

44. In the first refraction at A.

$$\mu_2 = 3/2, \mu_1 = 1, u = 0, R = \infty$$

$$\text{So, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow v = 0 \text{ since } (R \Rightarrow \infty \text{ and } u = 0)$$

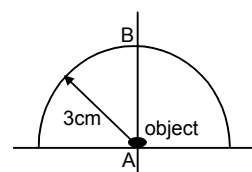
\therefore The image will be formed at the point, Now for the second refraction at B,

$$u = -3 \text{ cm, } R = -3 \text{ cm, } \mu_1 = 3/2, \mu_2 = 1$$

$$\text{So, } \frac{1}{v} + \frac{3}{2 \times 3} = \frac{1 - 1.5}{-3} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$$

$$\Rightarrow v = -3 \text{ cm, } \therefore \text{ There will be no shift in the final image.}$$



45. Thickness of glass = 3 cm,
- $\mu_g = 1.5$

$$\text{Image shift} = 3 \left(1 - \frac{1}{1.5} \right)$$

[Treating it as a simple refraction problem because the upper surface is flat and the spherical surface is in contact with the object]

$$= 3 \times \frac{0.5}{1.5} = 1 \text{ cm.}$$

The image will appear 1 cm above the point P.

46. As shown in the figure,
- $OQ = 3r$
- ,
- $OP = r$

$$\text{So, } PQ = 2r$$

For refraction at APB

$$\text{We know, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1.5}{v} - \frac{1}{-2r} = \frac{0.5}{r} = \frac{1}{2r} \quad [\text{because } u = -2r]$$

$$\Rightarrow v = \infty$$

For the reflection in concave mirror

$$u = \infty$$

$$\text{So, } v = \text{focal length of mirror} = r/2$$

For the refraction of APB of the reflected image.

$$\text{Here, } u = -3r/2$$

$$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{-0.5}{-r} \quad [\text{Here, } \mu_1 = 1.5 \text{ and } \mu_2 = 1 \text{ and } R = -r]$$

$$\Rightarrow v = -2r$$

As, negative sign indicates images are formed inside APB. So, image should be at C.

So, the final image is formed on the reflecting surface of the sphere.

47. a) Let the pin is at a distance of
- x
- from the lens.

$$\text{Then for 1}^{\text{st}} \text{ refraction, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\text{Here } \mu_2 = 1.5, \mu_1 = 1, u = -x, R = -60 \text{ cm}$$

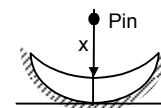
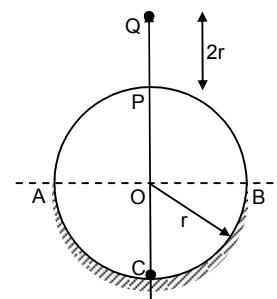
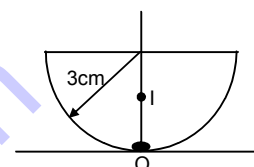
$$\therefore \frac{1.5}{v} - \frac{1}{-x} = \frac{0.5}{-60}$$

$$\Rightarrow 120(1.5x + v) = -vx \quad \dots(1)$$

$$\Rightarrow v(120 + x) = -180x$$

$$\Rightarrow v = \frac{-180x}{120 + x}$$

This image distance is again object distance for the concave mirror.



$$u = \frac{-180x}{120+x}, f = -10 \text{ cm } (\therefore f = R/2)$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v_1} = \frac{1}{-10} - \frac{-(120+x)}{180x}$$

$$\Rightarrow \frac{1}{v_1} = \frac{120+x-18x}{180x} \Rightarrow v_1 = \frac{180x}{120-17x}$$

Again the image formed is refracted through the lens so that the image is formed on the object taken in the 1st refraction. So, for 2nd refraction.

According to sign conversion $v = -x$, $\mu_2 = 1$, $\mu_1 = 1.5$, $R = -60$

$$\text{Now, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad [u = \frac{180x}{120-17x}]$$

$$\Rightarrow \frac{1}{-x} - \frac{1.5}{180x}(120-17x) = \frac{-0.5}{-60}$$

$$\Rightarrow \frac{1}{x} + \frac{120-17x}{120x} = \frac{-1}{120}$$

Multiplying both sides with 120 m, we get

$$120 + 120 - 17x = -x$$

$$\Rightarrow 16x = 240 \Rightarrow x = 15 \text{ cm}$$

\therefore Object should be placed at 15 cm from the lens on the axis.

48. For the double convex lens

$f = 25 \text{ cm}$, $R_1 = R$ and $R_2 = -2R$ (sign convention)

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{25} = (1.5 - 1) \left(\frac{1}{R} - \frac{1}{-2R} \right) = 0.5 \left(\frac{3R}{2} \right)$$

$$\Rightarrow \frac{1}{25} = \frac{3}{4R} \Rightarrow R = 18.75 \text{ cm}$$

$R_1 = 18.75 \text{ cm}$, $R_2 = 2R = 37.5 \text{ cm}$.

