E10-1 $l = rp = mvr = (13.7\times10^{-3}\text{kg})(380\,\text{m/s})(0.12\,\text{m}) = 0.62\,\text{kg}\cdot\text{m}^2/\text{s}.$

E10-2 (a) $\vec{\mathbf{l}} = m\vec{\mathbf{r}}\times\vec{\mathbf{v}}$, or

$$\vec{\mathbf{l}} = m(yv_z - zv_y)\hat{\mathbf{i}} + m(zv_x - xv_z)\hat{\mathbf{j}} + m(xv_y - yv_x)\hat{\mathbf{k}}.$$

(b) If $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ exist only in the xy plane then $z = v_z = 0$, so only the uk term survives.

E10-3 If the angular momentum $\vec{\mathbf{l}}$ is constant in time, then $d\vec{\mathbf{l}}/dt = 0$. Trying this on Eq. 10-1,

$$\begin{aligned}
\frac{d\vec{\mathbf{l}}}{dt} &= \frac{d}{dt}\left(\vec{\mathbf{r}}\times\vec{\mathbf{p}}\right),\\
&= \frac{d}{dt}\left(\vec{\mathbf{r}}\times m\vec{\mathbf{v}}\right),\\
&= m\frac{d\vec{\mathbf{r}}}{dt}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\frac{d\vec{\mathbf{v}}}{dt},\\
&= m\vec{\mathbf{v}}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\vec{\mathbf{a}}.
\end{aligned}$$

Now the cross product of a vector with itself is zero, so the first term vanishes. But in the exercise we are told the particle has constant velocity, so $\vec{\mathbf{a}} = 0$, and consequently the second term vanishes. Hence, $\vec{\mathbf{l}}$ is constant for a single particle if $\vec{\mathbf{v}}$ is constant.

E10-4 (a) $L = \sum l_i$; $l_i = r_i m_i v_i$. Putting the numbers in for each planet and then summing (I won't bore you with the arithmetic details) yields $L = 3.15\times10^{43}\text{kg}\cdot\text{m}^2/\text{s}$.

(b) Jupiter has $l = 1.94\times10^{43}\text{kg}\cdot\text{m}^2/\text{s}$, which is 61.6% of the total.

E10-5 $l = mvr = m(2\pi r/T)r = 2\pi(84.3\,\text{kg})(6.37\times10^6\text{m})^2/(86400\,\text{s}) = 2.49\times10^{11}\text{kg}\cdot\text{m}^2/\text{s}.$

E10-6 (a) Substitute and expand:

$$\begin{aligned}
\vec{\mathbf{L}} &= \sum(\vec{\mathbf{r}}_{\text{cm}} + \vec{\mathbf{r}}_i')\times(m_i\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{p}}_i'),\\
&= \sum(m_i\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{p}}_i' + m_i\vec{\mathbf{r}}_i'\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'),\\
&= M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \vec{\mathbf{r}}_{\text{cm}}\times(\sum\vec{\mathbf{p}}_i') + (\sum m_i\vec{\mathbf{r}}_i')\times\vec{\mathbf{v}}_{\text{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'.
\end{aligned}$$

(b) But $\sum\vec{\mathbf{p}}_i' = 0$ and $\sum m_i\vec{\mathbf{r}}_i' = 0$, because these two quantities are *in* the center of momentum and center of mass. Then

$$\vec{\mathbf{L}} = M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i' = \vec{\mathbf{L}}' + M\vec{\mathbf{r}}_{\text{cm}}\times\vec{\mathbf{v}}_{\text{cm}}.$$

E10-7 (a) Substitute and expand:

$$\vec{\mathbf{p}}_i' = m_i\frac{d\vec{\mathbf{r}}_i'}{dt} = m_i\frac{d\vec{\mathbf{r}}_i}{dt} - m_i\frac{d\vec{\mathbf{r}}_{\text{cm}}}{dt} = \vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\text{cm}}.$$

(b) Substitute and expand:

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum\frac{d\vec{\mathbf{r}}_i'}{dt}\times\vec{\mathbf{p}}_i' + \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt} = \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt}.$$

The first term vanished because $\vec{\mathbf{v}}_i'$ is parallel to $\vec{\mathbf{p}}_i'$.

