2018

MATHEMATICS

(Two hours and a half)

Answers to this Paper must be written on the paper provided separately.

*You will **not** be allowed to write during the first 15 minutes.*

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

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*Attempt **all** questions from Section A and **any four** questions from **Section B**.*

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables are provided.

SECTION—A (40 Marks)

*(Attempt **all** questions from this Section)*

Question 1.

(a) *Find the value of 'x' and 'y' if :* **[3]**

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) *Sonia had recurring deposit account in a bank and deposited ₹ 600 per month for $2^1/_2$ years. If the rate of interest was 10% p.a., find the maturity value of this account.* **[3]**

(c) *Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is :* **[4]**

(i) *a prime number.*

(ii) *a number divisible by 4.*

(iii) *a number that is a multiple of 6.*

(iv) *an odd number.*

Answer.

(a) We have, $$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

On comparing both sides, we get

$$\Rightarrow \quad 2x+6 = 10, \quad 2y-5 = 15$$

$$\Rightarrow \quad 2x = 10-6, \quad 2y = 15+5$$

$$\Rightarrow \quad x = \frac{4}{2}, \quad y = \frac{20}{2}$$

$$\therefore \quad x = 2, y = 10.$$ **Ans.**

(b) Here, P = ₹ 600, $n = 2\frac{1}{2}$ years = 30 months, $r = 10\%$

$$\therefore \quad \text{Interest, I} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= 600 \times \frac{30 \times 31}{2 \times 12} \times \frac{10}{100}$$

$$= ₹\ 2325$$

$$\therefore \quad \text{M.V.} = Pn + I = 600 \times 30 + 2325 = ₹\ 20325.$$ **Ans.**

(c) Here, $S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$$\therefore \quad n(S) = 10$$

(i) Let A be the event of getting a prime number.

$$\therefore \quad A = \{2\}$$

$$\Rightarrow \quad n(A) = 1$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{10}$$ **Ans.**

(ii) Let B be the event of getting a no. divisible by 4.

$$\therefore \quad B = \{4, 8, 12, 16, 20\}$$

$$\therefore \quad n(B) = 5$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$ **Ans.**

(iii) Let C be the event of getting a no. which is multiple of 6.

$$\therefore \quad C = \{6, 12, 18\}$$

$$\therefore \quad n(C) = 3$$

$$\therefore \quad P(C) = \frac{n(C)}{n(S)} = \frac{3}{10}$$ **Ans.**

(iv) Let D be the event of getting an odd number.

$$\therefore \quad D = \{\ \}$$

$$\therefore \quad n(D) = 0$$

$$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{0}{10} = 0$$ **Ans.**

Question 2.

(a) *The circumference of the base of a cylindrical vessel is 132 cm and its height is 25cm. Find the* **[3]**

(i) *radius of the cylinder*

(ii) *volume of cylinder.* $\left(\text{use } \pi = \frac{22}{7}\right)$

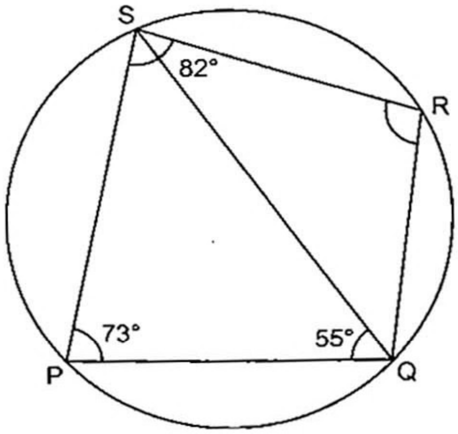
(b) *If $(k - 3)$, $(2k + 1)$ and $(4k + 3)$ are three consecutive terms of an A.P., find the value of k.* **[3]**

(c) *PQRS is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate :* **(4)**

(i) $\angle QRS$

(ii) $\angle RQS$

(iii) $\angle PRQ$



Answer.

(a) Given, circumference of base of cylinder = 132 cm, height of cylinder, $h = 25$ cm.

(i) Let r be the radius of cylinder

$$\therefore \quad 2\pi r = 132$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$$

$$\Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$ **Ans.**

(ii) Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times (21)^2 \times 25 = 34650 \text{ cm}^3$. **Ans.**

(b) Given, $(k-3), (2k+1), (4k+3)$ are 3 consecutive terms of an A.P.

$$\therefore \quad (2k+1)-(k-3) = (4k+3)-(2k+1)$$

$$\Rightarrow \quad 2k+1-k+3 = 4k+3-2k-1$$

$$\Rightarrow \quad k+4 = 2k+2$$

$$\Rightarrow \quad k-2k = 2-4$$

$$\Rightarrow \quad -k = -2$$

$$\Rightarrow \quad k = 2$$ **Ans.**

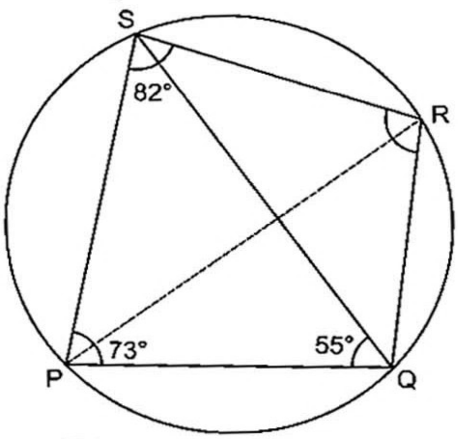
(c) Given, $\angle QPS = 73°$, $\angle PQS = 55°$, $\angle PSR = 82°$