49. $R_1 = +20 \text{ cm}$; $R_2 = +30 \text{ cm}$; $\mu = 1.6$

a) If placed in air :

$$\frac{1}{f} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \left(\frac{1.6}{1} - 1 \right) \left(\frac{1}{20} - \frac{1}{30} \right)$$

$$\Rightarrow f = 60/6 = 100 \text{ cm}$$

b) If placed in water :

$$\frac{1}{f} = (\mu_w - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \left(\frac{1.6}{1.33} - 1 \right) \left(\frac{1}{20} - \frac{1}{30} \right)$$

$$\Rightarrow f = 300 \text{ cm}$$

50. Given $\mu = 1.5$

Magnitude of radii of curvatures = 20 cm and 30 cm

The 4types of possible lens are as below.

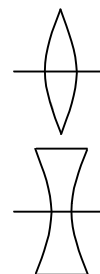
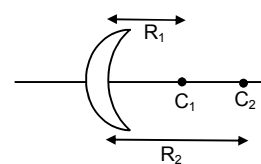
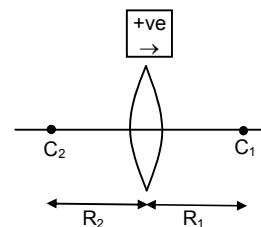
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Case (1) : (Double convex) [$R_1 = +ve$, $R_2 = -ve$]

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{20} - \frac{1}{-30} \right) \Rightarrow f = 24 \text{ cm}$$

Case (2) : (Double concave) [$R_1 = -ve$, $R_2 = +ve$]

$$\frac{1}{f} = (1.5 - 1) \left(\frac{-1}{20} - \frac{1}{30} \right) \Rightarrow f = -24 \text{ cm}$$



Case (3) : (Concave concave) [$R_1 = -ve, R_2 = -ve$]

$$\frac{1}{f} = (15 - 1) \left(\frac{1}{-20} - \frac{1}{-30} \right) \Rightarrow f = -120 \text{ cm}$$

Case (4) : (Concave convex) [$R_1 = +ve, R_2 = +ve$]

$$\frac{1}{f} = (15 - 1) \left(\frac{1}{20} - \frac{1}{30} \right) \Rightarrow f = +120 \text{ cm}$$

51. a) When the beam is incident on the lens from medium μ_1 .

$$\text{Then } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \text{ or } \frac{\mu_2}{v} - \frac{\mu_1}{(-\infty)} = \frac{\mu_2 - \mu_1}{R}$$

$$\text{or } \frac{1}{v} = \frac{\mu_2 - \mu_1}{\mu_2 R} \text{ or } v = \frac{\mu_2 R}{\mu_2 - \mu_1}$$

$$\text{Again, for 2}^{\text{nd}} \text{ refraction, } \frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$$

$$\text{or, } \frac{\mu_3}{v} = - \left[\frac{\mu_3 - \mu_2}{R} - \frac{\mu_2}{\mu_2 R} (\mu_2 - \mu_1) \right] \Rightarrow - \left[\frac{\mu_3 - \mu_2 - \mu_2 + \mu_1}{R} \right]$$

$$\text{or, } v = - \left[\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1} \right]$$

$$\text{So, the image will be formed at } = \frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$$

b) Similarly for the beam from μ_3 medium the image is formed at $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$.

52. Given that, $f = 10 \text{ cm}$

a) When $u = -9.5 \text{ cm}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9.8} = \frac{-0.2}{98}$$

$$\Rightarrow v = -490 \text{ cm}$$

$$\text{So, } \Rightarrow m = \frac{v}{u} = \frac{-490}{-9.8} = 50 \text{ cm}$$

So, the image is erect and virtual.

b) When $u = -10.2 \text{ cm}$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{-10.2} = \frac{102}{0.2}$$

$$\Rightarrow v = 510 \text{ cm}$$

$$\text{So, } m = \frac{v}{u} = \frac{510}{-9.8}$$

The image is real and inverted.

53. For the projector the magnification required is given by

$$m = \frac{v}{u} = \frac{200}{3.5} \Rightarrow u = 17.5 \text{ cm}$$

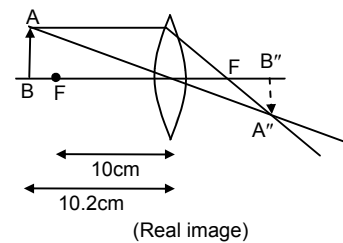
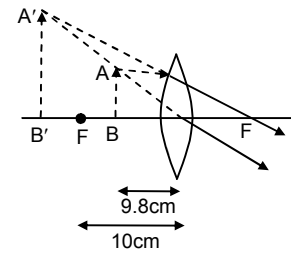
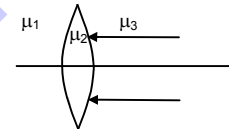
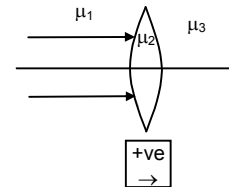
[$35 \text{ mm} > 23 \text{ mm}$, so the magnification is calculated taking object size 35 mm]

