E2-1 Add the vectors as is shown in Fig. 2-4. If $\vec{\mathbf{a}}$ has length $a = 4$ m and $\vec{\mathbf{b}}$ has length $b = 3$ m then the sum is given by $\vec{\mathbf{s}}$. The cosine law can be used to find the magnitude s of $\vec{\mathbf{s}}$,

$$s^2 = a^2 + b^2 - 2ab\cos\theta,$$

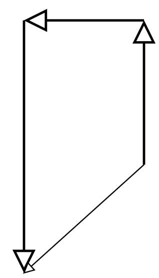
where θ is the angle between sides a and b in the figure.

(a) $(7\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = -1.0$, and $\theta = 180°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the same direction.

(b) $(1\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 1.0$, and $\theta = 0°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the opposite direction.

(c) $(5\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 0$, and $\theta = 90°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing at right angles to each other.

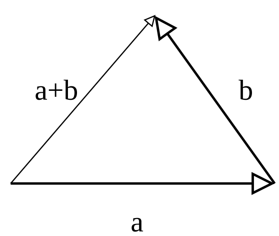
E2-2 (a) Consider the figures below.

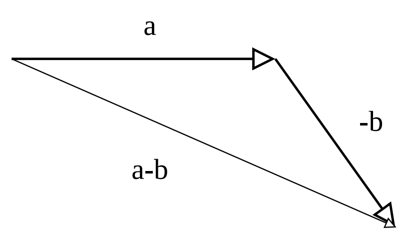


(b) Net displacement is 2.4 km west, $(5.2 - 3.1 = 2.1)$ km south. A bird would fly

$$\sqrt{2.4^2 + 2.1^2}\text{ km} = 3.2\text{ km}.$$

E2-3 Consider the figure below.





E2-4 (a) The components are $(7.34)\cos(252°) = -2.27\hat{\mathbf{i}}$ and $(7.34)\sin(252°) = -6.98\hat{\mathbf{j}}$.

(b) The magnitude is $\sqrt{(-25)^2 + (43)^2} = 50$; the direction is $\theta = \tan^{-1}(43/-25) = 120°$. We did need to choose the correct quadrant.

E2-5 The components are given by the trigonometry relations

$$O = H\sin\theta = (3.42\text{ km})\sin 35.0° = 1.96\text{ km}$$

and

$$A = H\cos\theta = (3.42\text{ km})\cos 35.0° = 2.80\text{ km}.$$

The stated angle is measured from the east-west axis, counter clockwise from east. So O is measured against the north-south axis, with north being positive; A is measured against east-west with east being positive.

Since her individual steps are displacement vectors which are only north-south or east-west, she must eventually take enough north-south steps to equal 1.96 km, and enough east-west steps to equal 2.80 km. Any individual step can only be along one or the other direction, so the minimum total will be 4.76 km.

E2-6 Let $\vec{\mathbf{r}}_\mathrm{f} = 124\hat{\mathbf{i}}$ km and $\vec{\mathbf{r}}_\mathrm{i} = (72.6\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}})$ km. Then the ship needs to travel

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} = (51.4\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}}) \text{ km}.$$

Ship needs to travel $\sqrt{51.4^2 + 31.4^2}$ km $= 60.2$ km in a direction $\theta = \tan^{-1}(31.4/51.4) = 31.4°$ west of north.

E2-7 (a) In unit vector notation we need only add the components; $\vec{\mathbf{a}}+\vec{\mathbf{b}} = (5\hat{\mathbf{i}}+3\hat{\mathbf{j}})+(-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}) = (5-3)\hat{\mathbf{i}} + (3+2)\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$.

(b) If we define $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$ and write the magnitude of $\vec{\mathbf{c}}$ as c, then $c = \sqrt{c_x^2 + c_y^2} = \sqrt{2^2 + 5^2} = 5.39$. The direction is given by $\tan\theta = c_y/c_x$ which gives an angle of 68.2°, measured counterclockwise from the positive x-axis.

E2-8 (a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4-1)\hat{\mathbf{i}} + (-3+1)\hat{\mathbf{j}} + (1+4)\hat{\mathbf{k}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

(b) $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4 - -1)\hat{\mathbf{i}} + (-3-1)\hat{\mathbf{j}} + (1-4)\hat{\mathbf{k}} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$.

(c) Rearrange, and $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$, or $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (-1-4)\hat{\mathbf{i}} + (1 - -3)\hat{\mathbf{j}} + (4-1)\hat{\mathbf{k}} = -5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

E2-9 (a) The magnitude of $\vec{\mathbf{a}}$ is $\sqrt{4.0^2 + (-3.0)^2} = 5.0$; the direction is $\theta = \tan^{-1}(-3.0/4.0) = 323°$.

(b) The magnitude of $\vec{\mathbf{b}}$ is $\sqrt{6.0^2 + 8.0^3} = 10.0$; the direction is $\theta = \tan^{-1}(6.0/8.0) = 36.9°$.

(c) The resultant vector is $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4.0 + 6.0)\hat{\mathbf{i}} + (-3.0 + 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is $\sqrt{(10.0)^2 + (5.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(5.0/10.0) = 26.6°$.

(d) The resultant vector is $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4.0 - 6.0)\hat{\mathbf{i}} + (-3.0 - 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\sqrt{(-2.0)^2 + (-11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(-11.0/-2.0) = 260°$.

(e) The resultant vector is $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (6.0 - 4.0)\hat{\mathbf{i}} + (8.0 - -3.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ is $\sqrt{(2.0)^2 + (11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(11.0/2.0) = 79.7°$.

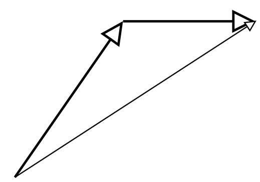
E2-10 (a) Find components of $\vec{\mathbf{a}}$; $a_x = (12.7)\cos(28.2°) = 11.2$, $a_y = (12.7)\sin(28.2°) = 6.00$. Find components of $\vec{\mathbf{b}}$; $b_x = (12.7)\cos(133°) = -8.66$, $b_y = (12.7)\sin(133°) = 9.29$. Then

$$\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (11.2 - 8.66)\hat{\mathbf{i}} + (6.00 + 9.29)\hat{\mathbf{j}} = 2.54\hat{\mathbf{i}} + 15.29\hat{\mathbf{j}}.$$

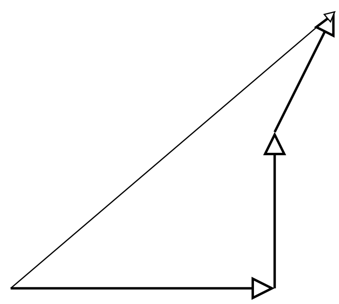
(b) The magnitude of $\vec{\mathbf{r}}$ is $\sqrt{2.54^2 + 15.29^2} = 15.5$.

(c) The angle is $\theta = \tan^{-1}(15.29/2.54) = 80.6°$.

E2-11 Consider the figure below.



E2-12 Consider the figure below.



E2-13 Our axes will be chosen so that $\hat{\mathbf{i}}$ points toward 3 O'clock and $\hat{\mathbf{j}}$ points toward 12 O'clock.
(a)
The two relevant positions are $\vec{\mathbf{r}}_{\mathrm{i}} = (11.3\ \mathrm{cm})\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_{\mathrm{f}} = (11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_{\mathrm{f}} - \vec{\mathbf{r}}_{\mathrm{i}} \\ &= (11.3\ \mathrm{cm})\hat{\mathbf{j}} - (11.3\ \mathrm{cm})\hat{\mathbf{i}}. \end{aligned}$$

(b)
The two relevant positions are now $\vec{\mathbf{r}}_{\mathrm{i}} = (11.3\ \mathrm{cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_{\mathrm{f}} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_{\mathrm{f}} - \vec{\mathbf{r}}_{\mathrm{i}} \\ &= (11.3\ \mathrm{cm})\hat{\mathbf{j}} - (-11.3\ \mathrm{cm})\hat{\mathbf{j}} \\ &= (22.6\ \mathrm{cm})\hat{\mathbf{j}}. \end{aligned}$$

(c)
The two relevant positions are now $\vec{\mathbf{r}}_{\mathrm{i}} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_{\mathrm{f}} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_{\mathrm{f}} - \vec{\mathbf{r}}_{\mathrm{i}} \\ &= (-11.3\ \mathrm{cm})\hat{\mathbf{j}} - (-11.3\ \mathrm{cm})\hat{\mathbf{j}} \\ &= (0\ \mathrm{cm})\hat{\mathbf{j}}. \end{aligned}$$

E2-14 (a) The components of $\vec{\mathbf{r}}_1$ are

$$r_{1x} = (4.13\,\mathrm{m})\cos(225^\circ) = -2.92\,\mathrm{m}$$

and

$$r_{1y} = (4.13\,\mathrm{m})\sin(225^\circ) = -2.92\,\mathrm{m}.$$

The components of $\vec{\mathbf{r}}_2$ are

$$r_{1x} = (5.26\,\mathrm{m})\cos(0^\circ) = 5.26\,\mathrm{m}$$

and

$$r_{1y} = (5.26\,\mathrm{m})\sin(0^\circ) = 0\,\mathrm{m}.$$

The components of $\vec{\mathbf{r}}_3$ are

$$r_{1x} = (5.94\,\mathrm{m})\cos(64.0^\circ) = 2.60\,\mathrm{m}$$

and

$$r_{1y} = (5.94\,\mathrm{m})\sin(64.0^\circ) = 5.34\,\mathrm{m}.$$

(b) The resulting displacement is

$$\left[(-2.92 + 5.26 + 2.60)\hat{\mathbf{i}} + (-2.92 + 0 + 5.34)\hat{\mathbf{j}}\right]\,\mathrm{m} = (4.94\hat{\mathbf{i}} + 2.42\hat{\mathbf{j}})\,\mathrm{m}.$$

(c) The magnitude of the resulting displacement is $\sqrt{4.94^2 + 2.42^2}\,\mathrm{m} = 5.5\,\mathrm{m}$. The direction of the resulting displacement is $\theta = \tan^{-1}(2.42/4.94) = 26.1^\circ$. (d) To bring the particle back to the starting point we need only reverse the answer to (c); the magnitude will be the same, but the angle will be 206°.

