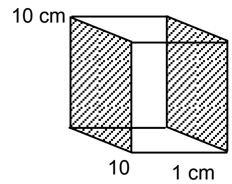
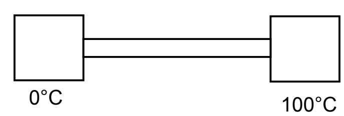
CHAPTER 28
HEAT TRANSFER



1. $t_1 = 90°C$, $t_2 = 10°C$
   $l = 1\text{ cm} = 1 \times 10^{-3}\text{ m}$
   $A = 10\text{ cm} \times 10\text{ cm} = 0.1 \times 0.1\text{ m}^2 = 1 \times 10^{-2}\text{ m}^2$
   $K = 0.80$ w/m-°C
   $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{8 \times 10^{-1} \times 1 \times 10^{-2} \times 80}{1 \times 10^{-2}} = 64 \text{ J/s} = 64 \times 60\ 3840 \text{ J.}$$
2. $t = 1\text{ cm} = 0.01\text{ m}$, $A = 0.8\text{ m}^2$
   $\theta_1 = 300$, $\theta_2 = 80$
   $K = 0.025$,
   $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.025 \times 0.8 \times (30030)}{0.01} = 440 \text{ watt.}$$
3. $K = 0.04$ J/m-5°C, $A = 1.6\text{ m}^2$
   $t_1 = 97°F = 36.1°C$ $t_2 = 47°F = 8.33°C$
   $l = 0.5\text{ cm} = 0.005\text{ m}$
   $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{4 \times 10^{-2} \times 1.6 \times 27.78}{5 \times 10^{-3}} = 356 \text{ J/s}$$
4. $A = 25\text{ cm}^2 = 25 \times 10^{-4}\text{ m}^2$
   $l = 1\text{ mm} = 10^{-3}\text{ m}$
   $K = 50$ w/m-°C
   $\frac{Q}{t}$ = Rate of conversion of water into steam
   $$= \frac{100 \times 10^{-3} \times 2.26 \times 10^6}{1 \text{ min}} = \frac{10^{-1} \times 2.26 \times 10^6}{60} = 0.376 \times 10^4$$
   $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \Rightarrow 0.376 \times 10^4 = \frac{50 \times 25 \times 10^{-4} \times (\theta - 100)}{10^{-3}}$$
   $$\Rightarrow \theta = \frac{10^{-3} \times 0.376 \times 10^4}{50 \times 25 \times 10^{-4}} = \frac{10^5 \times 0.376}{50 \times 25} = 30.1 \approx 30$$
5. $K = 46$ w/m-s°C
   $l = 1\text{ m}$
   $A = 0.04\text{ cm}^2 = 4 \times 10^{-6}\text{ m}^2$
   $L_{\text{fussion ice}} = 3.36 \times 10^5$ j/Kg



   $$\frac{Q}{t} = \frac{46 \times 4 \times 10^{-6} \times 100}{1} = 5.4 \times 10^{-8} \text{ kg} \approx 5.4 \times 10^{-5} \text{ g.}$$
6. $A = 2400\text{ cm}^2 = 2400 \times 10^{-4}\text{ m}^2$
   $\ell = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$
   $K = 0.06$ w/m-°C
   $\theta_1 = 20°C$
   $\theta_2 = 0°C$
   $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{0.06 \times 2400 \times 10^{-4} \times 20}{2 \times 10^{-3}} = 24 \times 6 \times 10^{-1} \times 10 = 24 \times 6 = 144 \text{ J/sec}$$
   Rate in which ice melts $= \frac{m}{t} = \frac{Q}{t \times L} = \frac{144}{3.4 \times 10^5}$ Kg/h $= \frac{144 \times 3600}{3.4 \times 10^5}$ Kg/s = 1.52 kg/s.
7. $\ell = 1\text{ mm} = 10^{-3}\text{ m}$ $m = 10$ kg
   $A = 200\text{ cm}^2 = 2 \times 10^{-2}\text{ m}^2$
   $L_{vap} = 2.27 \times 10^6$ J/kg
   $K = 0.80$ J/m-s-°C

$dQ = 2.27 \times 10^6 \times 10$,

$$\frac{dQ}{dt} = \frac{2.27 \times 10^7}{10^5} = 2.27 \times 10^2 \text{ J/s}$$

Again we know

$$\frac{dQ}{dt} = \frac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{1 \times 10^{-3}}$$

So, $\dfrac{8 \times 2 \times 10^{-3}(42 - T)}{10^{-3}} = 2.27 \times 10^2$

$\Rightarrow 16 \times 42 - 16T = 227 \Rightarrow T = 27.8 \approx 28°C$

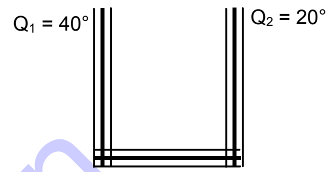
8. $K = 45$ w/m-°C

$\ell = 60$ cm $= 60 \times 10^{-2}$ m

$A = 0.2$ cm^2 $= 0.2 \times 10^{-4}$ m^2

Rate of heat flow,

$$= \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{60 \times 10^{-2}} = 30 \times 10^{-3}\ 0.03 \text{ w}$$



9. $A = 10$ cm^2, h = 10 cm

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{200 \times 10^{-3} \times 30}{1 \times 10^{-3}} = 6000$$

Since heat goes out from both surfaces. Hence net heat coming out.