Now, from lens formula,

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{1000} + \frac{1}{17.5} = \frac{1}{f}$$

$$\Rightarrow f = 17.19 \text{ cm.}$$



54. When the object is at 19 cm from the lens, let the image will be at, v_1 .

$$\Rightarrow \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-19} = \frac{1}{12}$$

$$\Rightarrow v_1 = 32.57 \text{ cm}$$

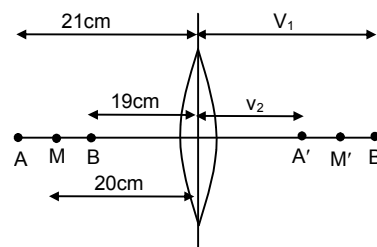
Again, when the object is at 21 cm from the lens, let the image will be at, v_2

$$\Rightarrow \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} + \frac{1}{21} = \frac{1}{12}$$

$$\Rightarrow v_2 = 28 \text{ cm}$$

$$\therefore \text{Amplitude of vibration of the image is } A = \frac{A'B'}{2} = \frac{v_1 - v_2}{2}$$

$$\Rightarrow A = \frac{32.57 - 28}{2} = 2.285 \text{ cm.}$$



55. Given, $u = -5 \text{ cm}$, $f = 8 \text{ cm}$

$$\text{So, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-5} = \frac{1}{8}$$

$$\Rightarrow v = -13.3 \text{ cm (virtual image).}$$

56. Given that,

$(-u) + v = 40 \text{ cm} = \text{distance between object and image}$

$h_o = 2 \text{ cm}$, $h_i = 1 \text{ cm}$

Since $\frac{h_i}{h_o} = \frac{v}{-u} = \text{magnification}$

$$\Rightarrow \frac{1}{2} = \frac{v}{-u} \Rightarrow u = -2v \quad \dots(1)$$

$$\text{Now, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$$

$$\Rightarrow \frac{3}{2v} = \frac{1}{f} \Rightarrow f = \frac{2v}{3} \quad \dots(2)$$

Again, $(-u) + v = 40$

$$\Rightarrow 3v = 40 \Rightarrow v = 40/3 \text{ cm}$$

$$\therefore f = \frac{2 \times 40}{3 \times 3} = 8.89 \text{ cm} = \text{focal length}$$

From eqn. (1) and (2)

$$u = -2v = -3f = -3(8.89) = 26.7 \text{ cm} = \text{object distance.}$$

57. A real image is formed. So, magnification $m = -2$ (inverted image)

$$\therefore \frac{v}{u} = -2 \Rightarrow v = -2u = (-2)(-18) = 36$$

$$\text{From lens formula, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$$

$$\Rightarrow f = 12 \text{ cm}$$

Now, for triple sized image $m = -3 = (v/u)$

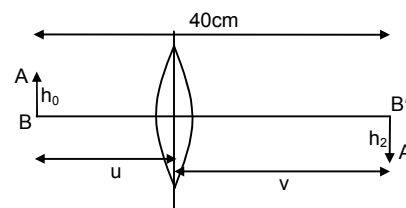
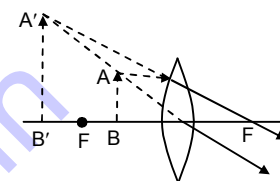
$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$$

$$\Rightarrow 3u = -48 \Rightarrow u = -16 \text{ cm}$$

So, object should be placed 16 cm from lens.

58. Now we have to calculate the image of A and B. Let the images be A' , B' . So, length of $A'B'$ = size of image.

For A, $u = -10 \text{ cm}$, $f = 6 \text{ cm}$



$$\text{Since, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-10} = \frac{1}{6}$$

$$\Rightarrow v = 15 \text{ cm} = OA'$$

$$\text{For B, } u = -12 \text{ cm, } f = 6 \text{ cm}$$

$$\text{Again, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{12}$$

$$\Rightarrow v = 12 \text{ cm} = OB'$$

$$\therefore A'B' = OA' - OB' = 15 - 12 = 3 \text{ cm.}$$

So, size of image = 3 cm.

$$59. u = -1.5 \times 10^{11} \text{ m ; } f = +20 \times 10^{-2} \text{ m}$$

Since, f is very small compared to u , distance is taken as ∞ . So, image will be formed at focus.

$$\Rightarrow v = +20 \times 10^{-2} \text{ m}$$

$$\therefore \text{ We know, } m = \frac{v}{u} = \frac{h_{\text{image}}}{h_{\text{object}}}$$

$$\Rightarrow \frac{20 \times 10^{-2}}{1.5 \times 10^{11}} = \frac{D_{\text{image}}}{1.4 \times 10^9}$$

$$\Rightarrow D_{\text{image}} = 1.86 \text{ mm}$$

$$\text{So, radius} = \frac{D_{\text{image}}}{2} = 0.93 \text{ mm.}$$

$$60. \text{ Given, } P = 5 \text{ diopter (convex lens)}$$

$$\Rightarrow f = 1/5 \text{ m} = 20 \text{ cm}$$

Since, a virtual image is formed, u and v both are negative.