(i) $\angle QRS + \angle QPS = 180°$

(sum of opposite angles of a cyclic quadrilateral are supplementary)

$$\Rightarrow \quad \angle QRS + 73° = 180°$$

$$\Rightarrow \quad \angle QRS = 180° - 73° = 107°$$ **Ans.**



(ii) $\angle PQR + \angle PSR = 180°$

(sum of opposite angles of a cyclic quadrilateral are supplementary)

$$\Rightarrow \quad \angle PQR + 82° = 180°$$

$$\Rightarrow \quad \angle PQR = 180° - 82°$$

$$\Rightarrow \quad \angle PQR = 98°$$
$$\Rightarrow \quad \angle RQS + \angle PQS = 98°$$
$$\Rightarrow \quad \angle RQS + 55° = 98°$$
$$\Rightarrow \quad \angle RQS = 98° - 55° = 43° \qquad \textbf{Ans.}$$

(iii) $\angle PSQ + \angle QPS + \angle PQS = 180°$ (sum of angles of a triangle is 180°)

$$\Rightarrow \quad \angle PSQ + 73° + 55° = 180°$$
$$\Rightarrow \quad \angle PSQ = 180° - 128° = 52°$$

$\therefore \quad \angle PRQ = \angle PSQ$ (angles on same segment are equal)

$$\Rightarrow \quad \angle PRQ = 52°. \qquad \textbf{Ans.}$$

Question 3.

(a) *If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and 'b'.* **[3]**

(b) *Prove that* $\sqrt{\sec^2\theta + \text{cosec}^2\,\theta} = \tan\theta + \cot\theta$ **[3]**

(c) *Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data :* **[4]**

Runs scored	*3000–4000*	*4000–5000*	*5000–6000*	*6000–7000*	*7000–8000*	*8000–9000*	*9000–10000*
No. of batsmen	*4*	*18*	*9*	*6*	*7*	*2*	*4*

Answer.

(a) Let $\quad f(x) = x^3 + ax + b$

$\because$ $(x + 2)$ and $(x + 3)$ are factors of $f(x)$

$$f(-2) = 0$$
$$\Rightarrow \quad (-2)^3 + a(-2) + b = 0$$
$$\Rightarrow \quad -8 - 2a + b = 0$$
$$\Rightarrow \quad -2a + b = 8 \qquad \ldots(i)$$
$$f(-3) = 0$$
$$(-3)^3 + a(-3) + b = 0$$
$$-27 - 3a + b = 0$$
$$-3a + b = 27 \qquad \ldots(ii)$$

Subtracting equation (ii) from equation (i), we have,

$$a = -19$$

From equation (i),

$$-2 \times (-19) + b = 8$$
$$\Rightarrow \quad b = 8 - 38 = -30$$
$$\therefore \quad a = -19, b = -30. \qquad \textbf{Ans.}$$

(b) To prove, $\quad \sqrt{\sec^2\theta + \text{cosec}^2\,\theta} = \tan\theta + \cot\theta$

$$\therefore \quad \text{L.H.S.} = \sqrt{\sec^2\theta + \text{cosec}^2\,\theta}$$
$$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$$

$$= \sqrt{\tan^2 \theta + \cot^2 \theta + 2}$$

$$= \sqrt{\tan^2 \theta + \cot^2 \theta + 2 \tan \theta . \cot \theta}$$

$(\because \tan \theta . \cot \theta = 1)$

$$= \sqrt{(\tan \theta + \cot \theta)^2}$$

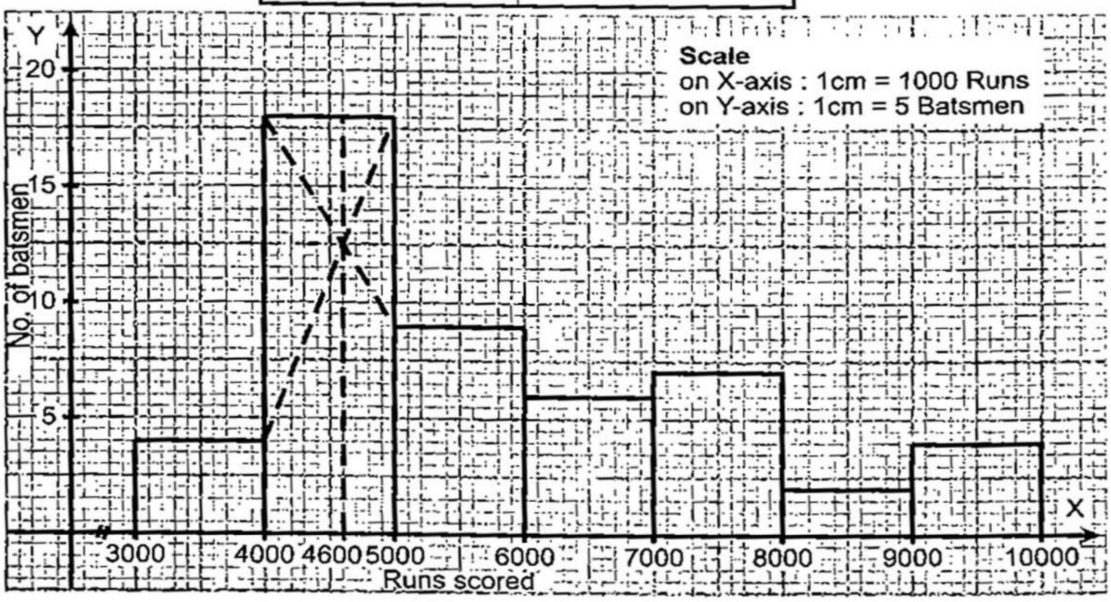
$$= \tan \theta + \cot \theta$$

$$= \text{R.H.S.}$$

Hence proved.