(c) Substitute and expand:

$$\begin{aligned}\frac{d\vec{\mathbf{L}}'}{dt} &= \sum \vec{\mathbf{r}}_i' \times \frac{d(\vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\rm cm})}{dt}, \\ &= \sum \vec{\mathbf{r}}_i' \times (m_i\vec{\mathbf{a}}_i - m_i\vec{\mathbf{a}}_{\rm c}{\rm m}), \\ &= \sum \vec{\mathbf{r}}_i' \times m_i\vec{\mathbf{a}}_i + \left(\sum m_i\vec{\mathbf{r}}_i'\right) \times \vec{\mathbf{a}}_{\rm c}{\rm m})\end{aligned}$$

The second term vanishes because of the definition of the center of mass. Then

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\mathbf{r}}_i' \times \vec{\mathbf{F}}_i,$$

where $\vec{\mathbf{F}}_i$ is the net force on the ith particle. The force $\vec{\mathbf{F}}_i$ may include both internal and external components. If there is an internal component, say between the ith and jth particles, then the torques from these two third law components will cancel out. Consequently,

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\tau}_i = \vec{\tau}_{\rm ext}.$$

E10-8 (a) Integrate.

$$\int \vec{\tau}\, dt = \int \frac{d\vec{\mathbf{L}}}{dt} dt = \int d\vec{\mathbf{L}} = \Delta\vec{\mathbf{L}}.$$

(b) If I is fixed, $\Delta L = I\Delta\omega$. Not only that,

$$\int \tau dt = \int Fr\, dt = r \int F\, dt = rF_{\rm av}\Delta t,$$

where we use the definition of average that depends on time.

E10-9 (a) $\vec{\tau}\Delta t = \Delta\vec{\mathbf{l}}$. The disk starts from rest, so $\Delta\vec{\mathbf{l}} = \vec{\mathbf{l}} - \vec{\mathbf{l}}_0 = \vec{\mathbf{l}}$. We need only concern ourselves with the magnitudes, so

$$l = \Delta l = \tau\Delta t = (15.8\,{\rm N{\cdot}m})(0.033\,{\rm s}) = 0.521\,{\rm kg{\cdot}m^2/s}.$$

(b) $\omega = l/I = (0.521\,{\rm kg{\cdot}m^2/s})/(1.22{\times}10^{-3}{\rm kg{\cdot}m^2}) = 427$ rad/s.

E10-10 (a) Let v_0 be the initial speed; the average speed while slowing to a stop is $v_0/2$; the time required to stop is $t = 2x/v_0$; the acceleration is $a = -v_0/t = -v_0^2/(2x)$. Then

$$a = -(43.3\,{\rm m/s})^2/[2(225\,{\rm m/s})] = -4.17\,{\rm m/s^2}.$$

(b) $\alpha = a/r = (-4.17\,{\rm m/s^2})/(0.247\,{\rm m}) = -16.9\ {\rm rad/s^2}$.
(c) $\tau = I\alpha = (0.155\,{\rm kg\cdot m^2})(-16.9\ {\rm rad/s^2}) = -2.62\,{\rm N\cdot m}$.

E10-11 Let $\vec{\mathbf{r}}_i = \vec{\mathbf{z}} + \vec{\mathbf{r}}_i'$. From the figure, $\vec{\mathbf{p}}_1 = -\vec{\mathbf{p}}_2$ and $\vec{\mathbf{r}}_1' = -\vec{\mathbf{r}}_2'$. Then

$$\begin{aligned}\vec{\mathbf{L}} &= \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = \vec{\mathbf{r}}_1 \times \vec{\mathbf{p}}_1 + \vec{\mathbf{r}}_2 \times \vec{\mathbf{p}}_2, \\ &= (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \times \vec{\mathbf{p}}_1, \\ &= (\vec{\mathbf{r}}_1' - \vec{\mathbf{r}}_2') \times \vec{\mathbf{p}}_1, \\ &= 2\vec{\mathbf{r}}_1' \times \vec{\mathbf{p}}_1.\end{aligned}$$

Since $\vec{\mathbf{r}}_1'$ and $\vec{\mathbf{p}}_1$ both lie in the xy plane then $\vec{\mathbf{L}}$ must be along the z axis.