$= \dfrac{\Delta Q}{\Delta t} = 6000 \times 2 = 12000$, $\dfrac{\Delta Q}{\Delta t} = MS\dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow 6000 \times 2 = 10^{-3} \times 10^{-1} \times 1000 \times 4200 \times \dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow \dfrac{\Delta\theta}{\Delta t} = \dfrac{72000}{420} = 28.57$

So, in 1 Sec. 28.57°C is dropped

Hence for drop of 1°C $\dfrac{1}{28.57}$ sec. = 0.035 sec. is required

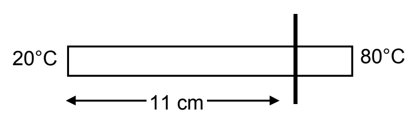
10. $\ell = 20$ cm $= 20 \times 10^{-2}$ m

$A = 0.2$ cm^2 $= 0.2 \times 10^{-4}$ m^2

$\theta_1 = 80°C$, $\theta_2 = 20°C$, $K = 385$

(a) $\dfrac{Q}{t} = \dfrac{KA(\theta_1 - \theta_2)}{\ell} = \dfrac{385 \times 0.2 \times 10^{-4}(80 - 20)}{20 \times 10^{-2}} = 385 \times 6 \times 10^{-4} \times 10 = 2310 \times 10^{-3} = 2.31$

(b) Let the temp of the 11 cm point be θ



$\dfrac{\Delta\theta}{\Delta l} = \dfrac{Q}{tKA}$

$\Rightarrow \dfrac{\Delta\theta}{\Delta l} = \dfrac{2.31}{385 \times 0.2 \times 10^{-4}}$

$\Rightarrow \dfrac{\theta - 20}{11 \times 10^{-2}} = \dfrac{2.31}{385 \times 0.2 \times 10^{-4}}$

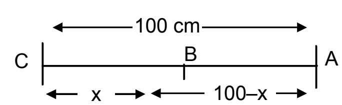
$\Rightarrow \theta - 20 = \dfrac{2.31 \times 10^4}{385 \times 0.2} \times 11 \times 10^{-2} = 33$

$\Rightarrow \theta = 33 + 20 = 53$

11. Let the point to be touched be ‘B’

No heat will flow when, the temp at that point is also 25°C

i.e. $Q_{AB} = Q_{BC}$



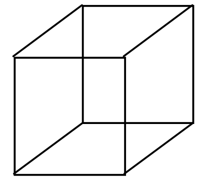
So, $\dfrac{KA(100 - 25)}{100 - x} = \dfrac{KA(25 - 0)}{x}$

$\Rightarrow 75\,x = 2500 - 25\,x \Rightarrow 100\,x = 2500 \Rightarrow x = 25$ cm from the end with 0°C

12. $V = 216\ cm^3$

$a = 6$ cm, Surface area $= 6\ a^2 = 6 \times 36\ m^2$

$t = 0.1$ cm $\quad \dfrac{Q}{t} = 100$ W,



$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell}$$

$$\Rightarrow 100 = \frac{K \times 6 \times 36 \times 10^{-4} \times 5}{0.1 \times 10^{-2}}$$

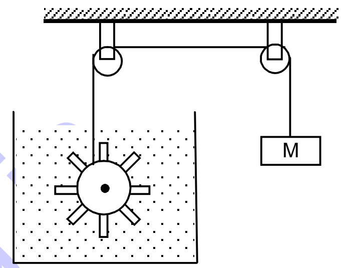
$$\Rightarrow K = \frac{100}{6 \times 36 \times 5 \times 10^{-1}} = 0.9259\ W/m°C \approx 0.92\ W/m°C$$

13. Given $\theta_1 = 1°C$, $\quad \theta_2 = 0°C$

$K = 0.50$ w/m-°C, $\quad d = 2$ mm $= 2 \times 10^{-3}$ m

$A = 5 \times 10^{-2}\ m^2$, $\quad v = 10$ cm/s $= 0.1$ m/s

Power = Force × Velocity = Mg × v



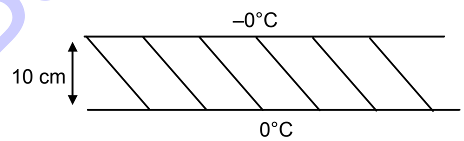
$$\text{Again Power} = \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\text{So, } Mgv = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\Rightarrow M = \frac{KA(\theta_1 - \theta_2)}{dvg} = \frac{5 \times 10^{-1} \times 5 \times^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10} = 12.5 \text{ kg.}$$

14. $K = 1.7$ W/m-°C $\quad f_w = 1000\ Kg/m^3$

$L_{ice} = 3.36 \times 10^5$ J/kg $\quad T = 10$ cm $= 10 \times 10^{-2}$ m



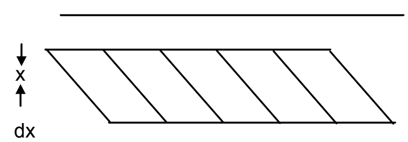
$$\text{(a) } \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} \Rightarrow \frac{\ell}{t} = \frac{KA(\theta_1 - \theta_2)}{Q} = \frac{KA(\theta_1 - \theta_2)}{mL}$$

$$= \frac{KA(\theta_1 - \theta_2)}{Atf_w L} = \frac{1.7 \times [0 - (-10)]}{10 \times 10^{-2} \times 1000 \times 3.36 \times 10^5}$$

$$= \frac{17}{3.36} \times 10^{-7} = 5.059 \times 10^{-7} \approx 5 \times 10^{-7} \text{ m/sec}$$

(b) let us assume that x length of ice has become formed to form a small strip of ice of length dx, dt time is required.

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{Adx f\omega L}{dt} = \frac{KA(\Delta\theta)}{x}$$



$$\Rightarrow \frac{dx f\omega L}{dt} = \frac{K(\Delta\theta)}{x} \Rightarrow dt = \frac{xdx f\omega L}{K(\Delta\theta)}$$

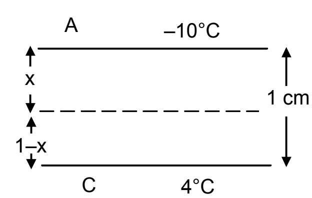
$$\Rightarrow \int_0^t dt = \frac{f\omega L}{K(\Delta\theta)} \int_0^t xdx \qquad \Rightarrow t = \frac{f\omega L}{K(\Delta\theta)} \left[\frac{x^2}{2}\right]_o^l = \frac{f\omega L}{K\Delta\theta} \frac{l^2}{2}$$

Putting values

$$\Rightarrow t = \frac{1000 \times 3.36 \times 10^5 \times (10 \times 10^{-2})^2}{1.7 \times 10 \times 2} = \frac{3.36}{2 \times 17} \times 10^6 \text{ sec.} = \frac{3.36 \times 10^6}{2 \times 17 \times 3600} \text{ hrs} = 27.45 \text{ hrs} \approx 27.5 \text{ hrs.}$$

15. let 'B' be the maximum level upto which ice is formed. Hence the heat conducted at that point from both the levels is the same.