(c)

Runs Scored	No. of batsmen
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	6
7000 – 8000	7
8000 – 9000	2
9000 – 10000	4



$\therefore$ Mode = 4600 **Ans.**

Question 4.

(a) *Solve the following inequation, write down the solution set and represent it on the real number line :* **[3]**

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

(b) *If the straight lines $3x - 5y = 7$ and $4x + ay + 9 = 0$ are perpendicular to one another, find the value of a.* **[3]**

(c) *Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places.* **[4]**

Answer.

(a) Given inequation is,

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

$\Rightarrow$ $-2 + 10x \leq 13x + 10;$ $13x + 10 < 24 + 10x$

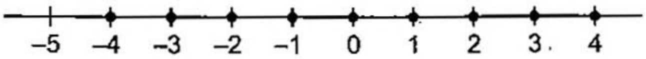
$$\Rightarrow \quad 10x - 13x \leq 10 + 2; \qquad 13x - 10x < 24 - 10$$

$$\Rightarrow \quad -3x \leq 12; \qquad 3x < 14$$

$$\Rightarrow \quad x \geq -4; \qquad x < \frac{14}{3}$$

$$\therefore \quad -4 \leq x < 4\frac{2}{3}$$

$\therefore$ Solution set = $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$



Ans.

(b) Given lines are $3x - 5y = 7$ and $4x + ay + 9 = 0$

$$\Rightarrow \quad -5y = -3x + 7 \quad \text{and} \quad ay = -4x - 9$$

$$\Rightarrow \quad y = \frac{3}{5}x - \frac{7}{5} \quad \text{and} \quad y = -\frac{4}{a}x - \frac{9}{a}$$

Comparing both equations with $y = mx + c$, we get,

$$m_1 = \frac{3}{5}, \qquad m_2 = -\frac{4}{a}$$

$\therefore$ The lines are perpendicular to each other,

$$m_1 \times m_2 = -1$$

$$\Rightarrow \quad \frac{3}{5} \times \left(-\frac{4}{a}\right) = -1$$

$$\Rightarrow \quad -\frac{12}{5} = -a$$

$$\Rightarrow \quad a = 2\frac{2}{5}.$$

Ans.

(c) We have, $x^2 + 7x = 7$

$$\Rightarrow \quad x^2 + 7x - 7 = 0$$

Comparing it with $ax^2 + bx + c = 0$, we have

$$a = 1, b = 7, c = -7$$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$= \frac{-7 \pm \sqrt{49 + 28}}{2}$$

$$= \frac{-7 \pm \sqrt{77}}{2}$$

$$= \frac{-7 \pm 8.775}{2}$$

$$= \frac{-7 + 8.775}{2} \text{ or } \frac{-7 - 8.775}{2}$$

$$= \frac{1.775}{2} \text{ or } \frac{-15.775}{2} = 0.8875 \text{ or } -7.8875$$

$= 0.89$ or -7.89 (correct to 2 decimal places) **Ans.**

SECTION—B (40 Marks)

*(Attempt any **four** questions from this Section)*

Question 5.

(a) *The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and common ratio of the series.* **[3]**

(b) *A man invests ₹ 22,500 in ₹ 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate :* **[3]**

(i) *The number of shares purchased*

(ii) *The annual dividend received*

(iii) *The rate of return he gets on his investment. Give your answer correct to the nearest whole number.*

(c) *Use graph paper for this question (Take 2 cm = 1 unit along both X and Y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B (2, – 2), C(0, – 1) and D(0, 1).* **[4]**

(i) *Reflect quadrilateral ABCD on the Y-axis and name it as A′B′CD.*

(ii) *Write down the coordinates of A′ and B′.*

(iii) *Name two points which are invariant under the above reflection.*

(iv) *Name the polygon A′B′CD.*

Answer.

(a) Let a be the first term and r be the common ratio of the given G.P.

$\therefore \quad T_4 = 16$ and $T_7 = 128$

$\Rightarrow \quad ar^3 = 16 \quad \ldots(i)$

and $\quad ar^6 = 128 \quad \ldots(ii)$

Dividing equation (ii) by equation (i), we get

$$\frac{ar^6}{ar^3} = \frac{128}{16}$$

$$\Rightarrow \quad r^3 = 8 \qquad \Rightarrow r = 2$$

$\therefore$ From equation (i), $a \times 2^3 = 16 \quad \Rightarrow a = \frac{16}{8} = 2$

$\therefore \quad a = 2, r = 2.$ **Ans.**

(b) Given, investment = ₹ 22,500, N.V. = ₹ 50, discount = 10%

$$\therefore \quad \text{M.V.} = ₹\left(50 - \frac{10}{100} \times 50\right) = ₹\,45.$$

Rate of dividend = 12%

(i) $\text{The no. of shares} = \frac{\text{Investment}}{\text{M.V.}}$

$$= \frac{22500}{45} = 500.$$

(ii) Annual dividend = Dividend per share × No. of shares

$$= \frac{12}{100} \times 50 \times 500 = ₹\,3000$$ **Ans.**

(iii) $\text{Rate of return} = \frac{\text{Dividend}}{\text{Investment}} \times 100\%$

$$= \frac{3000}{22500} \times 100\%$$
$$= 13.3\%$$
$$= 13\% \text{ (correct to the nearest whole no.)}$$ **Ans.**