$$\text{Given, } v/u = 4$$

$$\Rightarrow v = 4u \quad \dots(1)$$

$$\text{From lens formula, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{4u} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1-4}{4u} = -\frac{3}{4u}$$

$$\Rightarrow u = -15 \text{ cm}$$

\therefore Object is placed 15 cm away from the lens.

$$61. \text{ Let the object to placed at a distance } x \text{ from the lens further away from the mirror.}$$

For the concave lens (1st refraction)

$$u = -x, f = -20 \text{ cm}$$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-20} + \frac{1}{-x}$$

$$\Rightarrow v = -\left(\frac{20x}{x+20}\right)$$

So, the virtual image due to first refraction lies on the same side as that of object. ($A'B'$)

This image becomes the object for the concave mirror.

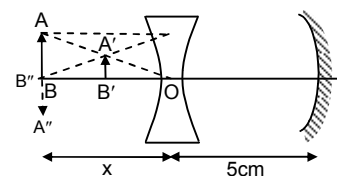
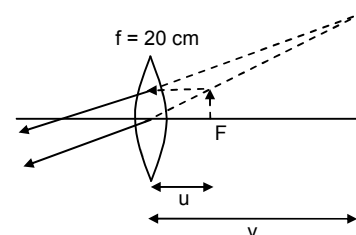
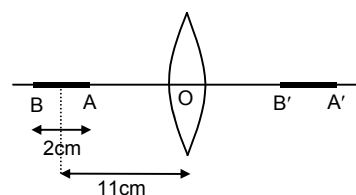
For the mirror,

$$u = -\left(5 + \frac{20x}{x+20}\right) = -\left(\frac{25x+100}{x+20}\right)$$

$$f = -10 \text{ cm}$$

From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{x+20}{25x+100}$$



$$\Rightarrow v = \frac{50(x+4)}{3x-20}$$

So, this image is formed towards left of the mirror.

Again for second refraction in concave lens,

$$u = -\left[5 - \frac{50(x+4)}{3x-20}\right] \text{ (assuming that image of mirror is formed between the lens and mirror)}$$

$v = +x$ (Since, the final image is produced on the object)

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{5 - \frac{50(x+4)}{3x-20}} = \frac{1}{-20}$$

$$\Rightarrow x = 60 \text{ cm}$$

The object should be placed at a distance 60 cm from the lens further away from the mirror.

So that the final image is formed on itself.

62. It can be solved in a similar manner like question no.61, by using the sign conversions properly. Left as an exercise for the student.
63. If the image in the mirror will form at the focus of the converging lens, then after transmission through the lens the rays of light will go parallel.

Let the object is at a distance x cm from the mirror

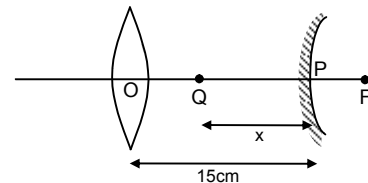
$\therefore u = -x$ cm ; $v = 25 - 15 = 10$ cm (because focal length of lens = 25 cm)

$f = 40$ cm

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{40}$$

$$\Rightarrow x = 400/30 = 40/3$$

\therefore The object is at distance $\left(15 - \frac{40}{3}\right) = \frac{5}{3} = 1.67$ cm from the lens.



64. The object is placed in the focus of the converging mirror.

There will be two images.

- a) One due to direct transmission of light through lens.
b) One due to reflection and then transmission of the rays through lens.

Case I : (S') For the image by direct transmission,

$u = -40$ cm, $f = 15$ cm

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-40}$$

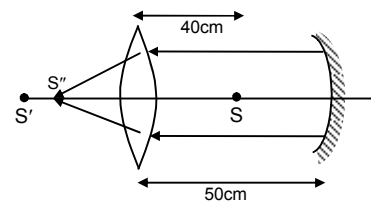
$$\Rightarrow v = 24 \text{ cm (left of lens)}$$

Case II : (S'') Since, the object is placed on the focus of mirror, after reflection the rays become parallel for the lens.

So, $u = \infty$

$$\Rightarrow f = 15 \text{ cm}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = 15 \text{ cm (left of lens)}$$

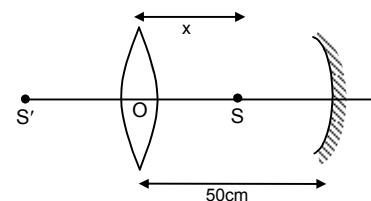


65. Let the source be placed at a distance ' x ' from the lens as shown, so that images formed by both coincide.

$$\text{For the lens, } \frac{1}{v_\ell} - \frac{1}{-x} = \frac{1}{15} \Rightarrow v_\ell = \frac{15x}{x-15} \quad \dots(1)$$

For the mirror, $u = -(50 - x)$, $f = -10$ cm

$$\text{So, } \frac{1}{v_m} + \frac{1}{-(50-x)} = -\frac{1}{10}$$



$$\Rightarrow \frac{1}{v_m} = \frac{1}{-(50-x)} - \frac{1}{10}$$

$$\text{So, } v_m = \frac{10(50-x)}{x-40} \quad \dots(2)$$

Since the lens and mirror are 50 cm apart,

$$v_l - v_m = 50 \Rightarrow \frac{15x}{x-15} - \frac{10(50-x)}{(x-40)} = 50$$

$$\Rightarrow x = 30 \text{ cm.}$$

So, the source should be placed 30 cm from the lens.