Let AB = x

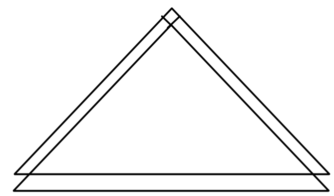


$$\text{i.e. } \frac{Q}{t}\text{ice} = \frac{Q}{t}\text{water} \qquad \Rightarrow \frac{K_{ice} \times A \times 10}{x} = \frac{K_{water} \times A \times 4}{(1 - x)}$$

$$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1 - x} \Rightarrow \frac{17}{x} = \frac{2}{1 - x}$$

$$\Rightarrow 17 - 17\,x = 2x \Rightarrow 19\,x = 17 \Rightarrow x = \frac{17}{19} = 0.894 \approx 89 \text{ cm}$$

16. $K_{AB} = 50$ j/m-s-°c $\theta_A = 40°C$
$K_{BC} = 200$ j/m-s-°c $\theta_B = 80°C$
$K_{AC} = 400$ j/m-s-°c $\theta_C = 80°C$
Length = 20 cm = 20×10^{-2} m
A = 1 cm^2 = 1×10^{-4} m^2

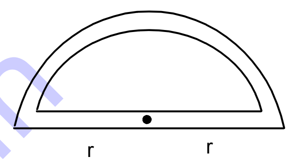


(a) $\dfrac{Q_{AB}}{t} = \dfrac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \dfrac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 1$ W.

(b) $\dfrac{Q_{AC}}{t} = \dfrac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \dfrac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 800 \times 10^{-2} = 8$

(c) $\dfrac{Q_{BC}}{t} = \dfrac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \dfrac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}} = 0$

17. We know $Q = \dfrac{KA(\theta_1 - \theta_2)}{d}$



$Q_1 = \dfrac{KA(\theta_1 - \theta_2)}{d_1}$, $Q_2 = \dfrac{KA(\theta_1 - \theta_2)}{d_2}$

$\dfrac{Q_1}{Q_2} = \dfrac{\dfrac{KA(\theta_1 - \theta_1)}{\pi r}}{\dfrac{KA(\theta_1 - \theta_1)}{2r}} = \dfrac{2r}{\pi r} = \dfrac{2}{\pi}$ [$d_1 = \pi r$, $d_2 = 2r$]

18. The rate of heat flow per sec.

$= \dfrac{dQ_A}{dt} = KA\dfrac{d\theta}{dt}$

The rate of heat flow per sec.

$= \dfrac{dQ_B}{dt} = KA\dfrac{d\theta_B}{dt}$

This part of heat is absorbed by the red.

$\dfrac{Q}{t} = \dfrac{ms\Delta\theta}{dt}$ where $\dfrac{d\theta}{dt}$ = Rate of net temp. variation

$\Rightarrow \dfrac{msd\theta}{dt} = KA\dfrac{d\theta_A}{dt} - KA\dfrac{d\theta_B}{dt}$ $\Rightarrow ms\dfrac{d\theta}{dt} = KA\left(\dfrac{d\theta_A}{dt} - \dfrac{d\theta_B}{dt}\right)$

$\Rightarrow 0.4 \times \dfrac{d\theta}{dt} = 200 \times 1 \times 10^{-4} (5 - 2.5)$ °C/cm

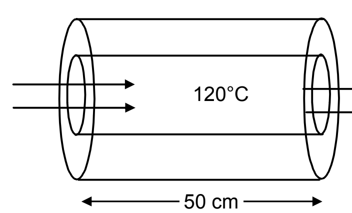
$\Rightarrow 0.4 \times \dfrac{d\theta}{dt} = 200 \times 10^{-4} \times 2.5$

$\Rightarrow \dfrac{d\theta}{dt} = \dfrac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}}$ °C/m = 1250×10^{-2} = 12.5 °C/m

19. Given

$K_{rubber} = 0.15$ J/m-s-°C $T_2 - T_1 = 90°C$

We know for radial conduction in a Cylinder



$\dfrac{Q}{t} = \dfrac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$

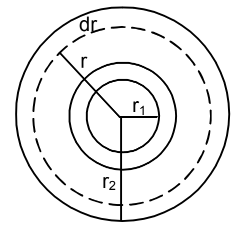
$= \dfrac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)} = 232.5 \approx 233$ j/s.

20. $\dfrac{dQ}{dt}$ = Rate of flow of heat

Let us consider a strip at a distance r from the center of thickness dr.

$\dfrac{dQ}{dt} = \dfrac{K \times 2\pi rd \times d\theta}{dr}$ [$d\theta$ = Temperature diff across the thickness dr]

$\Rightarrow C = \dfrac{K \times 2\pi rd \times d\theta}{dr}$ $\left[c = \dfrac{d\theta}{dr}\right]$



$\Rightarrow C\dfrac{dr}{r} = K2\pi d\ d\theta$

Integrating

$\Rightarrow C\displaystyle\int_{r_1}^{r_2}\dfrac{dr}{r} = K2\pi d \int_{\theta_1}^{\theta_2} d\theta$ $\Rightarrow C[\log r]_{r_1}^{r_2} = K2\pi d\ (\theta_2 - \theta_1)$

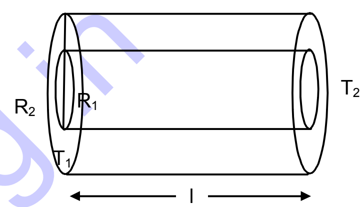
$\Rightarrow C(\log r_2 - \log r_1) = K2\pi d\ (\theta_2 - \theta_1) \Rightarrow C \log\left(\dfrac{r_2}{r_1}\right) = K2\pi d\ (\theta_2 - \theta_1)$

$\Rightarrow C = \dfrac{K2\pi d(\theta_2 - \theta_1)}{\log(r_2 / r_1)}$

21. $T_1 > T_2$

$A = \pi(R_2^2 - R_1^2)$

So, $Q = \dfrac{KA(T_2 - T_1)}{l} = \dfrac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$



Considering a concentric cylindrical shell of radius 'r' and thickness 'dr'. The radial heat flow through the shell

$H = \dfrac{dQ}{dt} = -KA\dfrac{d\theta}{dt}$ [(-)ve because as r – increases θ decreases]