E2-15 The components of the initial position are

$$r_{1x} = (12,000\ \mathrm{ft})\cos(40^\circ) = 9200\ \mathrm{ft}$$

and

$$r_{1y} = (12,000\ \mathrm{ft})\sin(40^\circ) = 7700\ \mathrm{ft}.$$

The components of the final position are

$$r_{2x} = (25,8000\ \mathrm{ft})\cos(163^\circ) = -24,700\ \mathrm{ft}$$

and

$$r_{2y} = (25,800\ \mathrm{ft})\sin(163^\circ) = 7540\ \mathrm{ft}.$$

The displacement is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left[(-24,700 - 9,200)\hat{\mathbf{i}} + (7,540 - 9,200)\hat{\mathbf{j}})\right] = (-33900\hat{\mathbf{i}} - 1660\hat{\mathbf{j}})\ \mathrm{ft}.$$

E2-16 (a) The displacement vector is $\vec{\mathbf{r}} = (410\hat{\mathbf{i}} - 820\hat{\mathbf{j}})$ mi, where positive x is east and positive y is north. The magnitude of the displacement is $\sqrt{(410)^2 + (-820)^2}$ mi $= 920$ mi. The direction is $\theta = \tan^{-1}(-820/410) = 300^\circ$.

(b) The average velocity is the displacement divided by the *total* time, 2.25 hours. Then

$$\vec{\mathbf{v}}_{\mathrm{av}} = (180\hat{\mathbf{i}} - 360\hat{\mathbf{j}})\ \mathrm{mi/hr}.$$

(c) The average speed is total distance over total time, or $(410 + 820)/(2.25)$ mi/hr $= 550$ mi/hr.

E2-17 (a) Evaluate $\vec{\mathbf{r}}$ when $t = 2$ s.

$$\begin{aligned}
\vec{\mathbf{r}} &= [(2\text{ m/s}^3)t^3 - (5\text{ m/s})t]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)t^4]\hat{\mathbf{j}} \\
&= [(2\text{ m/s}^3)(2\text{ s})^3 - (5\text{ m/s})(2\text{ s})]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)(2\text{ s})^4]\hat{\mathbf{j}} \\
&= [(16\text{ m}) - (10\text{ m})]\hat{\mathbf{i}} + [(6\text{ m}) - (112\text{ m})]\hat{\mathbf{j}} \\
&= [(6\text{ m})]\hat{\mathbf{i}} + [-(106\text{ m})]\hat{\mathbf{j}}.
\end{aligned}$$

(b) Evaluate:

$$\begin{aligned}
\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} &= [(2\text{ m/s}^3)3t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(7\text{ m/s}^4)4t^3]\hat{\mathbf{j}} \\
&= [(6\text{ m/s}^3)t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)t^3]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{v}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{v}} &= [(6\text{ m/s}^3)(2\text{ s})^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)(2\text{ s})^3]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}) - (5\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}} \\
&= [(19\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}},
\end{aligned}$$

for the velocity $\vec{\mathbf{v}}$ when $t = 2$ s.

(c) Evaluate

$$\begin{aligned}
\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} &= [(6\text{ m/s}^3)2t]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)3t^2]\hat{\mathbf{j}} \\
&= [(12\text{ m/s}^3)t]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)t^2]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{a}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{a}} &= [(12\text{ m/s}^3)(2\text{ s})]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)(2\ 2)^2]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}^2)]\hat{\mathbf{i}} + [-(336\text{ m/s}^2)]\hat{\mathbf{j}}.
\end{aligned}$$

E2-18 (a) Let *ui* point north, $\hat{\mathbf{j}}$ point east, and $\hat{\mathbf{k}}$ point up. The displacement is $(8.7\hat{\mathbf{i}} + 9.7\hat{\mathbf{j}} + 2.9\hat{\mathbf{k}})$ km. The average velocity is found by dividing each term by 3.4 hr; then

$$\vec{\mathbf{v}}_{\text{av}} = (2.6\hat{\mathbf{i}} + 2.9\hat{\mathbf{j}} + 0.85)\text{ km/hr}.$$

The magnitude of the average velocity is $\sqrt{2.6^2 + 2.9^2 + 0.85^2}$ km/hr $= 4.0$ km/hr.

(b) The horizontal velocity has a magnitude of $\sqrt{2.6^2 + 2.9^2}$ km/hr $= 3.9$ km/hr. The angle with the horizontal is given by $\theta = \tan^{-1}(0.85/3.9) = 13°$.

E2-19 (a) The derivative of the velocity is

$$\vec{\mathbf{a}} = [(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t]\hat{\mathbf{i}}$$

so the acceleration at $t = 3\,\text{s}$ is $\vec{\mathbf{a}} = (-18.0\,\text{m/s}^2)\hat{\mathbf{i}}$. (b) The acceleration is zero when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = 0$, or $t = 0.75\,\text{s}$. (c) The velocity is *never* zero; there is no way to "cancel" out the y component. (d) The speed equals 10 m/s when $10 = \sqrt{v_x^2 + 8^2}$, or $v_x = \pm 6.0\,\text{m/s}$. This happens when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = \pm 6.0\,\text{m/s}$, or when $t = 0\,\text{s}$.

E2-20 If v is constant then so is $v^2 = v_x^2 + v_y^2$. Take the derivative;

$$2v_x \frac{d}{dt} v_x + 2v_y \frac{d}{dt} v_y = 2(v_x a_x + v_y a_y).$$

But if the value is constant the derivative is zero.

E2-21 Let the actual flight time, as measured by the passengers, be T. There is some time difference between the two cities, call it ΔT = Namulevu time - Los Angeles time. The ΔT will be positive if Namulevu is east of Los Angeles. The time in Los Angeles can then be found from the time in Namulevu by subtracting ΔT.

The actual time of flight from Los Angeles to Namulevu is then the difference between when the plane lands (LA times) and when the plane takes off (LA time):

$$\begin{aligned} T &= (18{:}50 - \Delta T) - (12{:}50) \\ &= 6{:}00 - \Delta T, \end{aligned}$$

where we have written times in 24 hour format to avoid the AM/PM issue. The return flight time can be found from

$$\begin{aligned} T &= (18{:}50) - (1{:}50 - \Delta T) \\ &= 17{:}00 + \Delta T, \end{aligned}$$

where we have again changed to LA time for the purpose of the calculation.

(b) Now we just need to solve the two equations and two unknowns.

$$\begin{aligned} 17{:}00 + \Delta T &= 6{:}00 - \Delta T \\ 2\Delta T &= 6{:}00 - 17{:}00 \\ \Delta T &= -5{:}30. \end{aligned}$$

Since this is a negative number, Namulevu is located *west* of Los Angeles.

(a) $T = 6{:}00 - \Delta T = 11 : 30$, or eleven and a half hours.

(c) The distance traveled by the plane is given by $d = vt = (520 \text{ mi/hr})(11.5 \text{ hr}) = 5980$ mi. We'll draw a circle around Los Angeles with a radius of 5980 mi, and then we look for where it intersects with longitudes that would belong to a time zone ΔT away from Los Angeles. Since the Earth rotates once every 24 hours and there are 360 longitude degrees, then each hour corresponds to 15 longitude degrees, and then Namulevu must be located approximately $15^\circ \times 5.5 = 83^\circ$ west of Los Angeles, or at about longitude 160 east. The location on the globe is then latitude 5°, in the vicinity of Vanuatu.

When this exercise was originally typeset the times for the outbound and the inbound flights were inadvertently switched. I suppose that we could blame this on the airlines; nonetheless, when the answers were prepared for the back of the book the reversed numbers put Namulevu *east* of Los Angeles. That would put it in either the North Atlantic or Brazil.

E2-22 There is a three hour time zone difference. So the flight is seven hours long, but it takes 3 hr 51 min for the sun to travel same distance. Look for when the sunset distance has caught up with plane:

$$\begin{aligned} d_{\text{sunset}} &= d_{\text{plane}}, \\ v_{\text{sunset}}(t - 1{:}35) &= v_{\text{plane}} t, \\ (t - 1{:}35)/3{:}51 &= t/7{:}00, \end{aligned}$$

so $t = 3{:}31$ into flight.