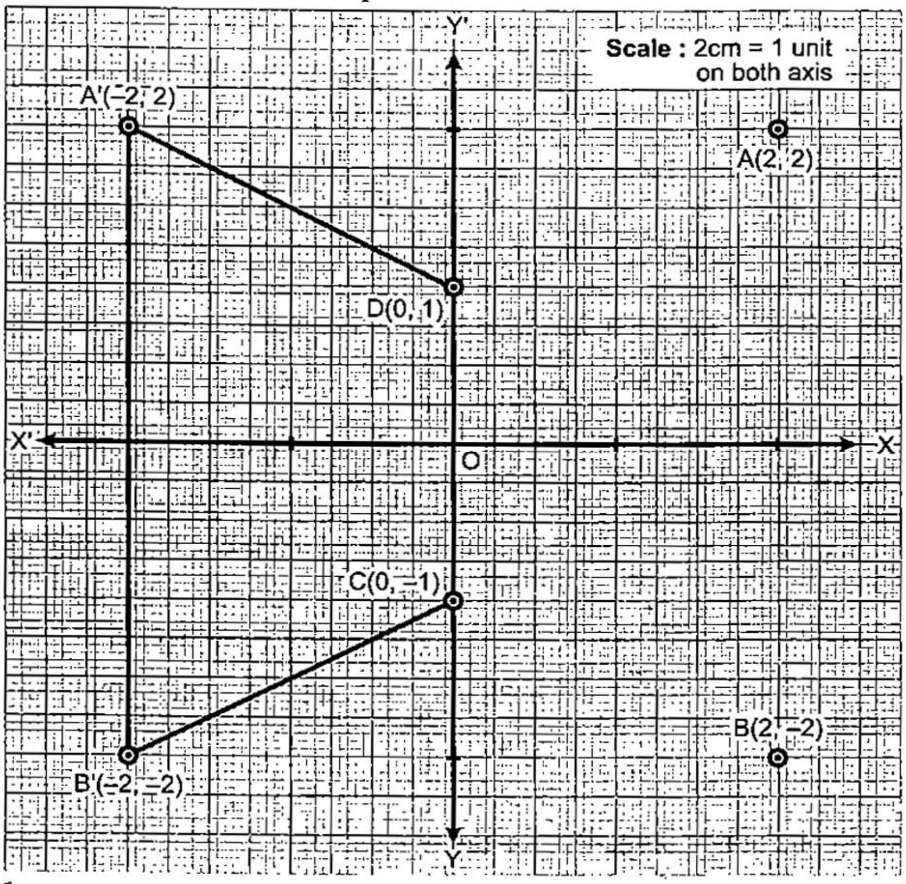
(c) (i) We have, A (2, 2), B (2, – 2), C (0, – 1), D (0, 1)

(ii) Coordinates of A′ = (– 2, 2)

Coordinates of B′ = (– 2, – 2)

(iii) Two invariant points are C(0, – 1) and D (0, 1)

(iv) A′B′CD is an isosceles trapezium.



Question 6.

(a) *Using properties of proportion, solve for x. Given that x is positive :* **[3]**

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

(b) *If* $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ *and C* $\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, *find* $AC + B^2 - 10C$. **[3]**

(c) *Prove that* $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$ **[4]**

Answer.

(a) Given, $$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$$ (using comp. and div.)

$$\Rightarrow \quad \frac{4x}{2\sqrt{4x^2-1}} = \frac{5}{3}$$

$$\Rightarrow \quad 10\sqrt{4x^2-1} = 12x$$

$$\Rightarrow \quad 100(4x^2-1) = 144x^2 \quad \text{(squaring both sides)}$$

$$\Rightarrow \quad 400\,x^2 - 100 = 144x^2$$

$$\Rightarrow \quad 400x^2 - 144x^2 = 100$$

$$\Rightarrow \quad 256x^2 = 100$$

$$\Rightarrow \quad x^2 = \frac{100}{256}$$

$$\Rightarrow \quad x^2 = \left(\frac{10}{16}\right)^2$$

$$\Rightarrow \quad x = \pm\frac{10}{16} = \pm\frac{5}{8}$$

$$\therefore \quad x = \frac{5}{8} \quad (\because x \text{ is positive}) \quad \textbf{Ans.}$$

(b) Given,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$\therefore \quad AC + B^2 - 10\,C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} + \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 40 \\ -9 & 73 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix} = \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix} \quad \textbf{Ans.}$$

(c) To prove, $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

$$\therefore \quad \text{L.H.S.} = (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2 = \text{R.H.S.}$$

Hence proved.

Question 7.

(a) *Find the value of k for which the following equation has equal roots :* **[3]**

$$x^2 + 4kx + (k^2 - k + 2) = 0$$

(b) *On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6 cm; BC = 8 cm and all angles are right angles. Find :* **[3]**

(i) *the actual length of the diagonal distance AC of the plot in km.*

(ii) *the actual area of the plot in sq. km.*

(c) *A(2, 5), B(– 1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the coordinates of 'M'. Hence find the equation of the line passing through the points C and M.* **[4]**

Answer.