$A = 2\pi rl$ $H = -2\pi rl\ K\dfrac{d\theta}{dt}$

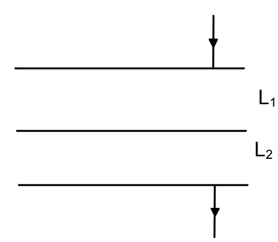
or $\displaystyle\int_{R_1}^{R_2}\dfrac{dr}{r} = -\dfrac{2\pi LK}{H}\int_{T_1}^{T_2} d\theta$

Integrating and simplifying we get

$H = \dfrac{dQ}{dt} = \dfrac{2\pi KL(T_2 - T_1)}{\text{Loge}(R_2 / R_1)} = \dfrac{2\pi KL(T_2 - T_1)}{\ln(R_2 / R_1)}$

22. Here the thermal conductivities are in series,

$\therefore \dfrac{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} \times \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}}{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} + \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}} = \dfrac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$



$\Rightarrow \dfrac{\dfrac{K_1}{l_1} \times \dfrac{K_2}{l_2}}{\dfrac{K_1}{l_1} + \dfrac{K_2}{l_2}} = \dfrac{K}{l_1 + l_2}$

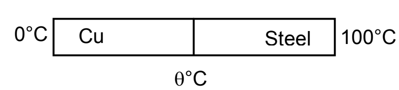
$\Rightarrow \dfrac{K_1K_2}{K_1l_2 + K_2l_1} = \dfrac{K}{l_1 + l_2} \Rightarrow K = \dfrac{(K_1K_2)(l_1 + l_2)}{K_1l_2 + K_2l_1}$

23. $K_{Cu} = 390$ w/m-°C $K_{St} = 46$ w/m-°C

Now, Since they are in series connection,

So, the heat passed through the crossections in the same.

So, $Q_1 = Q_2$

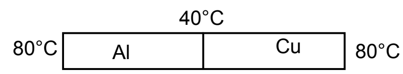


Or $\dfrac{K_{Cu} \times A \times (\theta - 0)}{l} = \dfrac{K_{St} \times A \times (100 - \theta)}{l}$

$\Rightarrow 390(\theta - 0) = 46 \times 100 - 46\ \theta \Rightarrow 436\ \theta = 4600$

$\Rightarrow \theta = \dfrac{4600}{436} = 10.55 \approx 10.6°C$

24. As the Aluminum rod and Copper rod joined are in parallel



$$\frac{Q}{t} = \left(\frac{Q}{t_1}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$

$$\Rightarrow \frac{KA(\theta_1-\theta_2)}{l} = \frac{K_1A(\theta_1-\theta_2)}{l} + \frac{K_2A(\theta_1-\theta_2)}{l}$$

$\Rightarrow K = K_1 + K_2 = (390 + 200) = 590$

$$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{l} = \frac{590\times 1\times 10^{-4}\times(60-20)}{1} = 590 \times 10^{-4} \times 40 = 2.36 \text{ Watt}$$

25. $K_{Al} = 200$ w/m-°C $\qquad K_{Cu} = 400$ w/m-°C

$A = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$

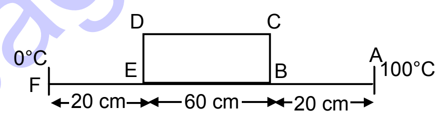
$l = 20 \text{ cm} = 2 \times 10^{-1}$ m

Heat drawn per second

$$= Q_{Al} + Q_{Cu} = \frac{K_{Al}\times A(80-40)}{l} + \frac{K_{Cu}\times A(80-40)}{l} = \frac{2\times 10^{-5}\times 40}{2\times 10^{-1}}[200+400] = 2.4 \text{ J}$$

Heat drawn per min = 2.4 × 60 = 144 J

26. $(Q/t)_{AB} = (Q/t)_{BE\text{ bent}} + (Q/t)_{BE}$



$$(Q/t)_{BE\text{ bent}} = \frac{KA(\theta_1-\theta_2)}{70} \qquad (Q/t)_{BE} = \frac{KA(\theta_1-\theta_2)}{60}$$

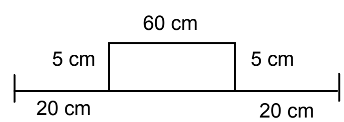
$$\frac{(Q/t)_{BE\text{ bent}}}{(Q/t)_{BE}} = \frac{60}{70} = \frac{6}{7}$$

$(Q/t)_{BE\text{ bent}} + (Q/t)_{BE} = 130$

$\Rightarrow (Q/t)_{BE\text{ bent}} + (Q/t)_{BE}\ 7/6 = 130$

$$\Rightarrow \left(\frac{7}{6}+1\right)(Q/t)_{BE\text{ bent}} = 130 \qquad \Rightarrow (Q/t)_{BE\text{ bent}} = \frac{130\times 6}{13} = 60$$

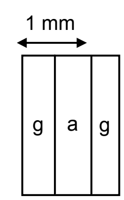
27. $\dfrac{Q}{t}\text{bent} = \dfrac{780\times A\times 100}{70}$



$$\frac{Q}{t}\text{str} = \frac{390\times A\times 100}{60}$$

$$\frac{(Q/t)\text{bent}}{(Q/t)\text{ str}} = \frac{780\times A\times 100}{70}\times\frac{60}{390\times A\times 100} = \frac{12}{7}$$

28. (a) $\dfrac{Q}{t} = \dfrac{KA(\theta_1-\theta_2)}{\ell} = \dfrac{1\times 2\times 1(40-32)}{2\times 10^{-3}} = 8000$ J/sec.