E2-23 The distance is

$$d = vt = (112 \text{ km/hr})(1\,\text{s})/(3600 \text{ s/hr}) = 31\,\text{m}.$$

E2-24 The time taken for the ball to reach the plate is

$$t = \frac{d}{v} = \frac{(18.4\,\text{m})}{(160 \text{ km/hr})}(3600 \text{ s/hr})/(1000 \text{ m/km}) = 0.414\,\text{s}.$$

E2-25 Speed is distance traveled divided by time taken; this is equivalent to the inverse of the slope of the line in Fig. 2-32. The line appears to pass through the origin and through the point (1600 km, 80×10^6 y), so the speed is $v = 1600$ km/80×10^6 y$= 2 \times 10^{-5}$ km/y. Converting,

$$v = 2 \times 10^{-5}\text{km/y}\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right) = 2 \text{ cm/y}$$

E2-26 (a) For Maurice Greene $v_{\text{av}} = (100\,\text{m})/(9.81\,\text{m}) = 10.2\,\text{m/s}$. For Khalid Khannouchi,

$$v_{\text{av}} = \frac{(26.219 \text{ mi})}{(2.0950 \text{ hr})}\left(\frac{1609\,\text{m}}{1 \text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 5.594\,\text{m/s}.$$

(b) If Maurice Greene ran the marathon with an average speed equal to his average sprint speed then it would take him

$$t = \frac{(26.219 \text{ mi})}{10.2\,\text{m/s}}\left(\frac{1609\,\text{m}}{1 \text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 1.149 \text{ hr},$$

or 1 hour, 9 minutes.

E2-27 The time saved is the difference,

$$\Delta t = \frac{(700 \text{ km})}{(88.5 \text{ km/hr})} - \frac{(700 \text{ km})}{(104.6 \text{ km/hr})} = 1.22 \text{ hr},$$

which is about 1 hour 13 minutes.

E2-28 The ground elevation will increase by 35 m in a horizontal distance of

$$x = (35.0\,\text{m})/\tan(4.3^\circ) = 465\,\text{m}.$$

The plane will cover that distance in

$$t = \frac{(0.465 \text{ km})}{(1300 \text{ km/hr})}\left(\frac{3600\,\text{s}}{1\text{hr}}\right) = 1.3\,\text{s}.$$

E2-29 Let $v_1 = 40$ km/hr be the speed up the hill, t_1 be the time taken, and d_1 be the distance traveled in that time. We similarly define $v_2 = 60$ km/hr for the down hill trip, as well as t_2 and d_2. Note that $d_2 = d_1$.

$v_1 = d_1/t_1$ and $v_2 = d_2/t_2$. $v_{\rm av} = d/t$, where d total distance and t is the total time. The total distance is $d_1 + d_2 = 2d_1$. The total time t is just the sum of t_1 and t_2, so

$$\begin{aligned} v_{\rm av} &= \frac{d}{t} \\ &= \frac{2d_1}{t_1 + t_2} \\ &= \frac{2d_1}{d_1/v_1 + d_2/v_2} \\ &= \frac{2}{1/v_1 + 1/v_2}, \end{aligned}$$

Take the reciprocal of both sides to get a simpler looking expression

$$\frac{2}{v_{\rm av}} = \frac{1}{v_1} + \frac{1}{v_2}.$$

Then the average speed is 48 km/hr.

E2-30 (a) Average speed is *total* distance divided by *total* time. Then

$$v_{\rm av} = \frac{(240\ \text{ft}) + (240\ \text{ft})}{(240\ \text{ft})/(4.0\ \text{ft/s}) + (240\ \text{ft})/(10\ \text{ft/s})} = 5.7\ \text{ft/s}.$$

(b) Same approach, but different information given, so

$$v_{\rm av} = \frac{(60\ \text{s})(4.0\ \text{ft/s}) + (60\ \text{s})(10\ \text{ft/s})}{(60\ \text{s}) + (60\ \text{s})} = 7.0\ \text{ft/s}.$$

E2-31 The distance traveled is the total area under the curve. The "curve" has four regions: (I) a triangle from 0 to 2 s; (II) a rectangle from 2 to 10 s; (III) a trapezoid from 10 to 12 s; and (IV) a rectangle from 12 to 16 s.

The area underneath the curve is the sum of the areas of the four regions.

$$d = \frac{1}{2}(2\,\text{s})(8\,\text{m/s}) + (8.0\,\text{s})(8\,\text{m/s}) + \frac{1}{2}(2\,\text{s})(8\,\text{m/s} + 4\,\text{m/s}) + (4.0\,\text{s})(4\,\text{m/s}) = 100\,\text{m}.$$

E2-32 The acceleration is the slope of a velocity-time curve,

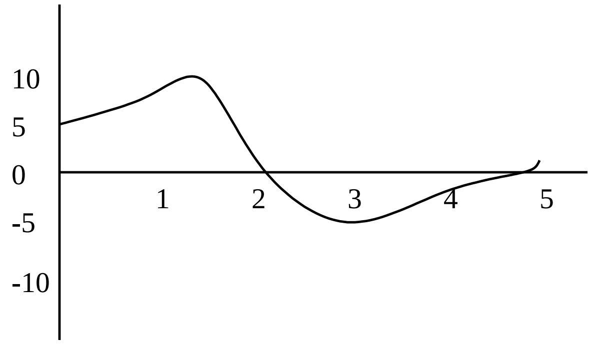
$$a = \frac{(8\,\text{m/s}) - (4\,\text{m/s})}{(10\,\text{s}) - (12\,\text{s})} = -2\,\text{m/s}^2.$$

E2-33 The initial velocity is $\vec{\mathbf{v}}_{\rm i} = (18\ \text{m/s})\hat{\mathbf{i}}$, the final velocity is $\vec{\mathbf{v}}_{\rm f} = (-30\ \text{m/s})\hat{\mathbf{i}}$. The average acceleration is then

$$\vec{\mathbf{a}}_{\rm av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_{\rm f} - \vec{\mathbf{v}}_{\rm i}}{\Delta t} = \frac{(-30\ \text{m/s})\hat{\mathbf{i}} - (18\ \text{m/s})\hat{\mathbf{i}}}{2.4\ \text{s}},$$

which gives $\vec{\mathbf{a}}_{\rm av} = (-20.0\ \text{m/s}^2)\hat{\mathbf{i}}$.

E2-34 Consider the figure below.



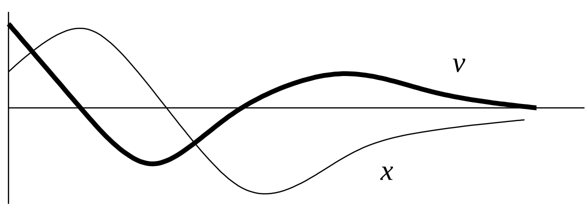
E2-35 (a) Up to A $v_x > 0$ and is constant. From A to B v_x is decreasing, but still positive. From B to C $v_x = 0$. From C to D $v_x < 0$, but $|v_x|$ is decreasing.

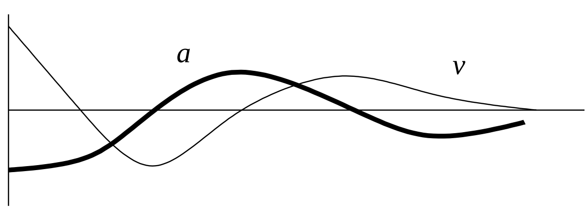
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

E2-36 (a) Up to A $v_x > 0$ and is decreasing. From A to B $v_x = 0$. From B to C $v_x > 0$ and is increasing. From C to D $v_x > 0$ and is constant.

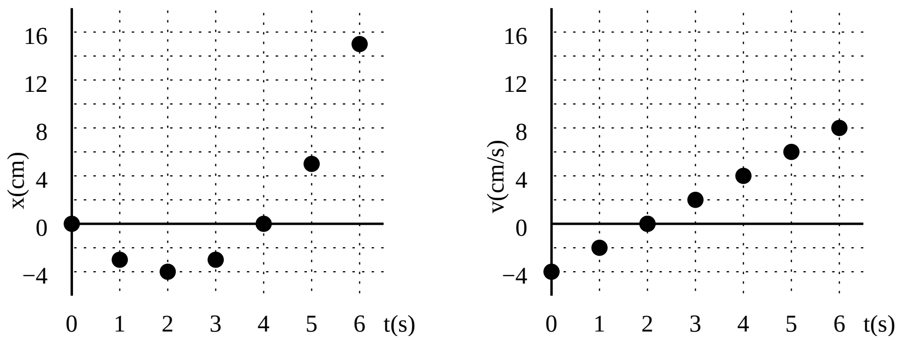
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

E2-37 Consider the figure below.





E2-38 Consider the figure below.



The acceleration is a constant 2 cm/s/s during the entire time interval.

E2-39 (a) A must have units of $\mathrm{m/s^2}$. B must have units of $\mathrm{m/s^3}$.

(b) The maximum positive x position occurs when $v_x = 0$, so

$$v_x = \frac{dx}{dt} = 2At - 3Bt^2$$

implies $v_x = 0$ when either $t = 0$ or $t = 2A/3B = 2(3.0\,\mathrm{m/s^2})/3(1.0\,\mathrm{m/s^3}) = 2.0\,\mathrm{s}$.

(c) Particle starts from rest, then travels in positive direction until $t = 2\,\mathrm{s}$, a distance of

$$x = (3.0\,\mathrm{m/s^2})(2.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^3 = 4.0\,\mathrm{m}.$$

Then the particle moves back to a final position of

$$x = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -16.0\,\mathrm{m}.$$

The total path followed was $4.0\,\mathrm{m} + 4.0\,\mathrm{m} + 16.0\,\mathrm{m} = 24.0\,\mathrm{m}$.

(d) The displacement is $-16.0\,\mathrm{m}$ as was found in part (c).

(e) The velocity is $v_x = (6.0\,\mathrm{m/s^2})t - (3.0\,\mathrm{m/s^3})t^2$. When $t = 0$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 1.0\,\mathrm{s}$, $v_x = 3.0\,\mathrm{m/s}$. When $t = 2.0\,\mathrm{s}$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 3.0\,\mathrm{s}$, $v_x = -9.0\,\mathrm{m/s}$. When $t = 4.0\,\mathrm{s}$, $v_x = -24.0\,\mathrm{m/s}$.

