{kl}{z^2}\right), \\ &= -\frac{2kl}{z^2}. \end{aligned}$$

P12-2 The ball just reaches the top, so $K_2 = 0$. Then $K_1 = U_2 - U_1 = mgL$, so $v_1 = \sqrt{2(mgL)/m} = \sqrt{2gL}$.

P12-3 Measure distances along the incline by x, where $x = 0$ is measured from the maximally compressed spring. The vertical position of the mass is given by $x\sin\theta$. For the spring $k = (268\,\text{N})/(0.0233\,\text{m}) = 1.15\times10^4\,\text{N/m}$. The total energy of the system is

$$\frac{1}{2}(1.15\times10^4\,\text{N/m})(0.0548\,\text{m})^2 = 17.3\,\text{J}.$$

(a) The block needs to have moved a vertical distance $x\sin(32.0^\circ)$, where

$$17.3\,\text{J} = (3.18\,\text{kg})(9.81\,\text{m/s}^2)x\sin(32.0^\circ),$$

or $x = 1.05\,\text{m}$.

(b) When the block hits the top of the spring the gravitational potential energy has changed by

$$\Delta U = -(3.18\,\text{kg})(9.81\,\text{m/s}^2)(1.05\,\text{m} - 0.0548\,\text{m})\sin(32.0^\circ) = 16.5,\text{J};$$

hence the speed is $v = \sqrt{2(16.5\,\text{J})/(3.18\,\text{kg})} = 3.22\,\text{m/s}$.

P12-4 The potential energy associated with the hanging part is

$$U = \int_{-L/4}^{0} \frac{M}{L} gy\, dy = \frac{Mg}{2L} y^2 \bigg|_{-L/4}^{0} = -\frac{MgL}{32},$$

so the work required is $W = MgL/32$.

P12-5 (a) Considering points P and Q we have

$$\begin{aligned} K_P + U_P &= K_Q + U_Q, \\ (0) + mg(5R) &= \frac{1}{2} mv^2 + mg(R), \\ 4mgR &= \frac{1}{2} mv^2, \\ \sqrt{8gR} &= v. \end{aligned}$$

There are two forces on the block, the normal force from the track,

$$N = \frac{mv^2}{R} = \frac{m(8gR)}{R} = 8mg,$$

and the force of gravity $W = mg$. They are orthogonal so

$$F_{\text{net}} = \sqrt{(8mg)^2 + (mg)^2} = \sqrt{65}\, mg$$

and the angle from the horizontal by

$$\tan\theta = \frac{-mg}{8mg} = -\frac{1}{8},$$

or $\theta = 7.13°$ below the horizontal.

(b) If the block *barely* makes it over the top of the track then the speed at the top of the loop (point S, perhaps?) is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$m{v_{\text{S}}}^2/R = mg.$$

Assume the block was released from point T. The energy conservation problem is then

$$\begin{aligned} K_T + U_T &= K_S + U_S, \\ (0) + mgy_T &= \frac{1}{2} mv_S^2 + mgy_S, \\ y_T &= \frac{1}{2}(R) + m(2R), \\ &= 5R/2. \end{aligned}$$

P12-6 The wedge slides to the left, the block to the right. Conservation of momentum requires $Mv_{\text{w}} + mv_{\text{b},x} = 0$. The block is constrained to move on the surface of the wedge, so

$$\tan\alpha = \frac{v_{\text{b},y}}{v_{\text{b},x} - v_{\text{w}}},$$

or

$$v_{\text{b},y} = v_{\text{b},x} \tan\alpha (1 + m/M).$$

Conservation of energy requires

$$\frac{1}{2}m{v_\mathrm{b}}^2 + \frac{1}{2}M{v_\mathrm{w}}^2 = mgh.$$

Combining,

$$\begin{aligned}
\frac{1}{2}m\left({v_{\mathrm{b},x}}^2 + {v_{\mathrm{b},y}}^2\right) + \frac{1}{2}M\left(\frac{m}{M}v_{\mathrm{b},x}\right)^2 &= mgh, \\
\left(\tan^2\alpha(1+m/M)^2 + 1 + \frac{m}{M}\right){v_{\mathrm{b},x}}^2 &= 2gh, \\
\left(\sin^2\alpha(M+m)^2 + M^2\cos^2\alpha + mM\cos^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha, \\
\left(M^2 + mM + mM\sin^2\alpha + m^2\sin^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha, \\
\left((M+m)(M+m\sin^2\alpha)\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,
\end{aligned}$$

or

$$v_{\mathrm{b},x} = M\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

Then

$$v_\mathrm{w} = -m\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

P12-7 $U(x) = -\int F_x\,dx = -Ax^2/2 - Bx^3/3$.

