MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper.*

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C***

***Section A:** Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

***Section B:** Internal choice has been provided in two questions of four marks each.*

***Section C:** Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 [10×2]

(i) The binary operation $*: R \times R \rightarrow R$ is defined as $a * b = 2a + b$.

Find $(2 * 3) * 4$.

(ii) If $A = \begin{pmatrix} 5 & a \\ b & 0 \end{pmatrix}$ and A is symmetric matrix, show that $a = b$

(iii) Solve: $3\tan^{-1}x + \cot^{-1}x = \pi$

(iv) Without expanding at any stage, find the value of:

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

(v) Find the value of constant 'k' so that the function f(x) defined as:

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

(vi) Find the approximate change in the volume 'V' of a cube of side x metres caused by decreasing the side by 1%.

(vii) Evaluate : $\int \frac{x^3+5x^2+4x+1}{x^2} dx$.

(viii) Find the differential equation of the family of concentric circles $x^2+y^2=a^2$

(ix) If A and B are events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, then find:

(a) $P(A/B)$

(b) $P(B/A)$

(x) In a race, the probabilities of A and B winning the race are $\frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability of neither of them winning the race.

Question 2 [4]

If the function $f(x) = \sqrt{2x-3}$ is invertible then find its inverse. Hence prove that $(fof^{-1})(x) = x$.

Question 3 [4]

If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, prove that $a + b + c = abc$.

Question 4 [4]

Use properties of determinants to solve for x:

$$\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0 \text{ and } x \neq 0.$$

Question 5 [4]

(a) Show that the function $f(x) = \begin{cases} x^2, x \leq 1 \\ \frac{1}{x}, x > 1 \end{cases}$ is continuous at $x = 1$ but not differentiable.

OR

(b) Verify Rolle's theorem for the following function:
$f(x) = e^{-x} \sin x \ \ on \ \ [0,\pi]$

Question 6 [4]

If $x = tan\left(\frac{1}{a}\log y\right)$, prove that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

Question 7 [4]

Evaluate: $\int tan^{-1}\sqrt{x}\, dx$

Question 8 [4]

(a) Find the points on the curve $y = 4x^3 - 3x + 5$ at which the equation of the tangent is parallel to the x-axis.

OR

(b) Water is dripping out from a conical funnel of semi-verticle angle $\frac{\pi}{4}$ at the uniform rate of 2 cm^2/sec in the surface, through a tiny hole at the vertex of the bottom. When the slant height of the water level is 4 cm, find the rate of decrease of the slant height of the water.

Question 9 [4]

(a) Solve: $sinx\, \frac{dy}{dx} - y = sinx.\tan\frac{x}{2}$

OR

(b) The population of a town grows at the rate of 10% per year. Using differential equation, find how long will it take for the population to grow 4 times.

Question 10 [6]

(a) Using matrices, solve the following system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

OR

(b) Using elementary transformation, find the inverse of the matrix:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Question 11 [4]

A speaks truth in 60% of the cases, while B in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

Question 12 [6]

A cone is inscribed in a sphere of radius 12 cm. If the volume of the cone is maximum, find its height.

Question 13 [6]

(a) Evaluate: $\int \frac{x-1}{\sqrt{x^2-x}}\,dx$

OR

(b) Evaluate: $\int_0^{\pi/2} \frac{\cos^2 x}{1+\sin x \cos x}\,dx$

Question 14 [6]

From a lot of 6 items containing 2 defective items, a sample of 4 items are drawn at random. Let the random variable X denote the number of defective items in the sample. If the sample is drawn without replacement, find:

(a) The probability distribution of X

(b) Mean of X

(c) Variance of X

SECTION B (20 Marks)

Question 15 [3×2]

(a) Find λ if the scalar projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

(b) The Cartesian equation of a line is: $2x - 3 = 3y + 1 = 5 - 6z$. Find the vector equation of a line passing through $(7, -5, 0)$ and parallel to the given line.

(c) Find the equation of the plane through the intersection of the planes

$\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 9$ and $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 3$ and passing through the origin.

Question 16 [4]

(a) If A, B, C are three non-collinear points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, respectively, then show that the length of the perpendicular from C on AB is $\frac{|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{b}\times\vec{a})|}{|\vec{b}-\vec{a}|}$

OR

(b) Show that the four points A, B, C and D with position vectors $4\hat{\imath}+5\hat{\jmath}+\hat{k}$, $-\hat{\jmath}-\hat{k}$, $3\hat{\imath}+9\hat{\jmath}+4\hat{k}$ and $4(-\hat{\imath}+\hat{\jmath}+\hat{k})$ respectively, are coplanar.

Question 17 [4]

(a) Draw a rough sketch of the curve and find the area of the region bounded by curve $y^2 = 8x$ and the line $x = 2$.

OR

(b) Sketch the graph of $y = |x+4|$. Using integration, find the area of the region bounded by the curve $y = |x+4|$ and $x = -6$ and $x = 0$.

Question 18 [6]

Find the image of a point having position vector : $3\hat{\imath}-2\hat{\jmath}+\hat{k}$ in the Plane $\vec{r}.(3\hat{\imath}-\hat{\jmath}+4\hat{k}) = 2$.

SECTION C (20 Marks)

Question 19 [3×2]

(a) Given the total cost function for x units of a commodity as:

$$C(x) = \frac{1}{3}x^3 + 3x^2 - 16x + 2.$$

Find:

(i) Marginal cost function

(ii) Average cost function

(b) Find the coefficient of correlation from the regression lines:

$$x - 2y + 3 = 0 \text{ and } 4x - 5y + 1 = 0.$$

(c) The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find the range of values of the output x, for which AC is increasing.

Question 20

(a) Find the line of regression of y on x from the following table.

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the value of y when $x = 6$.

OR

(b) From the given data:

Variable	x	y
Mean	6	8
Standard Deviation	4	6

and correlation coefficient: $\frac{2}{3}$. Find:

(i) Regression coefficients b_{yx} and b_{xy}

(ii) Regression line x on y

(iii) Most likely value of x when $y = 14$

Question 21 [4]

(a) A product can be manufactured at a total cost $C(x) = \frac{x^2}{100} + 100x + 40$, where x is the number of units produced. The price at which each unit can be sold is given by $P = \left(200 - \frac{x}{400}\right)$. Determine the production level x at which the profit is maximum. What is the price per unit and total profit at the level of production?

OR

(b) A manufacturer's marginal cost function is $\frac{500}{\sqrt{2x+25}}$. Find the cost involved to increase production from 100 units to 300 units.

Question 22 [6]

A manufacturing company makes two types of teaching aids A and B of Mathematics for Class X. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹ 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Formulate this as Linear Programming Problem and solve it. Identify the feasible region from the rough sketch.