(f) The acceleration is the time derivative of the velocity,

$$a_x = \frac{dv_x}{dt} = (6.0\,\mathrm{m/s^2}) - (6.0\,\mathrm{m/s^3})t.$$

When $t = 0\,\mathrm{s}$, $a_x = 6.0\,\mathrm{m/s^2}$. When $t = 1.0\,\mathrm{s}$, $a_x = 0.0\,\mathrm{m/s^2}$. When $t = 2.0\,\mathrm{s}$, $a_x = -6.0\,\mathrm{m/s^2}$. When $t = 3.0\,\mathrm{s}$, $a_x = -12.0\,\mathrm{m/s^2}$. When $t = 4.0\,\mathrm{s}$, $a_x = -18.0\,\mathrm{m/s^2}$.

(g) The distance traveled was found in part (a) to be $-20\,\mathrm{m}$ The average speed during the time interval is then $v_{x,\mathrm{av}} = (-20\,\mathrm{m})/(2.0\,\mathrm{s}) = -10\,\mathrm{m/s}$.

E2-40 $v_{0x} = 0$, $v_x = 360$ km/hr $= 100\,\mathrm{m/s}$. Assuming constant acceleration the average velocity will be

$$v_{x,\mathrm{av}} = \frac{1}{2}(100\,\mathrm{m/s} + 0) = 50\,\mathrm{m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (1800\,\mathrm{m})/(50\,\mathrm{m/s}) = 36\,\mathrm{s}.$$

The acceleration is

$$a_x = 2x/t^2 = 2(1800\,\mathrm{m})/(36.0\,\mathrm{s})^2 = 2.78\,\mathrm{m/s^2}.$$

E2-41 (a) Apply Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (3.0 \times 10^7 \text{ m/s}) &= (0) + (9.8 \text{ m/s}^2)t, \\ 3.1 \times 10^6 \text{ s} &= t. \end{aligned}$$

(b) Apply Eq. 2-28 using an initial position of $x_0 = 0$,

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2} a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(9.8 \text{ m/s}^2)(3.1 \times 10^6 \text{ s})^2, \\ x &= 4.7 \times 10^{13} \text{ m}. \end{aligned}$$

E2-42 $v_{0x} = 0$ and $v_x = 27.8\,\text{m/s}$. Then

$$t = (v_x - v_{0x})/a = ((27.8\,\text{m/s}) - (0))\,/(50\,\text{m/s}^2) = 0.56\,\text{s}.$$

I want that car.

E2-43 The muon will travel for t seconds before it comes to a rest, where t is given by

$$t = (v_x - v_{0x})/a = \big((0) - (5.20 \times 10^6 \text{m/s})\big)\,/(-1.30 \times 10^{14} \text{m/s}^2) = 4 \times 10^{-8} \text{s}.$$

The distance traveled will be

$$x = \frac{1}{2} a_x t^2 + v_{0x} t = \frac{1}{2}(-1.30 \times 10^{14} \text{m/s}^2)(4 \times 10^{-8} \text{s})^2 + (5.20 \times 10^6 \text{m/s})(4 \times 10^{-8} \text{s}) = 0.104\,\text{m}.$$

E2-44 The average velocity of the electron was

$$v_{x,\text{av}} = \frac{1}{2}(1.5 \times 10^5 \text{ m/s} + 5.8 \times 10^6 \text{ m/s}) = 3.0 \times 10^6 \text{ m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (0.012\,\text{m})/(3.0 \times 10^6 \text{ m/s}) = 4.0 \times 10^{-9} \text{ s}.$$

The acceleration is

$$a_x = (v_x - v_{0x})/t = ((5.8 \times 10^6 \text{ m/s}) - (1.5 \times 10^5 \text{ m/s}))/(4.0 \times 10^{-9} \text{ s}) = 1.4 \times 10^{15} \text{m/s}^2.$$

E2-45 It will be easier to solve the problem if we change the units for the initial velocity,

$$v_{0x} = 1020 \frac{\text{km}}{\text{hr}} \left(\frac{1000 \text{ m}}{\text{km}}\right) \left(\frac{\text{hr}}{3600 \text{ s}}\right) = 283 \frac{\text{m}}{\text{s}},$$

and then applying Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (0) &= (283 \text{ m/s}) + a_x(1.4 \text{ s}), \\ -202 \text{ m/s}^2 &= a_x. \end{aligned}$$

The problem asks for this in terms of g, so

$$-202 \text{ m/s}^2 \left(\frac{g}{9.8 \text{ m/s}^2}\right) = 21g.$$

E2-46 Change miles to feet and hours to seconds. Then $v_x = 81$ ft/s and $v_{0x} = 125$ ft/s. The time is then

$$t = ((81 \text{ ft/s}) - (125 \text{ ft/s}))/(-17 \text{ ft/s}^2) = 2.6\,\text{s}.$$

E2-47 (a) The time to stop is

$$t = ((0 \text{ m/s}) - (24.6 \text{ m/s}))/(-4.92 \text{ m/s}^2) = 5.00\,\text{s}.$$

(b) The distance traveled is

$$x = \frac{1}{2}a_x t^2 + v_{0x}t = \frac{1}{2}(-4.92 \text{ m/s}^2)(5.00\,\text{s})^2 + (24.6 \text{ m/s})(5.00\,\text{s}) = 62\,\text{m}.$$

E2-48 Answer part (b) first. The average velocity of the arrow while decelerating is

$$v_{y,\text{av}} = \frac{1}{2}((0) + (260 \text{ ft/s})) = 130 \text{ ft/s}.$$

The time for the arrow to travel 9 inches (0.75 feet) is

$$t = (0.75 \text{ ft})/(130 \text{ ft/s}) = 5.8 \times 10^{-3}\text{s}.$$

(a) The acceleration of the arrow is then

$$a_y = (v_y - v_{0y})/t = ((0) - (260 \text{ ft/s}))/(5.8 \times 10^{-3}\text{s}) = -4.5 \times 10^4 \text{ ft/s}^2.$$

E2-49 The problem will be somewhat easier if the units are consistent, so we'll write the maximum speed as

$$1000\frac{\text{ft}}{\text{min}}\left(\frac{\text{min}}{60 \text{ s}}\right) = 16.7\,\frac{\text{ft}}{\text{s}}.$$

(a) We can find the time required for the acceleration from Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (16.7 \text{ ft/s}) &= (0) + (4.00 \text{ ft/s}^2)t, \\ 4.18 \text{ s} &= t. \end{aligned}$$

And from this and Eq 2-28 we can find the distance

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2}a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(4.00 \text{ ft/s}^2)(4.18 \text{ s})^2, \\ x &= 34.9 \text{ ft}. \end{aligned}$$

(b) The motion of the elevator is divided into three parts: acceleration from rest, constant speed motion, and deceleration to a stop. The total distance is given at 624 ft and in part (a) we found the distance covered during acceleration was 34.9 ft. By symmetry, the distance traveled during deceleration should also be 34.9 ft. The distance traveled at constant speed is then $(624 - 34.9 - 34.9)$ ft $= 554$ ft. The time required for the constant speed portion of the trip is found from Eq. 2-22, rewritten as

$$\Delta t = \frac{\Delta x}{v} = \frac{554 \text{ ft}}{16.7 \text{ ft/s}} = 33.2 \text{ s}.$$

The total time for the trip is the sum of times for the three parts: accelerating (4.18 s), constant speed (33.2 s), and decelerating (4.18 s). The total is 41.6 seconds.

E2-50 (a) The deceleration is found from

$$a_x = \frac{2}{t^2}(x - v_0 t) = \frac{2}{(4.0\,\text{s})^2}((34\,\text{m}) - (16\,\text{m/s})(4.0\,\text{s})) = -3.75\,\text{m/s}^2.$$

(b) The impact speed is

$$v_x = v_{0x} + a_x t = (16\,\text{m/s}) + (-3.75\,\text{m/s}^2)(4.0\,\text{s}) = 1.0\,\text{m/s}.$$

E2-51 Assuming the drops fall from rest, the time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1700\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 19\,\text{s}.$$

The velocity of the falling drops would be

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(19\,\text{s}) = 190\,\text{m/s},$$

or about 2/3 the speed of sound.

E2-52 Solve the problem out of order.

(b) The time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-120\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 4.9\,\text{s}.$$

(a) The speed at which the elevator hits the ground is

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(4.9\,\text{s}) = 48\,\text{m/s}.$$

(d) The time to fall half-way is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-60\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 3.5\,\text{s}.$$

(c) The speed at the half-way point is

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(3.5\,\text{s}) = 34\,\text{m/s}.$$

E2-53 The initial velocity of the "dropped" wrench would be zero. I choose vertical to be along the y axis with up as positive, which is the convention of Eq. 2-29 and Eq. 2-30. It turns out that it is much easier to solve part (b) before solving part (a).

(b) We solve Eq. 2-29 for the time of the fall.

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (-24.0\text{ m/s}) &= (0) - (9.8\text{ m/s}^2)t, \\ 2.45\text{ s} &= t. \end{aligned}$$

(a) Now we can use Eq. 2-30 to find the height from which the wrench fell.

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= y_0 + (0)(2.45\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(2.45\text{ s})^2, \\ 0 &= y_0 - 29.4\text{ m} \end{aligned}$$

We have set $y = 0$ to correspond to the final position of the wrench: on the ground. This results in an initial position of $y_0 = 29.4$ m; it is positive because the wrench was dropped from a point *above* where it landed.