(a) Given equation is, $x^2 + 4k\,x + (k^2 - k + 2) = 0$

Comparing it with $ax^2 + bx + c = 0$, we have,

$$a = 1, b = 4k, c = k^2 - k + 2.$$

$\therefore$ $D = b^2 - 4ac = (4k)^2 - 4 \times 1 \times (k^2 - k + 2)$

$= 16k^2 - 4k^2 + 4k - 8$

$= 12k^2 + 4k - 8$

$\because$ The roots of given equation are equal, so

$D = 0$

$\Rightarrow$ $12k^2 + 4k - 8 = 0$

$\Rightarrow$ $3k^2 + k - 2 = 0$

$\Rightarrow$ $3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow$ $3k\,(k + 1) - 2\,(k + 1) = 0$

$\Rightarrow$ $(k + 1)\,(3k - 2) = 0$

$\Rightarrow$ $k + 1 = 0$ or $3k - 2 = 0$

$\Rightarrow$ $k = -1$ or $k = \frac{2}{3}$

$\therefore$ $k = -1$ or $\frac{2}{3}$. **Ans.**

(b) Here, $1 : k = 1 : 50{,}000$

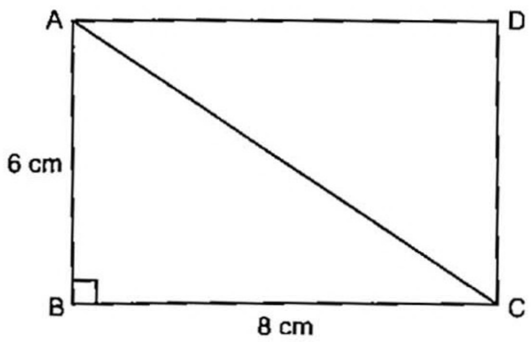
$AB = 6$ cm, $BC = 8$ cm

$\therefore$ $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100} = 10$ cm



(i) Actual length of AC = $k \times$ AC

$$= 50{,}000 \times 10 \text{ cm} = 5{,}00{,}000 \text{ cm}$$

$$= \frac{500000}{100000} \text{ km} = 5 \text{ km}.$$ **Ans.**

(ii) Area of rectangle ABCD $= 6 \times 8 = 48 \text{ cm}^2$

$\therefore$ Actual area $= k^2 \times$ Area of ABCD

$$= (50{,}000)^2 \times 48 \text{ cm}^2$$

$$= \frac{50{,}000 \times 50{,}000 \times 48}{1{,}00{,}000 \times 1{,}00{,}000} \text{ km}^2$$

$$= 12 \text{ km}^2.$$ **Ans.**

(c) Given vertices of triangle are, A (2, 5), B (– 1, 2), C (5, 8), AM : MB = 1 : 2.

$\because$ M is on AB, coordinates of M $= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$$= \left(\frac{1 \times (-1) + 2 \times 2}{1 + 2}, \frac{1 \times 2 + 2 \times 5}{1 + 2}\right)$$

$$= \left(\frac{-1 + 4}{3}, \frac{12}{3}\right) = (1, 4).$$ **Ans.**

The equation of line passing through C (5, 8) and M (1, 4) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\Rightarrow$ $$y - 8 = \frac{4 - 8}{1 - 5}(x - 5)$$

$\Rightarrow$ $$y - 8 = \frac{-4}{-4}(x - 5)$$

$\Rightarrow$ $$y - 8 = x - 5$$

$\Rightarrow$ $$x - y + 3 = 0.$$ **Ans.**

Question 8.

(a) *₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children.* **[3]**

(b) *If the mean of the following distribution is 24, find the value of 'a'.* **[3]**

Marks	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*
Number of students	*7*	*a*	*8*	*10*	*5*

(c) *Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120°.* **[4]**

(i) *Construct a circumcircle of ΔABC*

(ii) *Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.*

Answer.

(a)

$$\text{Total amount} = ₹\ 7{,}500$$

Let the original no. of children be x

$$\therefore \quad \text{Each receives} = \frac{7{,}500}{x}$$

If no. of children is $x - 20$,

$$\text{Each receives} = \frac{7{,}500}{x-20}$$

According to question

$$\frac{7{,}500}{x-20} - \frac{7{,}500}{x} = 100$$

$$\Rightarrow \quad 7{,}500\left(\frac{1}{x-20} - \frac{1}{x}\right) = 100$$

$$\Rightarrow \quad \frac{x - x + 20}{x(x-20)} = \frac{100}{7{,}500}$$

$$\Rightarrow \quad \frac{20}{x^2 - 20x} = \frac{1}{75}$$

$$\Rightarrow \quad x^2 - 20x = 1{,}500$$

$$\Rightarrow \quad x^2 - 20x - 1{,}500 = 0$$

$$\Rightarrow \quad x^2 - (50 - 30)x - 1{,}500 = 0$$

$$\Rightarrow \quad x^2 - 50x + 30x - 1{,}500 = 0$$

$$\Rightarrow \quad x(x - 50) + 30(x - 50) = 0$$

$$\Rightarrow \quad (x - 50)(x + 30) = 0$$

$$\Rightarrow \quad x - 50 = 0 \text{ or } x + 30 = 0$$

$$\Rightarrow \quad x = 50 \text{ or } x = -30$$

$$\therefore \quad x = 50 \ (\because x \text{ can not be negative})$$

$\therefore$ The original no. of children = 50. **Ans.**

(b)

Marks	Mid values (x)	No. of students (f)	fx
0 – 10	5	7	35
10 – 20	15	a	$15a$
20 – 30	25	8	200
30 – 40	35	10	350
40 – 50	45	5	225
		$\sum f = 30 + a$	$\sum fx = 15a + 810$

$$\therefore \quad \text{Mean} = \frac{\sum fx}{\sum f}$$

$$\Rightarrow \quad 24 = \frac{15a + 810}{a + 30}$$

$$\Rightarrow \quad 24a + 720 = 15a + 810$$

$$\Rightarrow \quad 24a - 15a = 810 - 720$$
$$\Rightarrow \quad 9a = 90$$
$$\Rightarrow \quad a = 10.$$
Ans.