E10-12 Expand:

$$\begin{aligned}
\vec{\mathbf{L}} &= \sum \vec{\mathbf{l}}_i = \sum \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i, \\
&= \sum m_i \vec{\mathbf{r}}_i \times \vec{\mathbf{v}}_i = \sum m_i \vec{\mathbf{r}}_i \times (\vec{\omega} \times \vec{\mathbf{r}}_i) \\
&= \sum m_i [(\vec{\mathbf{r}}_i \cdot \vec{\mathbf{r}}_i)\vec{\omega} - (\vec{\mathbf{r}}_i \cdot \vec{\omega})\vec{\mathbf{r}})i], \\
&= \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}} - (z_i x_i \omega)\hat{\mathbf{i}} - (z_i y_i \omega)\hat{\mathbf{j}}],
\end{aligned}$$

but if the body is symmetric about the z axis then the last two terms vanish, leaving

$$\vec{\mathbf{L}} = \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}}] = \sum m_i (x_i^2 + y_i^2)\vec{\omega} = I\vec{\omega}.$$

E10-13 An impulse of 12.8 N·s will change the linear momentum by 12.8 N·s; the stick starts from rest, so the final momentum *must* be 12.8 N·s. Since $p = mv$, we then can find $v = p/m =$ (12.8 N·s)/(4.42 kg) = 2.90 m/s.

Impulse is a vector, given by $\int \vec{\mathbf{F}} dt$. We can take the cross product of the impulse with the displacement vector $\vec{\mathbf{r}}$ (measured from the axis of rotation to the point where the force is applied) and get

$$\vec{\mathbf{r}} \times \int \vec{\mathbf{F}} dt \approx \int \vec{\mathbf{r}} \times \vec{\mathbf{F}} dt,$$

The two sides of the above expression are only equal if $\vec{\mathbf{r}}$ has a constant magnitude *and* direction. This won't be true, but if the force is of sufficiently short duration then it hopefully won't change much. The right hand side is an integral over a torque, and will equal the change in angular momentum of the stick.

The exercise states that the force is perpendicular to the stick, then $|\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = rF$, and the "torque impulse" is then (0.464 m)(12.8 N·s) = 5.94 kg·m/s. This "torque impulse" is equal to the change in the angular momentum, but the stick started from rest, so the final angular momentum of the stick is 5.94 kg·m/s.

But how fast is it rotating? We can use Fig. 9-15 to find the rotational inertia about the center of the stick: $I = \frac{1}{12}ML^2 = \frac{1}{12}(4.42 \text{ kg})(1.23 \text{ m})^2 = 0.557 \text{ kg·m}^2$. The angular velocity of the stick is $\omega = l/I = (5.94 \text{ kg·m/s})/(0.557 \text{ kg·m}^2) = 10.7$ rad/s.

E10-14 The point of rotation is the point of contact with the plane; the torque about that point is $\tau = rmg \sin\theta$. The angular momentum is $I\omega$, so $\tau = I\alpha$. In this case $I = mr^2/2 + mr^2$, the second term from the parallel axis theorem. Then

$$a = r\alpha = r\tau/I = mr^2 g \sin\theta/(3mr^2/2) = \frac{2}{3} g \sin\theta.$$

E10-15 From Exercise 8 we can immediately write

$$I_1(\omega_1 - \omega_0)/r_1 = I_2(\omega_2 - 0)/r_2,$$

but we also have $r_1\omega_1 = -r_2\omega_2$. Then

$$\omega_2 = -\frac{r_1 r_2 I_1 \omega_0}{r_1^2 I_2 - r_2^2 I_1}.$$

E10-16 (a) $\Delta\omega/\omega = (1/T_1 - 1/T_2)/(1/T_1) = -(T_2 - T_1)/T_2 = -\Delta T/T$, which in this case is $-(6.0\times10^{-3}\text{s})(8.64\times10^4\text{s}) = -6.9\times10^{-8}$.