(b) Resistance of glass $= \dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_g}$

Resistance of air $= \dfrac{\ell}{ak_a}$

Net resistance $= \dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_a}$

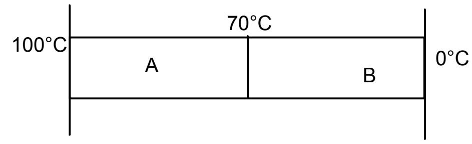
$$= \frac{\ell}{a}\left(\frac{2}{k_g}+\frac{1}{k_a}\right) = \frac{\ell}{a}\left(\frac{2k_a+k_g}{K_gk_a}\right)$$

$$= \frac{1\times 10^{-3}}{2}\left(\frac{2\times 0.025+1}{0.025}\right)$$

$$= \frac{1\times 10^{-3}\times 1.05}{0.05}$$

$$\frac{Q}{t} = \frac{\theta_1-\theta_2}{R} = \frac{8\times 0.05}{1\times 10^{-3}\times 1.05} = 380.9 \approx 381 \text{ W}$$

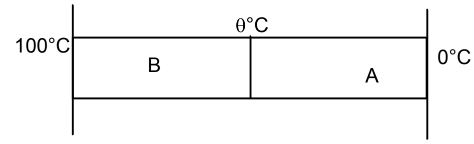
29. Now; Q/t remains same in both cases



In Case I : $\dfrac{K_A \times A \times (100-70)}{\ell} = \dfrac{K_B \times A \times (70-0)}{\ell}$

$\Rightarrow 30\ K_A = 70\ K_B$

In Case II : $\dfrac{K_B \times A \times (100-\theta)}{\ell} = \dfrac{K_A \times A \times (\theta-0)}{\ell}$



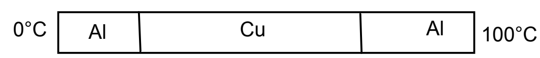
$\Rightarrow 100K_B - K_B\ \theta = K_A\ \theta$

$\Rightarrow 100K_B - K_B\ \theta = \dfrac{70}{30} K_B\ \theta$

$\Rightarrow 100 = \dfrac{7}{3}\theta + \theta$ $\qquad\qquad \Rightarrow \theta = \dfrac{300}{10} = 30°C$

30. $\theta_1 - \theta_2 = 100$

$\dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R}$



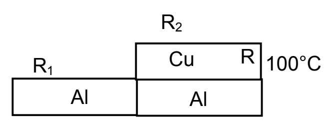
$R = R_1 + R_2 + R_3 = \dfrac{\ell}{aK_{Al}} + \dfrac{\ell}{aK_{Cu}} + \dfrac{\ell}{aK_{Al}} = \dfrac{\ell}{a}\left(\dfrac{2}{200} + \dfrac{1}{400}\right) = \dfrac{\ell}{a}\left(\dfrac{4+1}{400}\right) = \dfrac{\ell}{a}\dfrac{1}{80}$

$\dfrac{Q}{t} = \dfrac{100}{(\ell/a)(1/80)} \Rightarrow 40 = 80 \times 100 \times \dfrac{a}{\ell}$

$\Rightarrow \dfrac{a}{\ell} = \dfrac{1}{200}$

For (b)

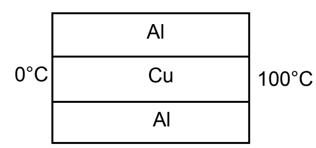
$R = R_1 + R_2 = R_1 + \dfrac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = R_{Al} + \dfrac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = \dfrac{\dfrac{l}{AK_{Al}} + \dfrac{l}{AK_{Cu}} + \dfrac{l}{AK_{Al}}}{\dfrac{l}{A_{Cu}} + \dfrac{l}{A_{Al}}}$



$= \dfrac{l}{AK_{Al}} + \dfrac{l}{A} + \dfrac{l}{K_{Al} + K_{Cu}} = \dfrac{l}{A}\left(\dfrac{1}{200} + \dfrac{1}{200+400}\right) = \dfrac{l}{A} \times \dfrac{4}{600}$

$\dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R} = \dfrac{100}{(l/A)(4/600)} = \dfrac{100 \times 600}{4}\dfrac{A}{l} = \dfrac{100 \times 600}{4} \times \dfrac{1}{200} = 75$

For (c)



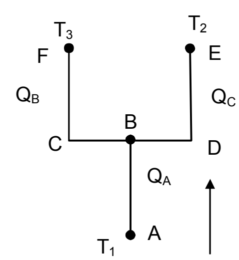
$\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{\dfrac{l}{aK_{Al}}} + \dfrac{1}{\dfrac{l}{aK_{Cu}}} + \dfrac{1}{\dfrac{l}{aK_{Al}}}$

$= \dfrac{a}{l}(K_{Al} + K_{Cu} + K_{Al}) = \dfrac{a}{l}(2 \times 200 + 400) = \dfrac{a}{l}(800)$

$\Rightarrow R = \dfrac{l}{a} \times \dfrac{1}{800}$

$\Rightarrow \dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R} = \dfrac{100 \times 800 \times a}{l}$

$= \dfrac{100 \times 800}{200} = 400\ W$

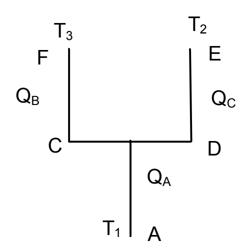


31. Let the temp. at B be T

$\dfrac{Q_A}{t} = \dfrac{Q_B}{t} + \dfrac{Q_C}{t}$ $\qquad \Rightarrow \dfrac{KA(T_1 - T)}{l} = \dfrac{KA(T - T_3)}{l + (l/2)} + \dfrac{KA(T - T_2)}{l + (l/2)}$

$\Rightarrow \dfrac{T_1 - T}{l} = \dfrac{T - T_3}{3l/2} + \dfrac{T - T_2}{3l/2}$ $\qquad \Rightarrow 3T_1 - 3T = 4T - 2(T_2 + T_3)$

$\Rightarrow -7T = -3T_1 - 2(T_2 + T_3)$ $\qquad \Rightarrow T = \dfrac{3T_1 + 2(T_2 + T_3)}{7}$



32. The temp at the both ends of bar F is same
Rate of Heat flow to right = Rate of heat flow through left
$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$

$$\Rightarrow \frac{K_A(T_1 - T)A}{l} + \frac{K_C(T_1 - T)A}{l} = \frac{K_B(T - T_2)A}{l} + \frac{K_D(T - T_2)A}{l}$$

$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$
$\Rightarrow T_1 - T = 2T - 2T_2$

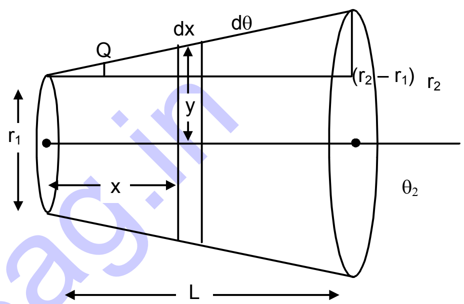
$$\Rightarrow T = \frac{T_1 + 2T_2}{3}$$