(c) Given, BC = 5 cm, AB = 6.5 cm, $\angle ABC = 120°$

Steps of construction :

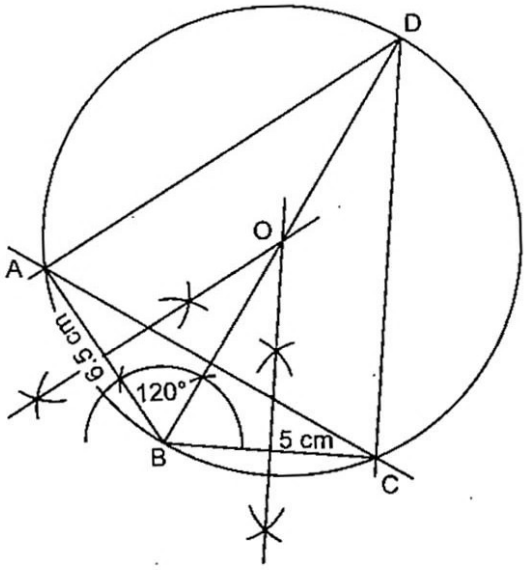
(i) Construct ΔABC with given data.

(ii) Draw perpendicular bisectors of BC and AB which meet at O.

(iii) Taking O as centre and OB as radius draw circumcircle of ΔABC passing through A, B and C.

(iv) Draw angle bisector of $\angle ABC$ as BD which meets circle at D.

(v) Join AD and CD. ABCD is the required cyclic quadrilateral.



Question 9.

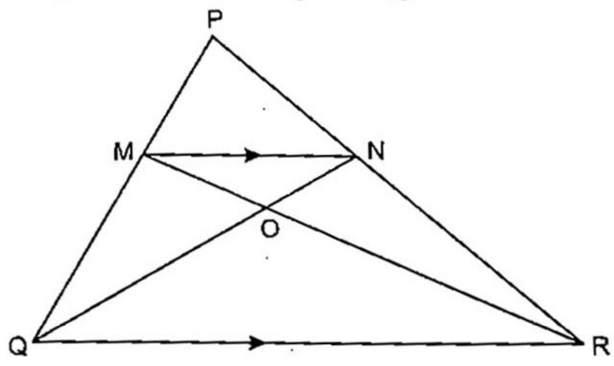
(a) *Priyanka has a recurring deposit account of ₹ 1000 per month at 10% per annum. If she gets ₹ 5550 as interest at the time of maturity, find the total time for which the account was held.* **[3]**

(b) *In ΔPQR, MN is parallel to QR and* $\frac{PM}{MQ} = \frac{2}{3}$ **[3]**

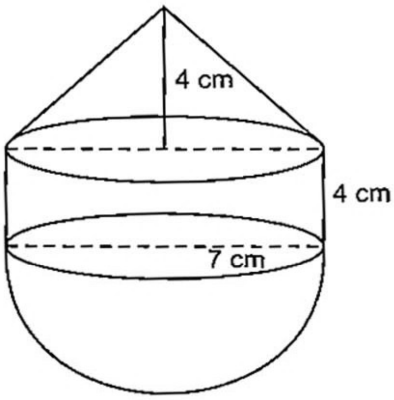
(i) *Find* $\frac{MN}{QR}$

(ii) *Prove that ΔOMN and ΔORQ are similar.*

(iii) *Find, Area of ΔOMN : Area of ΔORQ.*



(c) *The following figure represents a solid consisting of right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid.* **[4]**



Answer.

(a) Given, P = ₹ 1,000, $r = 10\%$, I = ₹ 5550, $n = ?$

$$\therefore \quad I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \quad 5550 = 1{,}000 \times \frac{n^2 + n}{24} \times \frac{10}{100}$$

$$\Rightarrow \quad 555 = \frac{5}{12}(n^2 + n)$$

$$\Rightarrow \quad 5n^2 + 5n = 6660$$

$$\Rightarrow \quad 5(n^2 + n) = 6660$$

$$\Rightarrow \quad n^2 + n = 1332$$

$$\Rightarrow \quad n^2 + n - 1332 = 0$$

$$\Rightarrow \quad n^2 + 37n - 36n - 1332 = 0$$

$$\Rightarrow \quad n(n + 37) - 36(n + 37) = 0$$

$$\Rightarrow \quad (n + 37)(n - 36) = 0$$

$$\Rightarrow \quad n + 37 = 0 \text{ or } n - 36 = 0$$

$$\Rightarrow \quad n = -37 \text{ or } n = 36$$

$$\therefore \quad n = 36 \; (\because n \text{ can not be negative})$$

$\therefore$ Required time = 36 months = 3 years. **Ans.**

(b) Given, MN ∥ QR,

$$\frac{PM}{MQ} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{PM}{PM + MQ} = \frac{2}{2 + 3}$$

$$\Rightarrow \quad \frac{PM}{PQ} = \frac{2}{5}$$

(i) In ΔPMN and ΔPQR,

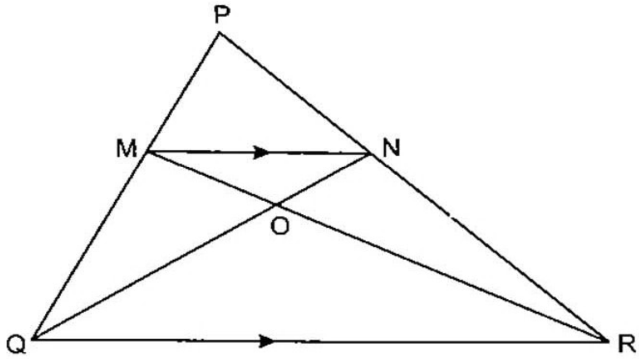
$\angle P = \angle P$ (common angle)