66. Given that, $f_1 = 15 \text{ cm}$, $F_m = 10 \text{ cm}$, $h_o = 2 \text{ cm}$

The object is placed 30 cm from lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

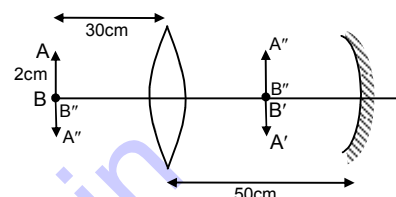
$$\Rightarrow v = \frac{uf}{u+f}$$

Since, $u = -30 \text{ cm}$ and $f = 15 \text{ cm}$

So, $v = 30 \text{ cm}$

So, real and inverted image ($A'B'$) will be formed at 30 cm from the lens and it will be of same size as the object. Now, this real image is at a distance 20 cm from the concave mirror. Since, $f_m = 10 \text{ cm}$, this real image is at the centre of curvature of the mirror. So, the mirror will form an inverted image $A''B''$ at the same place of same size.

Again, due to refraction in the lens the final image will be formed at AB and will be of same size as that of object. ($A''B''$)



67. For the lens, $f = 15 \text{ cm}$, $u = -30 \text{ cm}$

$$\text{From lens formula, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \Rightarrow v = 30 \text{ cm}$$

The image is formed at 30 cm of right side due to lens only.

Again, shift due to glass slab is,

$$= \Delta t = \left(1 - \frac{1}{\mu_g}\right)t \text{ [since, } \mu_g = 1.5 \text{ and } t = 1 \text{ cm]}$$

$$= 1 - (2/3) = 0.33 \text{ cm}$$

\therefore The image will be formed at $30 + 0.33 = 30.33 \text{ cm}$ from the lens on right side.

68. Let, the parallel beam is first incident on convex lens.

$d =$ diameter of the beam $= 5 \text{ mm}$

Now, the image due to the convex lens should be formed on its focus (point B)

So, for the concave lens,

$u = +10 \text{ cm}$ (since, the virtual object is on the right of concave lens)

$f = -10 \text{ cm}$

$$\text{So, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{1}{10} = 0 \Rightarrow v = \infty$$

So, the emergent beam becomes parallel after refraction in concave lens.

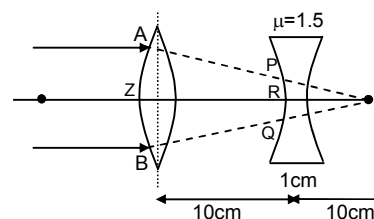
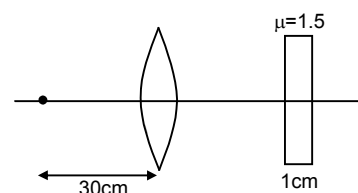
As shown from the triangles XYB and PQB ,

$$\frac{PQ}{XY} = \frac{RB}{ZB} = \frac{10}{20} = \frac{1}{2}$$

So, $PQ = \frac{1}{2} \times 5 = 2.5 \text{ mm}$

So, the beam diameter becomes 2.5 mm.

Similarly, it can be proved that if the light is incident of the concave side, the beam diameter will be 1 cm.



69. Given that, f_1 = focal length of converging lens = 30 cm

f_2 = focal length of diverging lens = -20 cm

and d = distance between them = 15 cm

Let, F = equivalent focal length

$$\text{So, } \therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{30} + \left(-\frac{1}{20}\right) - \left(\frac{15}{30(-20)}\right) = \frac{1}{120}$$

$$\Rightarrow F = 120 \text{ cm}$$

\Rightarrow The equivalent lens is a converging one.

Distance from diverging lens so that emergent beam is parallel (image at infinity),

$$d_1 = \frac{dF}{f_1} = \frac{15 \times 120}{30} = 60 \text{ cm}$$

It should be placed 60 cm left to diverging lens

\Rightarrow Object should be placed $(120 - 60) = 60$ cm from diverging lens.

$$\text{Similarly, } d_2 = \frac{dF}{f_2} = \frac{15 \times 120}{20} = 90 \text{ cm}$$

So, it should be placed 90 cm right to converging lens.

\Rightarrow Object should be placed $(120 + 90) = 210$ cm right to converging lens.

70. a) **First lens :**

$u = -15$ cm, $f = 10$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{15}\right) = \frac{1}{10}$$

$$\Rightarrow v = 30 \text{ cm}$$

So, the final image is formed 10 cm right of second lens.

b) **m for 1st lens :**

$$\frac{v}{u} = \frac{h_{\text{image}}}{h_{\text{object}}} \Rightarrow \left(\frac{30}{-15}\right) = \frac{h_{\text{image}}}{5 \text{ mm}}$$

$$\Rightarrow h_{\text{image}} = -10 \text{ mm (inverted)}$$

Second lens :

$u = -(40 - 30) = -10$ cm ; $f = 5$ cm

[since, the image of 1st lens becomes the object for the second lens].