(a) $U = -(-3.00\,\mathrm{N/m})(2.26\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(2.26\,\mathrm{m})^3/3 = 26.9\,\mathrm{J}$.

(b) There are two points to consider:

$$\begin{aligned}
U_1 &= -(-3.00\,\mathrm{N/m})(4.91\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(4.91\,\mathrm{m})^3/3 = 233\,\mathrm{J}, \\
U_2 &= -(-3.00\,\mathrm{N/m})(1.77\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(1.77\,\mathrm{m})^3/3 = 13.9\,\mathrm{J}, \\
K_1 &= \frac{1}{2}(1.18\mathrm{kg})(4.13\,\mathrm{m/s})^2 = 10.1\,\mathrm{J}.
\end{aligned}$$

Then

$$v_2 = \sqrt{\frac{2(10.1\,\mathrm{J} + 233\,\mathrm{J} - 13.9\,\mathrm{J})}{(1.18\,\mathrm{kg})}} = 19.7\,\mathrm{m/s}.$$

P12-8 Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$.

(a) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m})} = 4.85\,\mathrm{m/s}$.

(b) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.45\,\mathrm{m} - 0.45\,\mathrm{m})} = 2.43\,\mathrm{m/s}$.

P12-9 Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$. If the ball *barely* swings around the top of the peg then the speed at the top of the loop is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$mv^2/R = mg.$$

The energy conservation problem is then

$$\begin{aligned}
mv^2 &= 2mg(L - 2(L-d)) = 2mg(2d - L) \\
mg(L-d) &= 2mg(2d - L), \\
d &= 3L/5.
\end{aligned}$$

P12-10 The speed at the top and the speed at the bottom are related by

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR.$$

The magnitude of the net force is $F = mv^2/R$, the tension at the top is

$$T_\mathrm{t} = m{v_\mathrm{t}}^2/R - mg,$$

while tension at the bottom is

$$T_\mathrm{b} = m{v_\mathrm{b}}^2/R + mg,$$

The difference is

$$\Delta T = 2mg + m({v_\mathrm{b}}^2 - {v_\mathrm{t}}^2)/R = 2mg + 4mg = 6mg.$$

P12-11 Let the angle θ be measured from the horizontal to the point on the hemisphere where the boy is located. There are then two components to the force of gravity— a component tangent to the hemisphere, $W_{||} = mg\cos\theta$, and a component directed radially toward the center of the hemisphere, $W_\perp = mg\sin\theta$.

While the boy is in contact with the hemisphere the motion is circular so

$$mv^2/R = W_\perp - N.$$

When the boy leaves the surface we have $mv^2/R = W_\perp$, or $mv^2 = mgR\sin\theta$. Now for energy conservation,

$$\begin{aligned} K + U &= K_0 + U_0, \\ \frac{1}{2}mv^2 + mgy &= \frac{1}{2}m(0)^2 + mgR, \\ \frac{1}{2}gR\sin\theta + mgy &= mgR, \\ \frac{1}{2}y + y &= R, \\ y &= 2R/3. \end{aligned}$$

P12-12 (a) To be in contact at the top requires $m{v_\mathrm{t}}^2/R = mg$. The speed at the bottom would be given by energy conservation

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR,$$

so $v_\mathrm{b} = \sqrt{5gR}$ is the speed at the bottom that will allow the object to make it around the circle without loosing contact.

(b) The particle will lose contact with the track if $mv^2/R \leq mg\sin\theta$. Energy conservation gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgR(1 + \sin\theta)$$

for points above the half-way point. Then the condition for "sticking" to the track is

$$\frac{1}{R}v_0^2 - 2g(1 + \sin\theta) \leq g\sin\theta,$$

or, if $v_0 = 0.775v_\mathrm{m}$,

$$5(0.775)^2 - 2 \leq 3\sin\theta,$$

or $\theta = \arcsin(1/3)$.