(b) Assuming conservation of angular momentum, $\Delta I/I = -\Delta\omega/\omega$. Then the fractional change would be 6.9×10^{-8}.

E10-17 The rotational inertia of a solid sphere is $I = \frac{2}{5}MR^2$; so as the sun collapses

$$\begin{aligned}\vec{\mathbf{L}}_\text{i} &= \vec{\mathbf{L}}_\text{f},\\ I_\text{i}\vec{\omega}_\text{i} &= I_\text{f}\vec{\omega}_\text{f},\\ \frac{2}{5}M{R_\text{i}}^2\vec{\omega}_\text{i} &= \frac{2}{5}M{R_\text{f}}^2\vec{\omega}_\text{f},\\ {R_\text{i}}^2\vec{\omega}_i &= {R_\text{f}}^2\vec{\omega}_f.\end{aligned}$$

The angular frequency is inversely proportional to the period of rotation, so

$$T_\text{f} = T_\text{i}\frac{{R_\text{f}}^2}{{R_\text{i}}^2} = (3.6\times10^4 \text{ min})\left(\frac{(6.37\times10^6\text{m})}{(6.96\times10^8\text{m})}\right)^2 = 3.0 \text{ min}.$$

E10-18 The final angular velocity of the train with respect to the tracks is $\omega_\text{tt} = Rv$. The conservation of angular momentum implies

$$0 = MR^2\omega + mR^2(\omega_\text{tt} + \omega),$$

or

$$\omega = \frac{-mv}{(m+M)R}.$$

E10-19 This is much like a center of mass problem.

$$0 = I_\text{p}\phi_\text{p} + I_\text{m}(\phi_\text{mp} + \phi_\text{p}),$$

or

$$\phi_\text{mp} = -\frac{(I_\text{p} + I_\text{m})\phi_\text{p}}{I_\text{m}} \approx -\frac{(12.6\,\text{kg}\cdot\text{m}^2)(25^\circ)}{(2.47\times10^{-3}\text{kg}\cdot\text{m}^2)} = 1.28\times10^{5\circ}.$$

That's 354 rotations!

E10-20 $\omega_\text{f} = (I_\text{i}/I_\text{f})\omega_\text{i} = [(6.13\,\text{kg}\cdot\text{m}^2)/(1.97\,\text{kg}\cdot\text{m}^2)](1.22 \text{ rev/s}) = 3.80 \text{ rev/s}.$

E10-21 We have two disks which are originally not in contact which then come into contact; there are no external torques. We can write

$$\begin{aligned}\vec{\mathbf{l}}_{1,\text{i}} + \vec{\mathbf{l}}_{2,\text{i}} &= \vec{\mathbf{l}}_{1,\text{f}} + \vec{\mathbf{l}}_{2,\text{f}},\\ I_1\vec{\omega}_{1,\text{i}} + I_2\vec{\omega}_{2,\text{i}} &= I_1\vec{\omega}_{1,\text{f}} + I_2\vec{\omega}_{2,\text{f}}.\end{aligned}$$

The final angular velocities of the two disks will be equal, so the above equation can be simplified and rearranged to yield

$$\omega_\text{f} = \frac{I_1}{I_1 + I_2}\omega_{1,\text{i}} = \frac{(1.27 \text{ kg}\cdot\text{m}^2)}{(1.27 \text{ kg}\cdot\text{m}^2) + (4.85 \text{ kg}\cdot\text{m}^2)}(824 \text{ rev/min}) = 171 \text{ rev/min}$$

E10-22 $l_\perp = l\cos\theta = mvr\cos\theta = mvh.$

E10-23 (a) $\omega_{\rm f} = (I_1/I_2)\omega_{\rm i}$, $I_1 = (3.66\,{\rm kg})(0.363\,{\rm m})^2 = 0.482\,{\rm kg}\cdot{\rm m}^2$. Then

$$\omega_{\rm f} = [(0.482\,{\rm kg}\cdot{\rm m}^2)/(2.88\,{\rm kg}\cdot{\rm m}^2)](57.7 \text{ rad/s}) = 9.66 \text{ rad/s},$$

with the same rotational sense as the original wheel.

(b) Same answer, since friction is an internal force internal here.