33. $\text{Tan } \phi = \frac{r_2 - r_1}{L} = \frac{(y - r_1)}{x}$

$\Rightarrow xr_2 - xr_1 = yL - r_1L$
Differentiating wr to 'x'

$$\Rightarrow r_2 - r_1 = \frac{Ldy}{dx} - 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{r_2 - r_1}{L} \Rightarrow dx = \frac{dyL}{(r_2 - r_1)} \qquad \ldots(1)$$



$$\text{Now } \frac{Q}{T} = \frac{K\pi y^2 d\theta}{dx} \Rightarrow \frac{\theta dx}{T} = k\pi y^2 d\theta$$

$$\Rightarrow \frac{\theta L dy}{r_2 r_1} = K\pi y^2 d\theta \qquad \text{from(1)}$$

$$\Rightarrow d\theta \ \frac{QLdy}{(r_2 - r_1)K\pi y^2}$$

Integrating both side

$$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \frac{QL}{(r_2 - r_1)k\pi} \int_{r_1}^{r_2} \frac{dy}{y}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{-1}{y}\right]_{r_1}^{r_2}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{r_2 - r_1}{r_1 + r_2}\right]$$

$$\Rightarrow Q = \frac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$$

34. $\frac{d\theta}{dt} = \frac{60}{10 \times 60} = 0.1°\text{C/sec}$

$$\frac{dQ}{dt} = \frac{KA}{d}(\theta_1 - \theta_2)$$

$$= \frac{KA \times 0.1}{d} + \frac{KA \times 0.2}{d} + \ldots\ldots. + \frac{KA \times 60}{d}$$

$$= \frac{KA}{d}(0.1 + 0.2 + \ldots\ldots.. + 60) = \frac{KA}{d} \times \frac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$$

[$\therefore$ a + 2a +……….+ na = n/2{2a + (n – 1)a}]

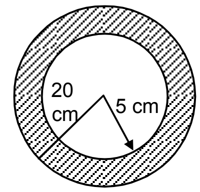
$$= \frac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \frac{200 \times 10^{-2} \times 300 \times 60.1}{20}$$

$= 3 \times 10 \times 60.1 = 1803 \text{ w} \approx 1800 \text{ w}$

35. $a = r_1 = 5 \text{ cm} = 0.05 \text{ m}$

$b = r_2 = 20 \text{ cm} = 0.2 \text{ m}$

$\theta_1 = T_1 = 50°C$ $\qquad\qquad$ $\theta_2 = T_2 = 10°C$



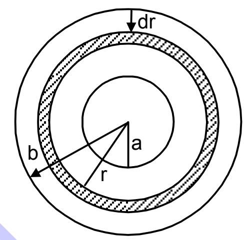
Now, considering a small strip of thickness 'dr' at a distance 'r'.

$A = 4\,\pi r^2$

$$H = -\,4\,\pi r^2 K \frac{d\theta}{dr} \qquad \text{[(–)ve because with increase of r, } \theta \text{ decreases]}$$

$$= \int_a^b \frac{dr}{r^2} = \frac{-4\pi K}{H}\int_{\theta_1}^{\theta_2} d\theta \qquad \text{On integration,}$$

$$H = \frac{dQ}{dt} = K\frac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$$



Putting the values we get

$$\frac{K \times 4 \times 3.14 \times 5 \times 20 \times 40 \times 10^{-3}}{15 \times 10^{-2}} = 100$$

$$\Rightarrow K = \frac{15}{4 \times 3.14 \times 4 \times 10^{-1}} = 2.985 \approx 3 \text{ w/m-°C}$$

36. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ $\qquad$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lms}$

Fall in Temp in $T_1 = \dfrac{KA(T_1 - T_2)}{Lms}$ $\qquad$ Final Temp. $T_1 \Rightarrow T_1 - \dfrac{KA(T_1 - T_2)}{Lms}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lms}$

$$\text{Final } \frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lms} - T_2 - \frac{KA(T_1 - T_2)}{Lms}$$

$$= (T_1 - T_2) - \frac{2KA(T_1 - T_2)}{Lms} = \frac{dT}{dt} = -\frac{2KA(T_1 - T_2)}{Lms} \Rightarrow \int_{(T_1 - T_2)}^{(T_1 - T_2)} \frac{dt}{(T_1 - T_2)} = \frac{-2KA}{Lms}dt$$

$$\Rightarrow Ln\frac{(T_1 - T_2)/2}{(T_1 - T_2)} = \frac{-2KAt}{Lms} \qquad \Rightarrow \ln(1/2) = \frac{-2KAt}{Lms} \qquad \Rightarrow \ln_2 = \frac{2KAt}{Lms} \qquad \Rightarrow t = \ln_2 \frac{Lms}{2KA}$$

37. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ $\qquad$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Fall in Temp in $T_1 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_2s_2}$ $\qquad$ Final Temp. $T_1 = T_1 - \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

$$\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1} - T_2 - \frac{KA(T_1 - T_2)}{Lm_2s_2} = (T_1 - T_2) - \left[\frac{KA(T_1 - T_2)}{Lm_1s_1} + \frac{KA(T_1 - T_2)}{Lm_2s_2}\right]$$

$$\Rightarrow \frac{dT}{dt} = -\frac{KA(T_1 - T_2)}{L}\left(\frac{1}{m_1s_1} + \frac{1}{m_2s_2}\right) \qquad \Rightarrow \frac{dT}{(T_1 - T_2)} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)dt$$

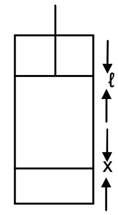
$$\Rightarrow \ln\Delta t = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t + C$$

At time $t = 0$, $T = T_0$, $\qquad$ $\Delta T = \Delta T_0$ $\qquad$ $\Rightarrow C = \ln\Delta T_0$

$$\Rightarrow \ln\frac{\Delta T}{\Delta T_0} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t \Rightarrow \frac{\Delta T}{\Delta T_0} = e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$$

$$\Rightarrow \Delta T = \Delta T_0\, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t} = (T_2 - T_1)\, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$$

38. $\frac{Q}{t} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_P dT}{dt} = \frac{KA(T_s - T_0)}{x}$



$\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_S - T_0)$

$\Rightarrow \frac{dT}{(T_S - T_0)} = -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_S - T_0)_{T_0}^{T} = -\frac{2KAdt}{5nRx}$