E2-54 (a) It is easier to solve the problem from the point of view of an object which falls from the highest point. The time to fall from the highest point is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-53.7\,\mathrm{m})}{(-9.81\,\mathrm{m/s^2})}} = 3.31\,\mathrm{s}.$$

The speed at which the object hits the ground is

$$v_y = a_y t = (-9.81\,\mathrm{m/s^2})(3.31\,\mathrm{s}) = -32.5\,\mathrm{m/s}.$$

But the motion is symmetric, so the object must have been launched up with a velocity of $v_y = 32.5\,\mathrm{m/s}$.

(b) Double the previous answer; the time of flight is 6.62 s.

E2-55 (a) The time to fall the first 50 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-50\,\mathrm{m})}{(-9.8\,\mathrm{m/s^2})}} = 3.2\,\mathrm{s}.$$

(b) The *total* time to fall 100 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-100\,\mathrm{m})}{(-9.8\,\mathrm{m/s^2})}} = 4.5\,\mathrm{s}.$$

The time to fall through the second 50 meters is the difference, 1.3 s.

E2-56 The rock returns to the ground with an equal, but opposite, velocity. The acceleration is then

$$a_y = ((-14.6\,\mathrm{m/s}) - (14.6\,\mathrm{m/s}))/(7.72\,\mathrm{s}) = 3.78\,\mathrm{m/s^2}.$$

That would put them on Mercury.

E2-57 (a) Solve Eq. 2-30 for the initial velocity. Let the distances be measured from the ground so that $y_0 = 0$.

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (36.8\text{ m}) &= (0) + v_{0y}(2.25\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(2.25\text{ s})^2, \\ 36.8\text{ m} &= v_{0y}(2.25\text{ s}) - 24.8\text{ m}, \\ 27.4\text{ m/s} &= v_{0y}. \end{aligned}$$

(b) Solve Eq. 2-29 for the velocity, using the result from part (a).

$$\begin{aligned} v_y &= v_{0y} - gt, \\ v_y &= (27.4\text{ m/s}) - (9.8\text{ m/s}^2)(2.25\text{ s}), \\ v_y &= 5.4\text{ m/s}. \end{aligned}$$

(c) We need to solve Eq. 2-30 to find the height to which the ball rises, but we don't know how long it takes to get there. So we first solve Eq. 2-29, because we do know the velocity at the highest point ($v_y = 0$).

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= (27.4\text{ m/s}) - (9.8\text{ m/s}^2)t, \\ 2.8\text{ s} &= t. \end{aligned}$$

And then we find the height to which the object rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ y &= (0) + (27.4\ \text{m/s})(2.8\ \text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2)(2.8\ \text{s})^2, \\ y &= 38.3\text{m}. \end{aligned}$$

This is the height as measured from the ground; so the ball rises $38.3 - 36.8 = 1.5$ m above the point specified in the problem.

E2-58 The time it takes for the ball to fall 2.2 m is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-2.2\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.67\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.67\,\text{s}) = -6.6\,\text{m/s}.$$

The ball then bounces up to a height of 1.9 m. It is easier to solve the falling part of the motion, and then apply symmetry. The time is would take to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1.9\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.62\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.62\,\text{s}) = -6.1\,\text{m/s}.$$

But we are interested in when the ball moves up, so $v_y = 6.1\,\text{m/s}$.

The acceleration while in contact with the ground is

$$a_y = ((6.1\,\text{m/s}) - (-6.6\,\text{m/s}))/(0.096\,\text{s}) = 130\,\text{m/s}^2.$$

E2-59 The position as a function of time for the first object is

$$y_1 = -\frac{1}{2}gt^2,$$

The position as a function of time for the second object is

$$y_2 = -\frac{1}{2}g(t - 1\,\text{s})^2$$

The difference,

$$\Delta y = y_2 - y_1 = frac12g\left((2\,\text{s})t - 1\right),$$

is the set equal to $10\,\text{m}$, so $t = 1.52\,\text{s}$.

E2-60 Answer part (b) first.

(b) Use the quadratic equation to solve

$$(-81.3\,\text{m}) = \frac{1}{2}(-9.81\,\text{m/s}^2)t^2 + (12.4\,\text{m/s})t$$

for time. Get $t = -3.0\,\text{s}$ and $t = 5.53\,\text{s}$. Keep the positive answer.

(a) Now find final velocity from

$$v_y = (-9.8\,\text{m/s}^2)(5.53\,\text{s}) + (12.4\,\text{m/s}) = -41.8\,\text{m/s}.$$

E2-61 The total time the pot is visible is 0.54 s; the pot is visible for 0.27 s on the way down. We'll define the initial position as the highest point and make our measurements from there. Then $y_0 = 0$ and $v_{0y} = 0$. Define t_1 to be the time at which the *falling* pot passes the top of the window y_1, then $t_2 = t_1 + 0.27$ s is the time the pot passes the bottom of the window $y_2 = y_1 - 1.1$ m. We have two equations we can write, both based on Eq. 2-30,

$$\begin{aligned} y_1 &= y_0 + v_{0y}t_1 - \frac{1}{2}gt_1^2, \\ y_1 &= (0) + (0)t_1 - \frac{1}{2}gt_1^2, \end{aligned}$$

and

$$\begin{aligned} y_2 &= y_0 + v_{0y}t_2 - \frac{1}{2}gt_2^2, \\ y_1 - 1.1 \text{ m} &= (0) + (0)t_2 - \frac{1}{2}g(t_1 + 0.27 \text{ s})^2, \end{aligned}$$

Isolate y_1 in this last equation and then set the two expressions equal to each other so that we can solve for t_1,

$$\begin{aligned} -\frac{1}{2}gt_1^2 &= 1.1 \text{ m} - \frac{1}{2}g(t_1 + 0.27 \text{ s})^2, \\ -\frac{1}{2}gt_1^2 &= 1.1 \text{ m} - \frac{1}{2}g(t_1^2 + [0.54 \text{ s}]t_1 + 0.073 \text{ s}^2), \\ 0 &= 1.1 \text{ m} - \frac{1}{2}g([0.54 \text{ s}]t_1 + 0.073 \text{ s}^2). \end{aligned}$$

This last line can be directly solved to yield $t_1 = 0.28$ s as the time when the falling pot passes the top of the window. Use this value in the first equation above and we can find $y_1 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.28 \text{ s})^2 = -0.38$ m. The negative sign is because the top of the window is beneath the highest point, so the pot must have risen to 0.38 m above the top of the window.

P2-1 (a) The net shift is $\sqrt{(22\,\text{m})^2 + (17\,\text{m})^2)} = 28\,\text{m}$.
(b) The vertical displacement is $(17\,\text{m})\sin(52^\circ) = 13\,\text{m}$.

P2-2 Wheel "rolls" through half of a turn, or $\pi r = 1.41\,\text{m}$. The vertical displacement is $2r = 0.90\,\text{m}$. The net displacement is

$$\sqrt{(1.41\,\text{m})^2 + (0.90\,\text{m})^2} = 1.67\,\text{m}0.$$

The angle is

$$\theta = \tan^{-1}(0.90\,\text{m})/(1.41\,\text{m}) = 33^\circ.$$

P2-3 We align the coordinate system so that the origin corresponds to the starting position of the fly and that all positions inside the room are given by positive coordinates.
(a) The displacement vector can just be written,

$$\Delta\vec{\mathbf{r}} = (10 \text{ ft})\hat{\mathbf{i}} + (12 \text{ ft})\hat{\mathbf{j}} + (14 \text{ ft})\hat{\mathbf{k}}.$$

(b) The magnitude of the displacement vector is $|\Delta\vec{\mathbf{r}}| = \sqrt{10^2 + 12^2 + 14^2}$ ft= 21 ft.

(c) The straight line distance between two points is the shortest possible distance, so the length of the path taken by the fly must be greater than or equal to 21 ft.

(d) If the fly walks it will need to cross two faces. The shortest path will be the diagonal across these two faces. If the lengths of sides of the room are l_1, l_2, and l_3, then the diagonal length across two faces will be given by

$$\sqrt{(l_1 + l_2)^2 + l_3^2},$$

where we want to choose the l_i from the set of 10 ft, 12 ft, and 14 ft that will minimize the length. The minimum distance is when $l_1 = 10$ ft, $l_2 = 12$ ft, and $l_3 = 14$. Then the minimal distance the fly would *walk* is 26 ft.

P2-4 Choose vector $\vec{\mathbf{a}}$ to lie on the x axis. Then $\vec{\mathbf{a}} = a\hat{\mathbf{i}}$ and $\vec{\mathbf{b}} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}}$ where $b_x = b\cos\theta$ and $b_y = b\sin\theta$. The sum then has components

$$r_x = a + b\cos\theta \text{ and } r_y = b\sin\theta.$$

Then

$$\begin{aligned} r^2 &= (a + b\cos\theta)^2 + (b\sin\theta)^2, \\ &= a^2 + 2ab\cos\theta + b^2. \end{aligned}$$

P2-5 (a) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(35.0 \text{ mi/hr})(t/2) + (55.0 \text{ mi/hr})(t/2)}{(t/2) + (t/2)} = 45.0 \text{ mi/hr}.$$

(b) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(d/2) + (d/2)}{(d/2)/(35.0 \text{ mi/hr}) + (d/2)/(55.0 \text{ mi/hr})} = 42.8 \text{ mi/hr}.$$

(c) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{d + d}{(d)/(45.0 \text{ mi/hr}) + (d)/(42.8 \text{ mi/hr})} = 43.9 \text{ mi/hr}$$

P2-6 (a) We'll do just one together. How about $t = 2.0\,\text{s}$?

$$x = (3.0\,\text{m/s})(2.0\,\text{s}) + (-4.0\,\text{m/s}^2)(2.0\,\text{s})^2 + (1.0\,\text{m/s}^3)(2.0\,\text{s})^3 = -2.0\,\text{m}.$$

The rest of the values are, starting from $t = 0$, $x = 0.0$ m, 0.0 m, -2.0 m, 0.0 m, and 12.0 m.