E10-24 (a) Assume the merry-go-round is a disk. Then conservation of angular momentum yields

$$(\frac{1}{2}m_{\rm m}R^2 + m_{\rm g}R^2)\omega + (m_{\rm r}R^2)(v/R) = 0,$$

or

$$\omega = -\frac{(1.13\,{\rm kg})(7.82\,{\rm m/s})/(3.72\,{\rm m})}{(827\,{\rm kg})/2 + (50.6\,{\rm kg})} = -5.12\times10^{-3}{\rm rad/s}.$$

(b) $v = \omega R = (-5.12\times10^{-3}{\rm rad/s})(3.72\,{\rm m}) = -1.90\times10^{-2}{\rm m/s}$.

E10-25 Conservation of angular momentum:

$$(m_{\rm m}k^2 + m_{\rm g}R^2)\omega = m_{\rm g}R^2(v/R),$$

so

$$\omega = \frac{(44.3\,{\rm kg})(2.92\,{\rm m/s})/(1.22\,{\rm m})}{(176\,{\rm kg})(0.916\,{\rm m})^2 + (44.3\,{\rm kg})(1.22\,{\rm m})^2} = 0.496 \text{ rad/s}.$$

E10-26 Use Eq. 10-22:

$$\omega_P = \frac{Mgr}{I\omega} = \frac{(0.492\,{\rm kg})(9.81\,{\rm m/s}^2)(3.88\times10^{-2}{\rm m})}{(5.12\times10^{-4}{\rm kg}\cdot{\rm m}^2)(2\pi 28.6 \text{ rad/s})} = 2.04 \text{ rad/s} = 0.324 \text{ rev/s}.$$

E10-27 The relevant precession expression is Eq. 10-22.

The rotational inertia will be a sum of the contributions from both the disk and the axle, but the radius of the axle is probably very small compared to the disk, probably as small as 0.5 cm. Since I is proportional to the radius squared, we expect contributions from the axle to be less than $(1/100)^2$ of the value for the disk. For the disk only we use

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(1.14 \text{ kg})(0.487 \text{ m})^2 = 0.135 \text{ kg}\cdot\text{m}^2.$$

Now for ω,

$$\omega = 975 \text{ rev/min}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 102 \text{ rad/s}.$$

Then $L = I\omega = 13.8$ kg·m^2/s.

Back to Eq. 10-22,

$$\omega_p = \frac{Mgr}{L} = \frac{(1.27 \text{ kg})(9.81 \text{ m/s}^2)(0.0610 \text{ m})}{13.8 \text{ kg}\cdot\text{m}^2/\text{s}} = 0.0551 \text{ rad/s}.$$

The *time* for one precession is

$$t = \frac{1\text{rev}}{\omega_p} = \frac{2\pi \text{ rad}}{(0.0551 \text{ rad/s})} = 114 \text{ s}.$$

P10-1 Positive z is out of the page.

(a) $\vec{\mathbf{l}} = rmv\sin\theta\hat{\mathbf{k}} = (2.91\,\mathrm{m})(2.13\,\mathrm{kg})(4.18\,\mathrm{m})\sin(147^\circ)\hat{\mathbf{k}} = 14.1\,\mathrm{kg}\cdot\mathrm{m}^2/s\hat{\mathbf{k}}$.

(b) $\vec{\tau} = rF\sin\theta\hat{\mathbf{k}} = (2.91\,\mathrm{m})(1.88\,\mathrm{N})\sin(26^\circ)\hat{\mathbf{k}} = 2.40\,\mathrm{N}\cdot\mathrm{m}\hat{\mathbf{k}}$.

P10-2 Regardless of where the origin is located one can orient the coordinate system so that the two paths lie in the xy plane and are both parallel to the y axis. The one of the particles travels along the path $x = vt$, $y = a$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_1 = mv\hat{\mathbf{i}}$. The other particle will then travel along a path $x = c - vt$, $y = a + d$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_2 = -mv\hat{\mathbf{i}}$. The angular momentum of the first particle is

$$\vec{\mathbf{l}}_1 = mvb\hat{\mathbf{j}} - mva\hat{\mathbf{k}},$$

while that of the second is

$$\vec{\mathbf{l}}_2 = -mvb\hat{\mathbf{j}} + mv(a+d)\hat{\mathbf{k}},$$

so the total is $\vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = mvd\hat{\mathbf{k}}$.