$\Rightarrow \ln\frac{T_S - T}{T_S - T_0} = -\frac{2KAdt}{5nRx} \Rightarrow T_S - T = (T_S - T_0)e^{-\frac{2KAt}{5nRx}}$

$\Rightarrow T = T_S - (T_S - T_0)e^{-\frac{2KAt}{5nRx}} = T_S + (T_S + T_0)e^{+\frac{2KAt}{5nRx}}$

$\Rightarrow \Delta T = T - T_0 = (T_S - T_0) + (T_S - T_0)e^{+\frac{2KAt}{5nRx}} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$

$\Rightarrow \frac{P_a AL}{nR} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$ $\quad$ $[p_a dv = nRdt \quad P_a Al = nRdt \quad dT = \frac{P_a AL}{nR}]$

$\Rightarrow L = \frac{nR}{P_a A}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$

39. $A = 1.6\ m^2$, $\quad$ $T = 37°C = 310\ K$, $\quad$ $\sigma = 6.0 \times 10^{-8}\ w/m^2\text{-}K^4$

Energy radiated per second

$= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887\ J$

40. $A = 12\ cm^2 = 12 \times 10^{-4}\ m^2$ $\quad$ $T = 20°C = 293\ K$

$e = 0.8$ $\quad$ $\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$\frac{Q}{t} = Ae\ \sigma T^4 = 12 \times 10^{-4}\ 0.8 \times 6 \times 10^{-8}\ (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42$

41. $E \rightarrow$ Energy radiated per unit area per unit time

Rate of heat flow $\rightarrow$ Energy radiated

(a) Per time = E × A

So, $E_{Al} = \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4}$ $\quad$ $\therefore$ 1 : 4

(b) Emissivity of both are same

$= \frac{m_1 S_1 dT_1}{m_2 S_2 dT_2} = 1$

$\Rightarrow \frac{dT_1}{dT_2} = \frac{m_2 S_2}{m_1 S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1 : 2 : 9$

42. $\frac{Q}{t} = Ae\ \sigma T^4$

$\Rightarrow T^4 = \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}}$

$\Rightarrow T = 1697.0 \approx 1700\ K$

43. (a) $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$, $\quad$ $T = 57°C = 330\ K$

$E = A\ \sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42\ J$

(b) $\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$, $\quad$ $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$\sigma = 6 \times 10^{-8}$ $\quad$ $T_1 = 473\ K$, $\quad$ $T_2 = 330\ K$

$= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4]$

$= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}]$

$= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58\ w$ $\quad$ from the ball.

44. $r = 1\text{ cm} = 1 \times 10^{-3}\text{ m}$

$A = 4\pi(10^{-2})^2 = 4\pi \times 10^{-4}\text{ m}^2$

$E = 0.3,\qquad \sigma = 6 \times 10^{-8}$

$\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$

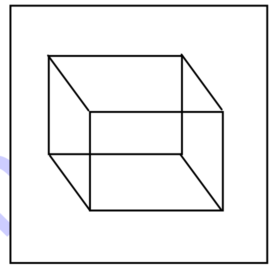
$= 0.3 \times 6 \times 10^{-8} \times 4\pi \times 10^{-4} \times [(100)^4 - (300)^4]$

$= 0.3 \times 6 \times 4\pi \times 10^{-12} \times [1 - 0.0081] \times 10^{12}$

$= 0.3 \times 6 \times 4 \times 3.14 \times 9919 \times 10^{-4}$

$= 4 \times 18 \times 3.14 \times 9919 \times 10^{-5} = 22.4 \approx 22\text{ W}$

45. Since the Cube can be assumed as black body



$e = \ell$

$\sigma = 6 \times 10^{-8}\text{ w/m}^2\text{-k}^4$

$A = 6 \times 25 \times 10^{-4}\text{ m}^2$

$m = 1\text{ kg}$

$s = 400\text{ J/kg-°K}$

$T_1 = 227°C = 500\text{ K}$

$T_2 = 27°C = 300\text{ K}$

$\Rightarrow ms\frac{d\theta}{dt} = e\sigma A(T_1^4 - T_2^4)$

$\Rightarrow \frac{d\theta}{dt} = \frac{e\sigma A\left(T_1^4 - T_2^4\right)}{ms}$

$= \frac{1 \times 6 \times 10^{-8} \times 6 \times 25 \times 10^{-4} \times [(500)^4 - (300)^4]}{1 \times 400}$

$= \frac{36 \times 25 \times 544}{400} \times 10^{-4} = 1224 \times 10^{-4} = 0.1224°C/s \approx 0.12°C/s.$

46. $Q = e\sigma A(T_2^4 - T_1^4)$

For any body, $210 = eA\sigma[(500)^4 - (300)^4]$

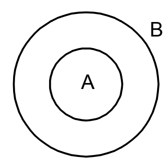
For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$

Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

47. $A_A = 20\text{ cm}^2,\qquad A_B = 80\text{ cm}^2$

$(mS)_A = 42\text{ J/°C},\qquad (mS)_B = 82\text{ J/°C},$

$T_A = 100°C,\qquad T_B = 20°C$



K_B is low thus it is a poor conducter and K_A is high.

Thus A will absorb no heat and conduct all

$\left(\frac{E}{t}\right)_A = \sigma A_A[(373)^4 - (293)^4] \qquad \Rightarrow (mS)_A\left(\frac{d\theta}{dt}\right)_A = \sigma A_A[(373)^4 - (293)^4]$

$\Rightarrow \left(\frac{d\theta}{dt}\right)_A = \frac{\sigma A_a\left[(373)^4 - (293)^4\right]}{(mS)_A} = \frac{6 \times 10^{-8}\left[(373)^4 - (293)^4\right]}{42} = 0.03\text{ °C/S}$

Similarly $\left(\frac{d\theta}{dt}\right)_B = 0.043\text{ °C/S}$

48. $\frac{Q}{t} = eAe(T_2^4 - T_1^4)$

$\Rightarrow \frac{Q}{At} = 1 \times 6 \times 10^{-8}[(300)^4 - (290)^4] \qquad = 6 \times 10^{-8}(81 \times 10^8 - 70.7 \times 10^8) = 6 \times 10.3$

$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$

$\Rightarrow \frac{Q}{tA} = \frac{K(\theta_1 - \theta_2)}{l} = \frac{K \times 17}{0.5} = 6 \times 10.3 = \frac{K \times 17}{0.5} \quad \Rightarrow K = \frac{6 \times 10.3 \times 0.5}{17} = 1.8$

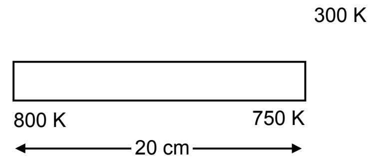
49. $\sigma = 6 \times 10^{-8}$ w/m^2-k^4

L = 20 cm = 0.2 m, K = ?