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{10}\right) = \frac{1}{5}$$

$$\Rightarrow v = 10 \text{ cm}$$

m for 2nd lens :

$$\frac{v}{u} = \frac{h_{\text{image}}}{h_{\text{object}}} \Rightarrow \left(\frac{10}{10}\right) = \frac{h_{\text{image}}}{-10}$$

$$\Rightarrow h_{\text{image}} = 10 \text{ mm (erect, real).}$$

c) So, size of final image = 10 mm

71. Let u = object distance from convex lens = -15 cm

v_1 = image distance from convex lens when alone = 30 cm

f_1 = focal length of convex lens

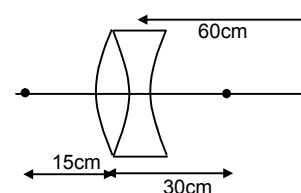
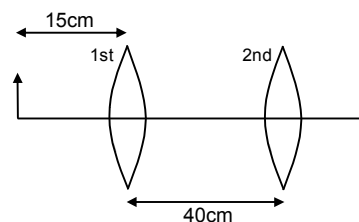
$$\text{Now, } \therefore \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

$$\text{or, } \frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15}$$

$$\text{or } f_1 = 10 \text{ cm}$$

Again, Let v = image (final) distance from concave lens = $+(30 + 30) = 60$ cm

v_1 = object distance from concave lens = +30 cm



f_2 = focal length of concave lens

$$\text{Now, } \therefore \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1}$$

$$\text{or, } \frac{1}{f_1} = \frac{1}{60} - \frac{1}{30} \Rightarrow f_2 = -60 \text{ cm.}$$

So, the focal length of convex lens is 10 cm and that of concave lens is 60 cm.

72. a) The beam will diverge after coming out of the two convex lens system because, the image formed by the first lens lies within the focal length of the second lens.

b) For 1st convex lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10}$ (since, $u = -\infty$)

or, $v = 10$ cm

for 2nd convex lens, $\frac{1}{v'} = \frac{1}{f} + \frac{1}{u}$

or, $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-(15-10)} = \frac{-1}{10}$

or, $v' = -10$ cm

So, the virtual image will be at 5 cm from 1st convex lens.

- c) If, F be the focal length of equivalent lens,

$$\text{Then, } \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{15}{100} = \frac{1}{20}$$

$$\Rightarrow F = 20 \text{ cm.}$$

73. Let us assume that it has taken time 't' from A to B.

$$\therefore AB = \frac{1}{2}gt^2$$

$$\therefore BC = h - \frac{1}{2}gt^2$$

This is the distance of the object from the lens at any time 't'.

Here, $u = -\left(h - \frac{1}{2}gt^2\right)$

$\mu_2 = \mu$ (given) and $\mu_1 = 1$ (air)

$$\text{So, } \Rightarrow \frac{\mu}{v} - \frac{1}{-\left(h - \frac{1}{2}gt^2\right)} = \frac{\mu - 1}{R}$$

$$\Rightarrow \frac{\mu}{v} = \frac{\mu - 1}{R} - \frac{1}{\left(h - \frac{1}{2}gt^2\right)} = \frac{(\mu - 1)\left(h - \frac{1}{2}gt^2\right) - R}{R\left(h - \frac{1}{2}gt^2\right)}$$

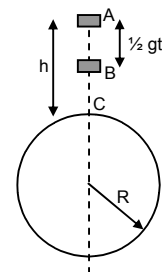
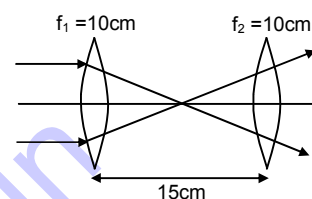
$$\text{So, } v = \text{image distance at any time 't'} = \frac{\mu R\left(h - \frac{1}{2}gt^2\right)}{(\mu - 1)\left(h - \frac{1}{2}gt^2\right) - R}$$

$$\text{So, velocity of the image} = V = \frac{dv}{dt} = \frac{d}{dt} \left[\frac{\mu R\left(h - \frac{1}{2}gt^2\right)}{(\mu - 1)\left(h - \frac{1}{2}gt^2\right) - R} \right] = \frac{\mu R^2 gt}{(\mu - 1)\left(h - \frac{1}{2}gt^2\right) - R} \text{ (can be found out).}$$

74. Given that, u = distance of the object = $-x$

f = focal length = $-R/2$

and, V = velocity of object = dx/dt

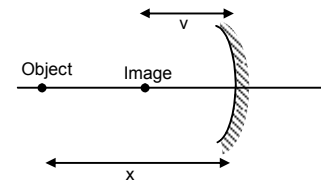


From mirror equation, $\frac{1}{-x} + \frac{1}{v} = -\frac{2}{R}$

$$\frac{1}{v} = -\frac{2}{R} + \frac{1}{x} = \frac{R-2x}{Rx} \Rightarrow v = \frac{Rx}{R-2x} = \text{Image distance}$$