P10-3 Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero.

If the force is applied a distance x above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = xF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l$, since initially the ball is not rotating.

For the ball to roll without slipping we need $v = \omega R$. We can start with this:

$$\begin{aligned} v &= \omega R, \\ \frac{p}{m} &= \frac{lR}{I}, \\ \frac{F\Delta t}{m} &= \frac{\tau\Delta t R}{I}, \\ \frac{F}{m} &= \frac{xFR}{I}. \end{aligned}$$

Then $x = I/mR$ is the condition for rolling without sliding from the start. For a solid sphere, $I = \frac{2}{5}mR^2$, so $x = \frac{2}{5}R$.

P10-4 The change in momentum of the block is $M(v_2 - v_1)$, this is equal to the magnitude the impulse delivered to the cylinder. According to E10-8 we can write $M(v_2 - v_1)R = I\omega_\mathrm{f}$. But in the end the box isn't slipping, so $\omega_f = v_2/R$. Then

$$Mv_2 - Mv_1 = (I/R^2)v_2,$$

or

$$v_2 = v_1/(1 + I/MR^2).$$

P10-5 Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero. Consequently, $F\Delta t = mv_0$.

If the force is applied a distance h above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = hF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l_0$, since initially the ball is not rotating. Consequently, the initial angular momentum of the ball is $l_0 = hmv_0 = I\omega_0$.

The ball originally slips while moving, but eventually it rolls. When it has begun to roll without slipping we have $v = R\omega$. Applying the results from E10-8,

$$m(v - v_0)R + I(\omega - \omega_0) = 0,$$

or

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

then, if $v = 9v_0/7$,

$$h = \left(\frac{9}{7} - 1\right)R + \frac{2}{5}R\left(\frac{9}{7}\right) = \frac{4}{5}R.$$

P10-6 (a) Refer to the previous answer. We now want $v = \omega = 0$, so

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

becomes

$$-v_0R - hv_0 = 0,$$

or $h = -R$. That'll scratch the felt.

(b) Assuming only a horizontal force then

$$v = \frac{(h + R)v_0}{R(1 + 2/5)},$$

which can only be negative if $h < -R$, which means hitting below the ball. Can't happen. If instead we allow for a downward component, then we can increase the "reverse English" as much as we want without increasing the initial forward velocity, and as such it would be possible to get the ball to move backwards.

P10-7 We assume the bowling ball is solid, so the rotational inertia will be $I = (2/5)MR^2$ (see Figure 9-15).

The normal force on the bowling ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the bowling ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

(a) The time requires for the ball to stop skidding is the time required to change the angular momentum to l, so

$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

Since we don't know v, we can't solve this for Δt. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k Mg} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned}\Delta t &= \frac{v_0 - v}{\mu_k g},\\ &= \frac{v_0 - 5\mu_k g\Delta t/2}{\mu_k g},\end{aligned}$$

$$
\begin{aligned}
2\mu_k g\Delta t &= 2v_0 - 5\mu_k g\Delta t,\\
\Delta t &= \frac{2v_0}{7\mu_k g},\\
&= \frac{2(8.50\,\mathrm{m/s})}{7(0.210)(9.81\,\mathrm{m/s^2})} = 1.18\,\mathrm{s}.
\end{aligned}
$$

(d) Use the expression for angular momentum and torque,

$$v = 5\mu_k g\Delta t/2 = 5(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})/2 = 6.08\,\mathrm{m/s}.$$

(b) The acceleration of the ball is $F/M = -\mu g$. The distance traveled is then given by

$$
\begin{aligned}
x &= \frac{1}{2}at^2 + v_0 t,\\
&= -\frac{1}{2}(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})^2 + (8.50\,\mathrm{m/s})(1.18\,\mathrm{s}) = 8.6\,\mathrm{m},
\end{aligned}
$$

(c) The angular acceleration is $\tau/I = 5\mu_k g/(2R)$. Then

$$
\begin{aligned}
\theta &= \frac{1}{2}\alpha t^2 + \omega_0 t,\\
&= \frac{5(0.210)(9.81\,\mathrm{m/s^2})}{4(0.11\,\mathrm{m})}(1.18\,\mathrm{s})^2 = 32.6\text{ rad} = 5.19\text{ revolutions.}
\end{aligned}
$$

P10-8 (a) $l = I\omega_0 = (1/2)MR^2\omega_0$.