$\angle PMN = \angle PQR$ (corresponding angles, MN ∥ QR)

$\therefore$ $\Delta PMN \sim \Delta PQR$ (AA axiom)

$$\therefore \quad \frac{MN}{QR} = \frac{PM}{PQ} \quad (\Delta PMN \sim \Delta PQR)$$

$$= \frac{2}{5}.$$ **Ans.**



(ii) In ΔOMN and ΔORQ,

$\angle MON = \angle QOR$ (vertically opposite angles)

$\angle OMN = \angle ORQ$ (Alternative angles, MN ∥ QR)

$\therefore$ $\angle OMN \sim \Delta ORQ$ (AA axiom) **Ans.**

(iii) $$\frac{\text{Area of } \Delta OMN}{\text{Area of } \Delta ORQ} = \frac{MN^2}{QR^2}$$

$$= \frac{2^2}{5^2} = \frac{4}{25}$$

$\Rightarrow$ Area of ΔOMN : Area of ΔORQ = 4 : 25. **Ans.**

(c) Given. common radius (r) = 7 cm.

Height of cylinder = Height of cone = h = 4 cm.

$\therefore$ Volume of solid = Volume of cone + Volume of cylinder + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left(\frac{1}{3}h + h + \frac{2}{3}r\right)$$

$$= \frac{22}{7} \times 7^2 \left(\frac{1}{3} \times 4 + 4 + \frac{2}{3} \times 7\right)$$

$$= 22 \times 7 \left(\frac{4}{3} + 4 + \frac{14}{3}\right)$$

$$= 154 \left(\frac{4 + 12 + 14}{3}\right)$$

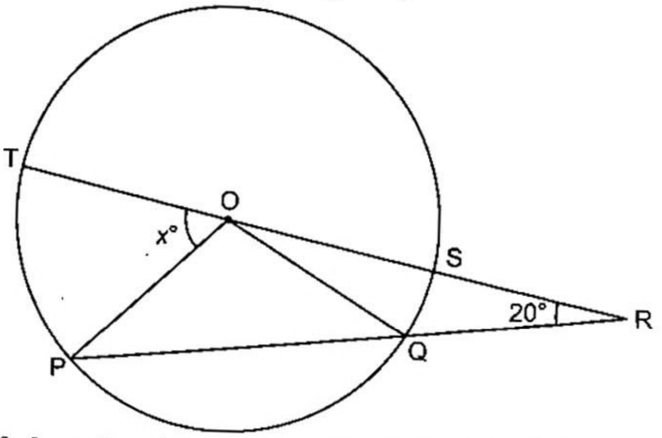
$$= 154 \times 10 = 1540 \text{ cm}^2.$$ **Ans.**

Question 10.

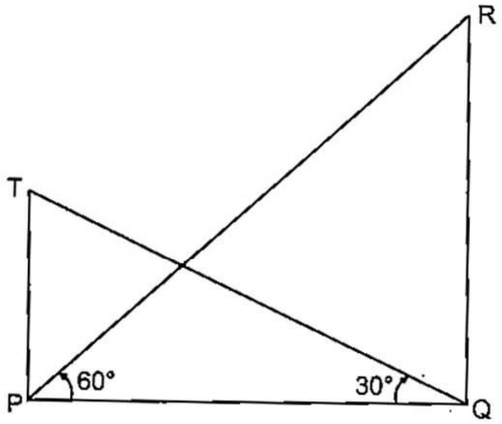
(a) *Use Remainder theorem to factorize the following polynomial :* **[3]**

$$2x^3 + 3x^2 - 9x - 10.$$

(b) *In the figure given below 'O' is the centre of the circle. If $QR = OP$ and $\angle ORP = 20°$. Find the value of 'x' giving reasons.* **[3]**



(c) *The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre.* **[4]**



Answer.

(a) Let $f(x) = 2x^3 + 3x^2 - 9x - 10$

For $x = 2$, $f(2) = 2 \times 2^3 + 3 \times 2^2 - 9 \times 2 - 10$

$= 16 + 12 - 18 - 10$

$= 28 - 28 = 0.$

$\therefore (x - 2)$ is a factor of $f(x)$

$x - 2)\ 2x^3 + 3x^2 - 9x - 10\ (2x^2 + 7x + 5$

$2x^3 - 4x^2$

$-\quad +$

$7x^2 - 9x$

$7x^2 - 14x$

$-\quad +$

$5x - 10$

$5x - 10$

$-\quad +$

$\times$

$\therefore \quad 2x^2 + 7x + 5 = 2x^2 + 5x + 2x + 5 = x(2x + 5) + 1(2x + 5)$

$$= (2x + 5)(x + 1)$$

$\therefore \quad f(x) = (x + 1)(x - 2)(2x + 5)$ **Ans.**

(b) Given, QR = OP, $\angle ORP = 20°$

$OP = OQ$ (radius of circle)