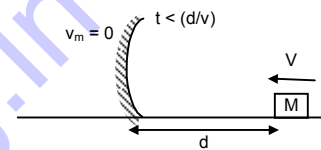
So, velocity of the image is given by,

$$\begin{aligned} V_1 &= \frac{dv}{dt} = \frac{\left[\frac{d}{dt}(xR)(R-2x)\right] - \left[\frac{d}{dt}(R-2x)\right][xR]}{(R-2x)^2} \\ &= \frac{R\left[\frac{dx}{dt}(R-2x)\right] - \left[-2\frac{dx}{dt}x\right]}{(R-2x)^2} = \frac{R[v(R-2x) + 2vx]}{(R-2x)^2} \\ &= \frac{VR^2}{(2x-R)^2} = \frac{R[VR-2xV+2xV]}{(R-2x)^2} \end{aligned}$$



75. a) When $t < d/V$, the object is approaching the mirror
As derived in the previous question,

$$\begin{aligned} V_{\text{image}} &= \frac{\text{Velocity of object} \times R^2}{[2 \times \text{distance between them} - R]^2} \\ \Rightarrow V_{\text{image}} &= \frac{VR^2}{[2(d-Vt) - R]^2} \quad [\text{At any time, } x = d - Vt] \end{aligned}$$



- b) After a time $t > d/V$, there will be a collision between the mirror and the mass.
As the collision is perfectly elastic, the object (mass) will come to rest and the mirror starts to move away with same velocity V .

At any time $t > d/V$, the distance of the mirror from the mass will be

$$x = V\left(t - \frac{d}{V}\right) = Vt - d$$

Here, $u = -(Vt - d) = d - Vt$; $f = -R/2$

$$\text{So, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{d-Vt} + \frac{1}{(-R/2)} = -\left[\frac{R+2(d-Vt)}{R(d-Vt)}\right]$$

$$\Rightarrow v = -\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right] = \text{Image distance}$$

So, Velocity of the image will be,

$$V_{\text{image}} = \frac{d}{dt}(\text{Image distance}) = \frac{d}{dt}\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right]$$

Let, $y = (d - Vt)$

$$\Rightarrow \frac{dy}{dt} = -V$$

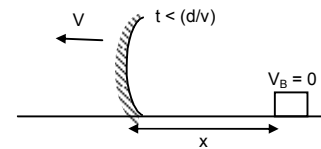
$$\text{So, } V_{\text{image}} = \frac{d}{dt}\left[\frac{Ry}{R+2y}\right] = \frac{(R+2y)R(-V) - Ry(+2)(-V)}{(R+2y)^2}$$

$$= -VR\left[\frac{R+2y-2y}{(R+2y)^2}\right] = \frac{-VR^2}{(R+2y)^2}$$

Since, the mirror itself moving with velocity V ,

$$\text{Absolute velocity of image} = V\left[1 - \frac{R^2}{(R+2y)^2}\right] \quad (\text{since, } V = V_{\text{mirror}} + V_{\text{image}})$$

$$= V\left[1 - \frac{R^2}{[2(Vt-d) - R]^2}\right]$$



76. Recoil velocity of gun = $V_g = \frac{mV}{M}$.

At any time 't', position of the bullet w.r.t. mirror = $Vt + \frac{mV}{M}t = \left(1 + \frac{m}{M}\right)Vt$

For the mirror, $u = -\left(1 + \frac{m}{M}\right)Vt = kVt$

v = position of the image

From lens formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} + \frac{1}{kVt} = \frac{1}{kVt} - \frac{1}{f} = \frac{f - kVt}{kVtf}$$

Let $\left(1 + \frac{m}{M} = k\right)$,

So, $v = \frac{kVtf}{-kVt + f} = \left(\frac{kVtf}{f - kVt}\right)$

So, velocity of the image with respect to mirror will be,

$$v_1 = \frac{dv}{dt} = \frac{d}{dt} \left[\frac{kVtf}{f - kVt} \right] = \frac{(f - kVt)kVf - kVtf(-kV)}{(f - kVt)^2} = \frac{kVt^2}{(f - kVt)^2}$$

Since, the mirror itself is moving at a speed of mV/M and the object is moving at 'V', the velocity of separation between the image and object at any time 't' will be,

$$v_s = V + \frac{mV}{M} + \frac{kVf^2}{(f - kVt)^2}$$

When, $t = 0$ (just after the gun is fired),

$$v_s = V + \frac{mV}{M} + kV = V + \frac{m}{M}V + \left(1 + \frac{m}{M}\right)V = 2\left(1 + \frac{m}{M}\right)V$$

77. Due to weight of the body suppose the spring is compressed by which is the mean position of oscillation.

$m = 50 \times 10^{-3}$ kg, $g = 10$ ms⁻², $k = 500$ Nm⁻², $h = 10$ cm = 0.1 m

For equilibrium, $mg = kx \Rightarrow x = mg/k = 10^{-3}$ m = 0.1 cm

So, the mean position is at $30 + 0.1 = 30.1$ cm from P (mirror).