(b) Always final minus initial. The answers are $x_\text{f} - x_\text{i} = -2.0\,\text{m} - 0.0\,\text{m} = -2.0\,\text{m}$ and $x_\text{f} - x_\text{i} = 12.0\,\text{m} - 0.0\,\text{m} = 12.0\,\text{m}$.

(c) Always displacement divided by (change in) time.

$$v_{\text{av}} = \frac{(12.0\,\text{m}) - (-2.0\,\text{m})}{(4.0\,\text{s}) - (2.0\,\text{s})} = 7.0\,\text{m/s},$$

and

$$v_{\text{av}} = \frac{(0.0\,\text{m}) - (0.0\,\text{m})}{(3.0\,\text{s}) - (0.0\,\text{s})} = 0.0\,\text{m/s}.$$

P2-7 (a) Assume the bird has no size, the trains have some separation, and the bird is just leaving one of the trains. The bird will be able to fly from one train to the other *before* the two trains collide, regardless of how close together the trains are. After doing so, the bird is now on the other train, the trains are still separated, so once again the bird can fly between the trains before they collide. This process can be repeated every time the bird touches one of the trains, so the bird will make an infinite number of trips between the trains.

(b) The trains collide in the middle; therefore the trains collide after (51 km)/(34 km/hr) = 1.5 hr. The bird was flying with constant speed this entire time, so the distance flown by the bird is (58 km/hr)(1.5 hr) = 87 km.

P2-8 (a) Start with a perfect square:

$$\begin{aligned}
(v_1 - v_2)^2 &> 0, \\
v_1^2 + v_2^2 &> 2v1v2, \\
(v_1^2 + v_2^2)t_1 t_2 &> 2v_1 v_2 t_1 t_2, \\
d_1^2 + d_2^2 + (v_1^2 + v_2^2)t_1 t_2 &> d_1^2 + d_2^2 + 2v_1 v_2 t_1 t_2, \\
(v_1^2 t_1 + v_2^2 t_2)(t_1 + t_2) &> (d_1 + d_2)^2, \\
\frac{v_1^2 t_1 + v_2^2 t_2}{d_1 + d_2} &> \frac{d_1 + d_2}{t_1 + t_2}, \\
\frac{v_1 d_1 + v_2 d_2}{d_1 + d_2} &> \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}
\end{aligned}$$

Actually, it only works if $d_1 + d_2 > 0$!

(b) If $v_1 = v_2$.

P2-9 (a) The average velocity during the time interval is $v_{\text{av}} = \Delta x/\Delta t$, or

$$v_{\text{av}} = \frac{(A + B(3\,\text{s})^3) - (A + B(2\,\text{s})^3)}{(3\,\text{s}) - (2\,\text{s})} = (1.50\ \text{cm/s}^3)(19\,\text{s}^3)/(1\,\text{s}) = 28.5\ \text{cm/s}.$$

(b) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2\,\text{s})^2 = 18\ \text{cm/s}$.

(c) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(3\,\text{s})^2 = 40.5\ \text{cm/s}$.

(d) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2.5\,\text{s})^2 = 28.1\ \text{cm/s}$.

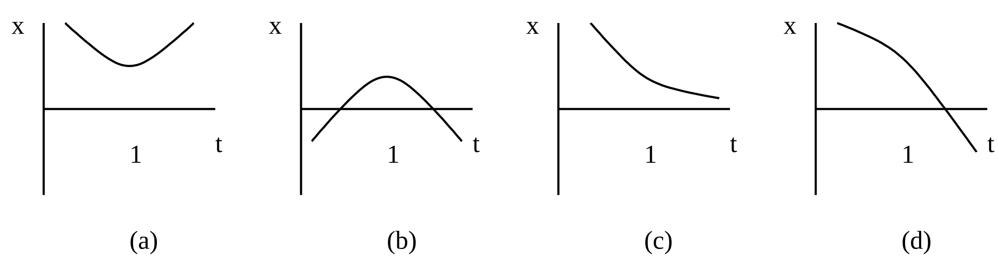
(e) The midway position is $(x_{\text{f}} + x_{\text{i}})/2$, or

$$x_{\text{mid}} = A + B[(3\,\text{s})^3 + (2\,\text{s})^3)]/2 = A + (17.5\,\text{s}^3)B.$$

This occurs when $t = \sqrt[3]{(17.5\,\text{s}^3)}$. The instantaneous velocity at this point is

$$v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(\sqrt[3]{(17.5\,\text{s}^3)})^2 = 30.3\ \text{cm/s}.$$

P2-10 Consider the figure below.



P2-11 (a) The average velocity is displacement divided by change in time,

$$v_{\rm av} = \frac{(2.0~{\rm m/s^3})(2.0~{\rm s})^3 - (2.0~{\rm m/s^3})(1.0~{\rm s})^3}{(2.0~{\rm s}) - (1.0~{\rm s})} = \frac{14.0~{\rm m}}{1.0~{\rm s}} = 14.0~{\rm m/s}.$$

The average acceleration is the change in velocity. So we need an expression for the velocity, which is the time derivative of the position,

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.0~{\rm m/s^3})t^3 = (6.0~{\rm m/s^3})t^2.$$

From this we find average acceleration

$$a_{\rm av} = \frac{(6.0~{\rm m/s^3})(2.0~{\rm s})^2 - (6.0~{\rm m/s^3})(1.0~{\rm s})^2}{(2.0~{\rm s}) - (1.0~{\rm s})} = \frac{18.0~{\rm m/s}}{1.0~{\rm s}} = 18.0~{\rm m/s^2}.$$

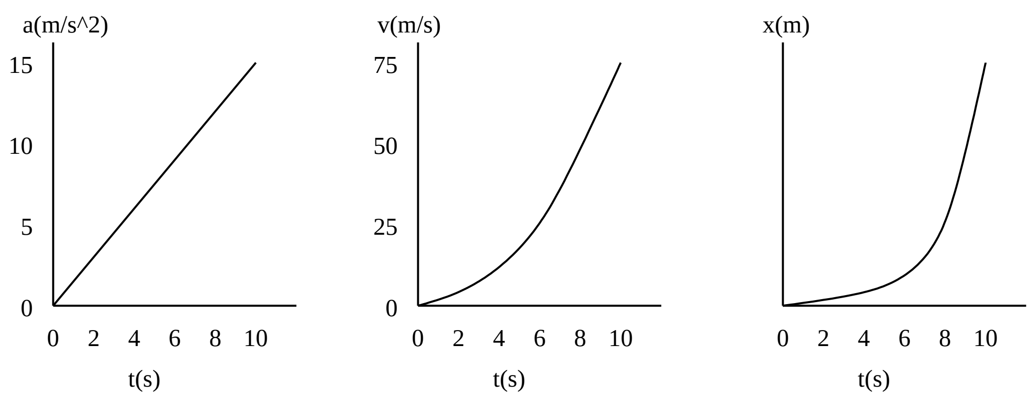
(b) The instantaneous velocities can be found directly from $v = (6.0~{\rm m/s^2})t^2$, so $v(2.0~{\rm s}) = 24.0$ m/s and $v(1.0~{\rm s}) = 6.0$ m/s. We can get an expression for the instantaneous acceleration by taking the time derivative of the velocity

$$a = \frac{dv}{dt} = \frac{d}{dt}(6.0~{\rm m/s^3})t^2 = (12.0~{\rm m/s^3})t.$$

Then the instantaneous accelerations are $a(2.0~{\rm s}) = 24.0~{\rm m/s^2}$ and $a(1.0~{\rm s}) = 12.0~{\rm m/s^2}$

(c) Since the motion is monotonic we expect the average quantities to be somewhere between the instantaneous values at the endpoints of the time interval. Indeed, that is the case.

P2-12 Consider the figure below.



P2-13 Start with $v_{\rm f} = v_{\rm i} + at$, but $v_{\rm f} = 0$, so $v_{\rm i} = -at$, then

$$x = \frac{1}{2}at^2 + v_{\rm i}t = \frac{1}{2}at^2 - at^2 = -\frac{1}{2}at^2,$$

so $t = \sqrt{-2x/a} = \sqrt{-2(19.2~{\rm ft})/(-32~{\rm ft/s^2})} = 1.10\,{\rm s}$. Then $v_{\rm i} = -(-32~{\rm ft/s^2})(1.10\,{\rm s}) = 35.2$ ft/s. Converting,

$$35.2~{\rm ft/s}(1/5280~{\rm mi/ft})(3600~{\rm s/h}) = 24~{\rm mi/h}.$$

P2-14 (b) The average speed during while traveling the 160 m is

$$v_{\rm av} = (33.0\,{\rm m/s} + 54.0\,{\rm m/s})/2 = 43.5\,{\rm m/s}.$$

The time to travel the 160 m is $t = (160\,{\rm m})/(43.5\,{\rm m/s}) = 3.68\,{\rm s}$.