P12-13 The rotational inertia is

$$I = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2.$$

Conservation of energy is

$$\frac{1}{2}I\omega^2 = 3Mg(L/2),$$

so $\omega = \sqrt{9g/(4L)}$.

P12-14 The rotational speed of the sphere is $\omega = v/r$; the rotational kinetic energy is $K_\mathrm{r} = \frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$.

(a) For the marble to stay on the track $mv^2/R = mg$ at the top of the track. Then the marble needs to be released from a point

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + 2mgR,$$

or $h = R/2 + R/5 + 2R = 2.7R$.

(b) Energy conservation gives

$$6mgR = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mgR,$$

or $mv^2/R = 50mg/7$. This corresponds to the horizontal force acting on the marble.

P12-15 $\frac{1}{2}mv_0^2 + mgy = 0$, where y is the distance beneath the rim, or $y = -r\cos\theta_0$. Then

$$v_0 = \sqrt{-2gy} = \sqrt{2gr\cos\theta_0}.$$

P12-16 (a) For E_1 the atoms will eventually move apart completely.

(b) For E_2 the moving atom will bounce back and forth between a closest point and a farthest point.

(c) $U \approx -1.2\times10^{-19}\mathrm{J}$.

(d) $K = E_1 - U \approx 2.2\times10^{-19}\mathrm{J}$.

(e) Find the slope of the curve, so

$$F \approx -\frac{(-1\times10^{-19}\mathrm{J}) - (-2\times10^{-19}\mathrm{J})}{(0.3\times10^{-9}\mathrm{m}) - (0.2\times10^{-9}\mathrm{m})} = -1\times10^{-9}\mathrm{N},$$

which would point toward the larger mass.

P12-17 The function needs to fall off at infinity in both directions; an exponential envelope would work, but it will need to have an $-x^2$ term to force the potential to zero on *both* sides. So we propose something of the form

$$U(x) = P(x)e^{-\beta x^2}$$

where $P(x)$ is a polynomial in x and β is a positive constant.

We proposed the *polynomial* because we need a symmetric function which has two zeroes. A quadratic of the form $\alpha x^2 - U_0$ would work, it has two zeroes, a minimum at $x = 0$, and is symmetric.

So our *trial* function is

$$U(x) = \left(\alpha x^2 - U_0\right) e^{-\beta x^2}.$$

This function should have *three* extrema. Take the derivative, and then we'll set it equal to zero,

$$\frac{dU}{dx} = 2\alpha x e^{-\beta x^2} - 2\left(\alpha x^2 - U_0\right)\beta x e^{-\beta x^2}.$$

Setting this equal to zero leaves two possibilities,

$$\begin{aligned} x &= 0, \\ 2\alpha - 2\left(\alpha x^2 - U_0\right)\beta &= 0. \end{aligned}$$

The first equation is trivial, the second is easily rearranged to give

$$x = \pm\sqrt{\frac{\alpha + \beta U_0}{\beta\alpha}}$$

These are the points $\pm x_1$. We can, if we wanted, try to find α and β from the picture, but you might notice we have one equation, $U(x_1) = U_1$ and two unknowns. It really isn't very illuminating to take this problem much farther, but we could.

(b) The force is the derivative of the potential; this expression was found above.

(c) As long as the energy is *less* than the two peaks, then the motion would be oscillatory, trapped in the well.

P12-18 (a) $F = -\partial U/\partial r$, or

$$F = -U_0\left(\frac{r_0}{r^2} + \frac{1}{r}\right)e^{-r/r_0}.$$

(b) Evaluate the force at the four points:

$$\begin{aligned} F(r_0) &= -2(U_0/r_0)e^{-1}, \\ F(2r_0) &= -(3/4)(U_0/r_0)e^{-2}, \\ F(4r_0) &= -(5/16)(U_0/r_0)e^{-4}, \\ F(10r_0) &= -(11/100)(U_0/r_0)e^{-10}. \end{aligned}$$

The ratios are then

$$\begin{aligned} F(2r_0)/F(r_0) &= (3/8)e^{-1} = 0.14, \\ F(4r_0)/F(r_0) &= (5/32)e^{-3} = 7.8\times 10^{-3}, \\ F(10r_0)/F(r_0) &= (11/200)e^{-9} = 6.8\times 10^{-6}. \end{aligned}$$