$\Rightarrow E = \dfrac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$

$\Rightarrow K = \dfrac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \dfrac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$

$\Rightarrow K = 73.993 \approx 74.$



50. v = 100 cc

$\Delta\theta = 5°C$

t = 5 min

For water

$\dfrac{mS\Delta\theta}{dt} = \dfrac{KA}{l}\Delta\theta$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 1000 \times 4200}{5} = \dfrac{KA}{l}$

For Kerosene

$\dfrac{ms}{at} = \dfrac{KA}{l}$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 800 \times 2100}{t} = \dfrac{KA}{l}$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 800 \times 2100}{t} = \dfrac{100 \times 10^{-3} \times 1000 \times 4200}{5}$

$\Rightarrow T = \dfrac{5 \times 800 \times 2100}{1000 \times 4200} = 2$ min

51. 50°C 45°C 40°C

Let the surrounding temperature be 'T'°C

Avg. t = $\dfrac{50 + 45}{2} = 47.5$

Avg. temp. diff. from surrounding

T = 47.5 – T

Rate of fall of temp = $\dfrac{50 - 45}{5} = 1$ °C/mm

From Newton's Law

1°C/mm = bA × t

$\Rightarrow bA = \dfrac{1}{t} = \dfrac{1}{47.5 - T}$ …(1)

In second case,

Avg, temp = $\dfrac{40 + 45}{2} = 42.5$

Avg. temp. diff. from surrounding

t′ = 42.5 – t

Rate of fall of temp = $\dfrac{45 - 40}{8} = \dfrac{5}{8}$ °C/mm

From Newton's Law

$\dfrac{5}{B} = bAt'$

$\Rightarrow \dfrac{5}{8} = \dfrac{1}{(47.5 - T)} \times (42.5 - T)$

By C & D [Componendo & Dividendo method]

We find, T = 34.1°C

52. Let the water eq. of calorimeter = m

$$\frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10} = \text{Rate of heat flow}$$

$$\frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18} = \text{Rate of flow}$$

$$\Rightarrow \frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10} = \frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18}$$

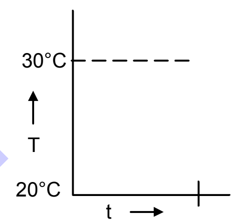
$\Rightarrow (m + 50 \times 10^{-3})18 = 10m + 1000 \times 10^{-3}$

$\Rightarrow 18m + 18 \times 50 \times 10^{-3} = 10m + 1000 \times 10^{-3}$

$\Rightarrow 8m = 100 \times 10^{-3}$ kg

$\Rightarrow m = 12.5 \times 10^{-3}$ kg = 12.5 g

53. In steady state condition as no heat is absorbed, the rate of loss of heat by conduction is equal to that of the supplied.



i.e. H = P

m = 1Kg, Power of Heater = 20 W, Room Temp. = 20°C

(a) $H = \frac{d\theta}{dt} = P = 20$ watt

(b) by Newton's law of cooling

$$\frac{-d\theta}{dt} = K(\theta - \theta_0)$$

$-20 = K(50 - 20) \Rightarrow K = 2/3$

Again, $\frac{-d\theta}{dt} = K(\theta - \theta_0) = \frac{2}{3} \times (30 - 20) = \frac{20}{3}$ w

(c) $\left(\frac{dQ}{dt}\right)_{20} = 0,$ $\left(\frac{dQ}{dt}\right)_{30} = \frac{20}{3}$ $\left(\frac{dQ}{dt}\right)_{avg} = \frac{10}{3}$

T = 5 min = 300 ′

Heat liberated = $\frac{10}{3} \times 300 = 1000$ J

Net Heat absorbed = Heat supplied – Heat Radiated = 6000 – 1000 = 5000 J

Now, $m\Delta\theta' = 5000$

$$\Rightarrow S = \frac{5000}{m\Delta\theta} = \frac{5000}{1 \times 10} = 500 \text{ J Kg}^{-1}{}^\circ\text{C}^{-1}$$

54. Given:

Heat capacity = m × s = 80 J/°C

$$\left(\frac{d\theta}{dt}\right)_{increase} = 2\ ^\circ\text{C/s}$$

$$\left(\frac{d\theta}{dt}\right)_{decrease} = 0.2\ ^\circ\text{C/s}$$

(a) Power of heater = $mS\left(\frac{d\theta}{dt}\right)_{increasing} = 80 \times 2 = 160$ W

(b) Power radiated = $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = 80 \times 0.2 = 16$ W

(c) Now $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = K(T - T_0)$

$\Rightarrow 16 = K(30 - 20)$ $\Rightarrow K = \frac{16}{10} = 1.6$

Now, $\frac{d\theta}{dt} = K(T - T_0) = 1.6 \times (30 - 25) = 1.6 \times 5 = 8$ W

(d) P.t = H $\Rightarrow$ 8 × t

55. $\frac{d\theta}{dt} = - K(T - T_0)$

Temp. at t = 0 is θ_1

(a) Max. Heat that the body can loose = $\Delta Q_m = ms(\theta_1 - \theta_0)$

($\therefore$ as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0)\times 9}{10}$

If it takes time t_1, for this process, the temp. at t_1

$= \theta_1 - (\theta_1 - \theta_0)\frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10} \times 1$

Now, $\frac{d\theta}{dt} = - K(\theta - \theta_1)$

Let $\theta = \theta_1$ at t = 0; & θ be temp. at time t

$\int_{\theta}^{\theta} \frac{d\theta}{\theta - \theta_o} = -K\int_0^t dt$

or, $\ln\frac{\theta - \theta_0}{\theta_1 - \theta_0} = - Kt$

or, $\theta - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$ …(2)

Putting value in the Eq (1) and Eq (2)

$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}$

$\Rightarrow t_1 = \frac{\ln 10}{k}$

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