(b) The initial speed is $v_0 = R\omega_0$. The chip decelerates in a time $t = v_0/g$, and during this time the chip travels with an average speed of $v_0/2$ through a distance of

$$y = v_{\mathrm{av}}t = \frac{v_0}{2}\frac{v_0}{g} = \frac{R^2\omega^2}{2g}.$$

(c) Loosing the chip won't change the angular velocity of the wheel.

P10-9 Since $L = I\omega = 2\pi I/T$ and L is constant, then $I \propto T$. But $I \propto R^2$, so $R^2 \propto T$ and

$$\frac{\Delta T}{T} = \frac{2R\Delta R}{R^2} = \frac{2\Delta R}{R}.$$

Then

$$\Delta T = (86400\,\mathrm{s})\frac{2(30\,\mathrm{m})}{(6.37\times 10^6\mathrm{m})} \approx 0.8\,\mathrm{s}.$$

P10-10 Originally the rotational inertia was

$$I_{\mathrm{i}} = \frac{2}{5}MR^2 = \frac{8\pi}{15}\rho_0 R^5.$$

The average density can be found from Appendix C. Now the rotational inertia is

$$I_{\mathrm{f}} = \frac{8\pi}{15}(\rho_1 - \rho_2)R_1^5 + \frac{8\pi}{15}\rho_2 R^5,$$

where ρ_1 is the density of the core, R_1 is the radius of the core, and ρ_2 is the density of the mantle. Since the angular momentum is constant we have $\Delta T/T = \Delta I/I$. Then

$$\frac{\Delta T}{T} = \frac{\rho_1 - \rho_2}{\rho_0}\frac{R_1^5}{R^5} + \frac{\rho_2}{\rho_0} - 1 = \frac{10.3 - 4.50}{5.52}\frac{3570^5}{6370^5} + \frac{4.50}{5.52} - 1 = -0.127,$$

so the day is getting longer.

P10-11 The cockroach initially has an angular speed of $\omega_{\mathrm{c,i}} = -v/r$. The rotational inertia of the cockroach about the axis of the turntable is $I_{\mathrm{c}} = mR^2$. Then conservation of angular momentum gives

$$\begin{aligned} l_{\mathrm{c,i}} + l_{\mathrm{s,i}} &= l_{\mathrm{c,f}} + l_{\mathrm{s,f}}, \\ I_{\mathrm{c}}\omega_{\mathrm{c,i}} + I_{\mathrm{s}}\omega_{\mathrm{s,i}} &= I_{\mathrm{c}}\omega_{\mathrm{c,f}} + I_{\mathrm{s}}\omega_{\mathrm{s,f}}, \\ -mR^2 v/r + I\omega &= (mR^2 + I)\omega_{\mathrm{f}}, \\ \omega_{\mathrm{f}} &= \frac{I\omega - mvR}{I + mR^2}. \end{aligned}$$

P10-12 (a) The skaters move in a circle of radius $R = (2.92\,\mathrm{m})/2 = 1.46\,\mathrm{m}$ centered midway between the skaters. The angular velocity of the system will be $\omega_{\mathrm{i}} = v/R = (1.38\,\mathrm{m/s})/(1.46\,\mathrm{m}) = 0.945$ rad/s.

(b) Moving closer will decrease the rotational inertia, so

$$\omega_{\mathrm{f}} = \frac{2MR_{\mathrm{i}}{}^2}{2MR_{\mathrm{f}}{}^2}\omega_{\mathrm{i}} = \frac{(1.46\,\mathrm{m})^2}{(0.470\,\mathrm{m})^2}(0.945\ \mathrm{rad/s}) = 9.12\ \mathrm{rad/s}.$$