(a) The acceleration is

$$a = \frac{2x}{t^2} - \frac{2v_i}{t} = \frac{2(160\,{\rm m})}{(3.68\,{\rm s})^2} - \frac{2(33.0\,{\rm m/s})}{(3.68\,{\rm s})} = 5.69\,{\rm m/s^2}.$$

(c) The time required to get up to a speed of 33 m/s is

$$t = v/a = (33.0\,{\rm m/s})/(5.69\,{\rm m/s^2}) = 5.80\,{\rm s}.$$

(d) The distance moved from start is

$$d = \frac{1}{2}at^2 = \frac{1}{2}(5.69\,{\rm m/s^2})(5.80\,{\rm s})^2 = 95.7\,{\rm m}.$$

P2-15 (a) The distance traveled during the reaction time happens at constant speed; $t_{\rm reac} = d/v = (15\,{\rm m})/(20\,{\rm m/s}) = 0.75\,{\rm s}$.

(b) The braking distance is proportional to the speed squared (look at the numbers!) and in this case is $d_{\rm brake} = v^2/(20\,{\rm m/s^2})$. Then $d_{\rm brake} = (25\,{\rm m/s})^2/(20\,{\rm m/s^2}) = 31.25\,{\rm m}$. The reaction time distance is $d_{\rm reac} = (25\,{\rm m/s})(0.75\,{\rm s}) = 18.75\,{\rm m}$. The stopping distance is then 50 m.

P2-16 (a) For the car $x_{\rm c} = a_{\rm c}t^2/2$. For the truck $x_{\rm t} = v_{\rm t}t$. Set both x_i to the same value, and then substitute the time from the truck expression:

$$x = a_{\rm c}t^2/2 = a_{\rm c}(x/v_{\rm t})^2/2,$$

or

$$x = 2{v_{\rm t}}^2/a_{\rm c} = 2(9.5\,{\rm m/s})^2/(2.2\,{\rm m/s}) = 82\,{\rm m}.$$

(b) The speed of the car will be given by $v_{\rm c} = a_{\rm c}t$, or

$$v_{\rm c} = a_{\rm c}t = a_{\rm c}x/v_{\rm t} = (2.2\,{\rm m/s})(82\,{\rm m})/(9.5\,{\rm m/s}) = 19\,{\rm m/s}.$$

P2-17 The runner covered a distance d_1 in a time interval t_1 during the acceleration phase and a distance d_2 in a time interval t_2 during the constant speed phase. Since the runner started from rest we know that the constant speed is given by $v = at_1$, where a is the runner's acceleration.

The distance covered during the acceleration phase is given by

$$d_1 = \frac{1}{2}at_1^2.$$

The distance covered during the constant speed phase can also be found from

$$d_2 = vt_2 = at_1t_2.$$

We want to use these two expressions, along with $d_1 + d_2 = 100$ m and $t_2 = (12.2\text{ s}) - t_1$, to get

$$\begin{aligned}100\text{ m} &= d_1 + d_2 = \frac{1}{2}at_1^2 + at_1(12.2\text{ s} - t_1),\\ &= -\frac{1}{2}at_1^2 + a(12.2\text{ s})t_1,\\ &= -(1.40\text{ m/s}^2)t_1^2 + (34.2\text{ m/s})t_1.\end{aligned}$$

This last expression is quadratic in t_1, and is solved to give $t_1 = 3.40\,\text{s}$ or $t_1 = 21.0\,\text{s}$. Since the race only lasted 12.2 s we can ignore the second answer.

(b) The distance traveled during the acceleration phase is then

$$d_1 = \frac{1}{2}at_1^2 = (1.40\ \text{m/s}^2)(3.40\,\text{s})^2 = 16.2\,\text{m}.$$

P2-18 (a) The ball will return to the ground with the same speed it was launched. Then the total time of flight is given by

$$t = (v_\text{f} - v_\text{i})/g = (-25\,\text{m/s} - 25\,\text{m/s})/(9.8\,\text{m/s}^2) = 5.1\,\text{s}.$$

(b) For small quantities we can think in terms of derivatives, so

$$\delta t = (\delta v_\text{f} - \delta v_\text{i})/g,$$

or $\tau = 2\epsilon/g$.

P2-19 Use $y = -gt^2/2$, but only keep the absolute value. Then $y_{50} = (9.8\,\text{m/s}^2)(0.05\,\text{s})^2/2 =$ 1.2 cm; $y_{100} = (9.8\,\text{m/s}^2)(0.10\,\text{s})^2/2 = 4.9$ cm; $y_{150} = (9.8\,\text{m/s}^2)(0.15\,\text{s})^2/2 = 11$ cm; $y_{200} = (9.8\,\text{m/s}^2)(0.20\,\text{s})^2/2 = 20$ cm; $y_{250} = (9.8\,\text{m/s}^2)(0.25\,\text{s})^2/2 = 31$ cm.

P2-20 The truck will move 12 m in $(12\,\text{m})/(55\ \text{km/h}) = 0.785\,\text{s}$. The apple will fall $y = -gt^2/2 = -(9.81\,\text{m/s}^2)(0.785\,\text{s})^2/2 = -3.02\,\text{m}$.

P2-21 The rocket travels a distance $d_1 = \frac{1}{2}at_1^2 = \frac{1}{2}(20\ \text{m/s}^2)(60\ \text{s})^2 = 36,000$ m during the acceleration phase; the rocket velocity at the end of the acceleration phase is $v = at = (20\ \text{m/s}^2)(60\ \text{s}) =$ 1200 m/s. The second half of the trajectory can be found from Eqs. 2-29 and 2-30, with $y_0 = 36,000$ m and $v_{0y} = 1200$ m/s.

(a) The highest point of the trajectory occurs when $v_y = 0$, so

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= (1200\ \text{m/s}) - (9.8\ \text{m/s}^2)t, \\ 122\ \text{s} &= t. \end{aligned}$$

This time is used to find the height to which the rocket rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ &= (36000\ \text{m}) + (1200\ \text{m/s})(122\text{s}) - \frac{1}{2}(9.8\ \text{m/s}^2)(122\ \text{s})^2 = 110000\ \text{m}. \end{aligned}$$

(b) The easiest way to find the total time of flight is to solve Eq. 2-30 for the time when the rocket has returned to the ground. Then

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= (36000\ \text{m}) + (1200\ \text{m/s})t - \frac{1}{2}(9.8\ \text{m/s}^2)t^2. \end{aligned}$$

This quadratic expression has two solutions for t; one is negative so we don't need to worry about it, the other is $t = 270\,\text{s}$. This is the free-fall part of the problem, to find the total time we need to add on the 60 seconds of accelerated motion. The total time is then 330 seconds.

P2-22 (a) The time required for the player to "fall" from the highest point a distance of $y = 15$ cm is $\sqrt{2y/g}$; the total time spent in the top 15 cm is twice this, or $2\sqrt{2y/g} = 2\sqrt{2(0.15\,\text{m})/(9.81\,\text{m/s})} = 0.350\,\text{s}$.

(b) The time required for the player to "fall" from the highest point a distance of 76 cm is $\sqrt{2(0.76\,\text{m})/(9.81\,\text{m/s})} = 0.394\,\text{s}$, the time required for the player to fall from the highest point a distance of $(76 - 15 = 61)$ cm is $\sqrt{2(0.61\,\text{m})/g} = 0.353\,\text{s}$. The time required to fall the bottom 15 cm is the difference, or 0.041 s. The time spent in the bottom 15 cm is twice this, or 0.081 s.

P2-23 (a) The average speed between A and B is $v_{\text{av}} = (v + v/2)/2 = 3v/4$. We can also write $v_{\text{av}} = (3.0\,\text{m})/\Delta t = 3v/4$. Finally, $v/2 = v - g\Delta t$. Rearranging, $v/2 = g\Delta t$. Combining all of the above,

$$\frac{v}{2} = g\left(\frac{4(3.0\,\text{m})}{3v}\right) \text{ or } v^2 = (8.0\,\text{m})g.$$

Then $v = \sqrt{(8.0\,\text{m})(9.8\,\text{m/s}^2)} = 8.85\,\text{m/s}$.

(b) The time to the highest point above B is $v/2 = gt$, so the distance above B is

$$y = -\frac{g}{2}t^2 + \frac{v}{2}t = -\frac{g}{2}\left(\frac{v}{2g}\right)^2 + \frac{v}{2}\left(\frac{v}{2g}\right) = \frac{v^2}{8g}.$$

Then $y = (8.85\,\text{m/s})^2/(8(9.8\,\text{m/s}^2)) = 1.00\,\text{m}$.

P2-24 (a) The time in free fall is $t = \sqrt{-2y/g} = \sqrt{-2(-145\,\text{m})/(9.81\,\text{m/s}^2)} = 5.44\,\text{s}$.

(b) The speed at the bottom is $v = -gt = -(9.81\,\text{m/s}^2)(5.44\,\text{s}) = -53.4\,\text{m/s}$.