Suppose, maximum compression in spring is δ .

Since, E.K.E. - I.K.E. = Work done

$$\Rightarrow 0 - 0 = mg(h + \delta) - \frac{1}{2}k\delta^2 \quad (\text{work energy principle})$$

$$\Rightarrow mg(h + \delta) = \frac{1}{2}k\delta^2 \Rightarrow 50 \times 10^{-3} \times 10(0.1 + \delta) = \frac{1}{2}500\delta^2$$

$$\text{So, } \delta = \frac{0.5 \pm \sqrt{0.25 + 50}}{2 \times 250} = 0.015 \text{ m} = 1.5 \text{ cm.}$$

From figure B,

Position of B is $30 + 1.5 = 31.5$ cm from pole.

Amplitude of the vibration = $31.5 - 30.1 = 1.4$.

Position A is $30.1 - 1.4 = 28.7$ cm from pole.

For A $u = -31.5$, $f = -12$ cm

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{31.5}$$

$$\Rightarrow v_A = -19.38 \text{ cm}$$

For B $f = -12$ cm, $u = -28.7$ cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{28.7}$$

$$\Rightarrow v_B = -20.62 \text{ cm}$$

The image vibrates in length $(20.62 - 19.38) = 1.24$ cm.

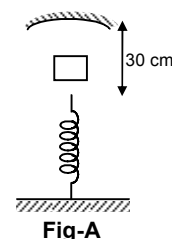
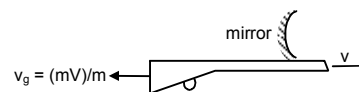


Fig-A

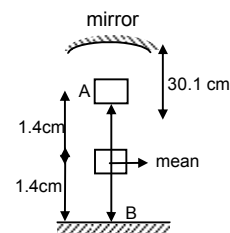


Fig-B

78. a) In time, $t = R/v$ the mass B must have moved $(v \times R/v) = R$ closer to the mirror stand

So, For the block B :

$$u = -R, f = -R/2$$

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{R} = -\frac{1}{R}$$

$\Rightarrow v = -R$ at the same place.

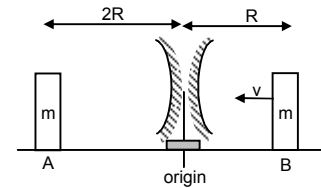
For the block A : $u = -2R, f = -R/2$

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{2R} = \frac{-3}{2R}$$

$\Rightarrow v = \frac{-2R}{3}$ image of A at $\frac{2R}{3}$ from PQ in the x-direction.

So, with respect to the given coordinate system,

\therefore Position of A and B are $\frac{-2R}{3}, R$ respectively from origin.



b) When $t = 3R/v$, the block B after colliding with mirror stand must have come to rest (elastic collision) and the mirror have travelled a distance R towards left from its initial position.

So, at this point of time,

For block A :

$$u = -R, f = -R/2$$

Using lens formula, $v = -R$ (from the mirror),

So, position $x_A = -2R$ (from origin of coordinate system)

For block B :

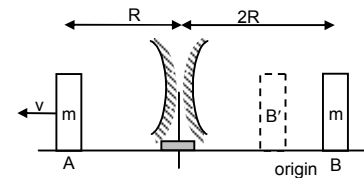
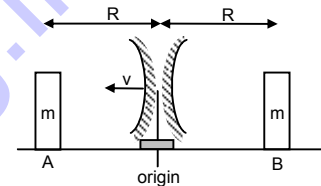
Image is at the same place as it is R distance from mirror. Hence, position of image is '0'.

Distance from PQ (coordinate system)

\therefore positions of images of A and B are $-2R, 0$ from origin.

c) Similarly, it can be proved that at time $t = 5R/v$,

the position of the blocks will be $-3R$ and $-4R/3$ respectively.



79. Let a = acceleration of the masses A and B (w.r.t. elevator). From the freebody diagrams,

$$T - mg + ma - 2m = 0 \quad \dots(1)$$

$$\text{Similarly, } T - ma = 0 \quad \dots(2)$$

$$\text{From (1) and (2), } 2ma - mg - 2m = 0$$

$$\Rightarrow 2ma = m(g + 2)$$

$$\Rightarrow a = \frac{10 + 2}{2} = \frac{12}{2} = 6 \text{ ms}^{-2}$$

so, distance travelled by B in $t = 0.2$ sec is,

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 6 \times (0.2)^2 = 0.12 \text{ m} = 12 \text{ cm.}$$

So, Distance from mirror, $u = -(42 - 12) = -30 \text{ cm}$; $f = +12 \text{ cm}$

$$\text{From mirror equation, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \frac{1}{12}$$

$$\Rightarrow v = 8.57 \text{ cm}$$

Distance between image of block B and mirror = 8.57 cm.