$\Rightarrow \quad OP = OQ = OR$

$\therefore \quad \angle QOS = \angle ORQ \; (\because QR = OQ)$

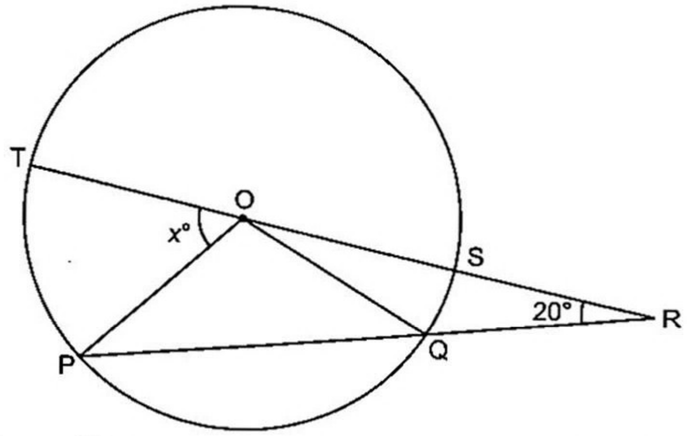
$= 20°.$

$\therefore \quad \angle OQP = \angle QOR + \angle ORQ$ (Exterior angle is equal to sum of interior opposite angles)

$= 20° + 20° = 40°$

$\therefore \quad \angle OPQ = \angle OQP \; (\because OP = OQ)$

$= 40°$



$\therefore \quad \angle POQ + \angle OPQ + \angle OQP = 180°$ (sum of angles in a triangle is180°)

$\Rightarrow \quad \angle POQ + 40° + 40° = 180°$

$\Rightarrow \quad \angle POQ = 180° - 80° = 100°$

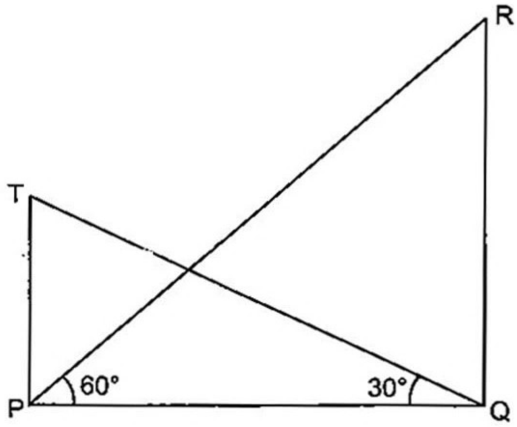
$\therefore \quad \angle POT + \angle POQ + \angle QOR = 180°$ (sum of angles on a straight line is 180°)

$\Rightarrow \quad x° + 100° + 20° = 180°$

$\Rightarrow \quad x° = 180° - 120° = 60°.$ **Ans.**

(c) Given, QR = 50 m, $\angle RPQ = 60°$, $\angle PQT = 30°$.

Let $\quad PT = x$ m, $PQ = y$ m



$\therefore$ In ΔPQR, $\tan 60° = \dfrac{QR}{PQ}$

$\Rightarrow \quad \sqrt{3} = \dfrac{50}{y}$

$\Rightarrow \quad y = \dfrac{50}{\sqrt{3}} \quad \ldots(i)$

In ΔPQT, $\quad \tan 30° = \dfrac{PT}{PQ}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{x}{y}$

$$\Rightarrow \quad x = \frac{y}{\sqrt{3}} \quad \ldots(ii)$$

$\therefore$ From equations (i) and (ii),

$$x = \frac{50/\sqrt{3}}{\sqrt{3}}$$

$$= \frac{50}{\sqrt{3} \times \sqrt{3}} = \frac{50}{3}$$

$$= 16.6$$

$= 17$m (correct to the nearest metre) **Ans.**

Question 11.

(a) *The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms.* **[4]**

(b) *Use Graph paper for this question.* **[6]**

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded :

Height in cm	*135–140*	*140–145*	*145–150*	*150–155*	*155–160*	*160–165*	*165–170*
No. of boys	*4*	*8*	*20*	*14*	*7*	*6*	*1*

Taking 2 cm = height of 10 cm along one axis and 2 cm =10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) *the median*

(ii) *lower quartile*

(iii) *if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.*

Answer.

(a) Let a be the first term and d be the common difference of given AP

$\therefore \quad T_4 = 22$ and $T_{15} = 66$

$\Rightarrow \quad a + 3d = 22 \quad \ldots(i)$

$a + 14d = 66 \quad \ldots(ii)$

Subtracting equation (i) from equation (ii), we get

$11d = 44 \Rightarrow d = 4$

From (i) $\quad a + 3 \times 4 = 22$

$\Rightarrow \quad a = 22 - 12 = 10$

$\therefore \quad a = 10, d = 4.$

Sum of series to 8 terms,

$$S_8 = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{8}{2}[2 \times 10 + (8-1) \cdot 4]$$

$$= 4(20 + 28)$$

$= 4 \times 48 = 192$ **Ans.**

(b)

Height in cm	**No. of Boys**	***c.f.***
135 – 140	4	4
140 – 145	8	12
145 – 150	20	32
150 – 155	14	46
155 – 160	7	53
160 – 165	6	59
165 – 170	1	60
	$n = 60$	

(i) Median $= \frac{n}{2}$th observation

$= \frac{60}{2}$th observation

$=$ 30th observation = 149 cm (from ogive)

(ii) Lower quartile $= \frac{n}{4}$th observation $= \frac{60}{4}$th observation

$=$ 15th observation = 146 cm (from ogive)

(iii) No. of boys whose height is less than 158 cm = 51

$\therefore$ No. of tall boys = 60 – 51 = 9. **Ans.**