(c) The time for deceleration is given by $v = -25gt$, or $t = -(-53.4\,\text{m/s})/(25 \times 9.81\,/\text{s}^2) = 0.218\,\text{s}$. The distance through which deceleration occurred is

$$y = \frac{25g}{2}t^2 + vt = (123\,\text{m/s}^2)(0.218\,\text{s})^2 + (-53.4\,\text{m/s})(0.218\,\text{s}) = -5.80\,\text{m}.$$

P2-25 Find the time she fell from Eq. 2-30,

$$(0 \text{ ft}) = (144 \text{ ft}) + (0)t - \frac{1}{2}(32 \text{ ft/s}^2)t^2,$$

which is a simple quadratic with solutions $t = \pm 3.0$ s. Only the positive solution is of interest. Use this time in Eq. 2-29 to find her speed when she hit the ventilator box,

$$v_y = (0) - (32 \text{ ft/s}^2)(3.0 \text{ s}) = -96 \text{ ft/s}.$$

This becomes the initial velocity for the deceleration motion, so her average speed during deceleration is given by Eq. 2-27,

$$v_{\text{av},y} = \frac{1}{2}(v_y + v_{0y}) = \frac{1}{2}((0) + (-96 \text{ ft/s})) = -48 \text{ ft/s}.$$

This average speed, used with the distance of 18 in (1.5 ft), can be used to find the time of deceleration

$$v_{\text{av},y} = \Delta y/\Delta t,$$

and putting numbers into the expression gives $\Delta t = 0.031$ s. We actually used $\Delta y = -1.5$ ft, where the negative sign indicated that she was still moving downward. Finally, we use this in Eq. 2-26 to find the acceleration,

$$(0) = (-96 \text{ ft/s}) + a(0.031 \text{ s}),$$

which gives $a = +3100$ ft/s^2. In terms of g this is $a = 97g$, which can be found by multiplying through by $1 = g/(32 \text{ ft/s}^2)$.

P2-26 Let the speed of the disk when it comes into contact with the ground be v_1; then the average speed during the deceleration process is $v_1/2$; so the time taken for the deceleration process is $t_1 = 2d/v_1$, where $d = -2\,\mathrm{mm}$. But d is also give by $d = at_1^2/2 + v_1 t_1$, so

$$d = \frac{100g}{2}\left(\frac{2d}{v_1}\right)^2 + v_1\left(\frac{2d}{v_1}\right) = 200g\frac{d^2}{v_1^2} + 2d,$$

or $v_1^2 = -200gd$. The negative signs *are* necessary!.

The disk was dropped from a height $h = -y$ and it first came into contact with the ground when it had a speed of v_1. Then the average speed is $v_1/2$, and we can repeat most of the above (except $a = -g$ instead of $100g$), and then the time to fall is $t_2 = 2y/v_1$,

$$y = \frac{g}{2}\left(\frac{2y}{v_1}\right)^2 + v_1\left(\frac{2y}{v_1}\right) = 2g\frac{y^2}{v_1^2} + 2y,$$

or $v_1^2 = -2gy$. The negative signs *are* necessary!.

Equating, $y = 100d = 100(-2\,\mathrm{mm}) = -0.2\,\mathrm{m}$, so $h = 0.2\,\mathrm{m}$. Note that although $100g$'s sounds like plenty, you still shouldn't be dropping your hard disk drive!

P2-27 Measure from the feet! Jim is 2.8 cm tall in the photo, so 1 cm on the photo is 60.7 cm in real-life. Then Jim has fallen a distance $y_1 = -3.04\,\mathrm{m}$ while Clare has fallen a distance $y_2 = -5.77\,\mathrm{m}$. Clare jumped first, and the time she has been falling is t_2; Jim jumped seconds, the time he has been falling is $t_1 = t_2 - \Delta t$. Then $y_2 = -gt_2^2/2$ and $y_1 = -gt_1^2/2$, or $t_2 = \sqrt{-2y_2/g} = \sqrt{-2(-5.77\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 1.08\,\mathrm{s}$ and $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(3.04\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 0.79\,\mathrm{s}$. So Jim waited $0.29\,\mathrm{s}$.

P2-28 (a) Assuming she starts from rest and has a speed of v_1 when she opens her chute, then her average speed while falling freely is $v_1/2$, and the time taken to fall $y_1 = -52.0\,\mathrm{m}$ is $t_1 = 2y_1/v_1$. Her speed v_1 is given by $v_1 = -gt_1$, or $v_1^2 = -2gy_1$. Then $v_1 = -\sqrt{-2(9.81\,\mathrm{m/s^2})(-52.0\,\mathrm{m})} = -31.9\,\mathrm{m/s}$. We must use the *negative* answer, because she fall down! The time in the air is then $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(52.0\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 3.26\,\mathrm{s}$.

Her final speed is $v_2 = -2.90\,\mathrm{m/s}$, so the time for the deceleration is $t_2 = (v_2 - v_1)/a$, where $a = 2.10\,\mathrm{m/s^2}$. Then $t_2 = (-2.90\,\mathrm{m/s} - -31.9\,\mathrm{m/s})/(2.10\,\mathrm{m/s^2}) = 13.8\,\mathrm{s}$.

Finally, the total time of flight is $t = t_1 + t_2 = 3.26\,\mathrm{s} + 13.8\,\mathrm{s} = 17.1\,\mathrm{s}$.

(b) The distance fallen during the deceleration phase is

$$y_2 = -\frac{g}{2}t_2^2 + v_1 t_2 = -\frac{(2.10\,\mathrm{m/s^2})}{2}(13.8\,\mathrm{s})^2 + (-31.9\,\mathrm{m/s})(13.8\,\mathrm{s}) = -240\,\mathrm{m}.$$

The total distance fallen is $y = y_1 + y_2 = -52.0\,\mathrm{m} - 240\,\mathrm{m} = -292\,\mathrm{m}$. It is negative because she was falling down.

P2-29 Let the speed of the bearing be v_1 at the top of the windows and v_2 at the bottom. These speeds are related by $v_2 = v_1 - gt_{12}$, where $t_{12} = 0.125\,\mathrm{s}$ is the time between when the bearing is at the top of the window and at the bottom of the window. The average speed is $v_{\mathrm{av}} = (v_1 + v_2)/2 = v_1 - gt_{12}/2$. The distance traveled in the time t_{12} is $y_{12} = -1.20\,\mathrm{m}$, so

$$y_{12} = v_{\mathrm{av}} t_{12} = v_1 t_{12} - gt_{12}^2/2,$$

and expression that can be solved for v_1 to yield

$$v_1 = \frac{y_{12} + gt_{12}^2/2}{t_{12}} = \frac{(-1.20\,\mathrm{m}) + (9.81\,\mathrm{m/s^2})(0.125\,\mathrm{s})^2/2}{(0.125\,\mathrm{s})} = -8.99\,\mathrm{m/s}.$$

Now that we know v_1 we can find the height of the building above the top of the window. The time the object has fallen to get to the top of the window is $t_1 = -v_1/g = -(-8.99\,\text{m/s})/(9.81\,\text{m/s}^2) = 0.916\,\text{m}$.

The total time for falling is then $(0.916\,\text{s}) + (0.125\,\text{s}) + (1.0\,\text{s}) = 2.04\,\text{s}$. Note that we remembered to divide the last time by two! The total distance from the top of the building to the bottom is then

$$y = -gt^2/2 = -(9.81\,\text{m/s}^2)(2.04\,\text{s})^2/2 = 20.4\,\text{m}.$$

P2-30 Each ball falls from a highest point through a distance of 2.0 m in

$$t = \sqrt{-2(2.0\,\text{m})/(9.8\,\text{m/s}^2)} = 0.639\,\text{s}.$$

The time each ball spends in the air is twice this, or 1.28 s. The frequency of tosses per ball is the reciprocal, $f = 1/T = 0.781\,\text{s}^{-1}$. There are five ball, so we multiply this by 5, but there are two hands, so we then divide that by 2. The tosses per hand per second then requires a factor 5/2, and the tosses per hand per minute is 60 times this, or 117.

P2-31 Assume each hand can toss n objects per second. Let τ be the amount of time that any one object is in the air. Then $2n\tau$ is the number of objects that are in the air at any time, where the "2" comes from the fact that (most?) jugglers have two hands. We'll estimate n, but τ can be found from Eq. 2-30 for an object which falls a distance h from rest:

$$0 = h + (0)t - \frac{1}{2}gt^2,$$

solving, $t = \sqrt{2h/g}$. But τ is twice this, because the object had to go up before it could come down. So the number of objects that can be juggled is

$$4n\sqrt{2h/g}$$

We estimate $n = 2$ tosses/second. So the maximum number of objects one could juggle to a height h would be

$$3.6\sqrt{h/\text{meters}}.$$

P2-32 (a) We need to look up the height of the leaning tower to solve this! If the height is $h = 56\,\text{m}$, then the time taken to fall a distance $h = -y_1$ is $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(-56\,\text{m})/(9.81\,\text{m/s}^2)} = 3.4\,\text{s}$. The second object, however, has only fallen a a time $t_2 = t_1 - \Delta t = 3.3\,\text{s}$, so the distance the second object falls is $y_2 = -gt_2^2/2 = -(9.81\,\text{m/s}^2)(3.3\,\text{s})^2/2 = 53.4$. The difference is $y_1 - y_2 = 2.9\,\text{m}$.

(b) If the vertical separation is $\Delta y = 0.01\,\text{m}$, then we can approach this problem in terms of differentials,

$$\delta y = at\,\delta t,$$

so $\delta t = (0.01\,\text{m})/[((9.81\,\text{m/s}^2)(3.4\,\text{s})] = 3\times 10^{-4}\,\text{s}$.

P2-33 Use symmetry, and focus on the path from the highest point downward. Then $\Delta t_U = 2t_U$, where t_U is the time from the highest point to the upper level. A similar expression exists for the lower level, but replace U with L. The distance from the highest point to the upper level is $y_U = -gt_U^2/2 = -g(\Delta t_U/2)^2/2$. The distance from the highest point to the lower level is $y_L = -gt_L^2/2 = -g(\Delta t_L/2)^2/2$. Now $H = y_U - y_L = -g\Delta t_U^2/8 - -g\Delta t_L^2/8$, which can be written as

$$g = \frac{8H}{\Delta t_L^2 - \Delta t_U^2}.$$